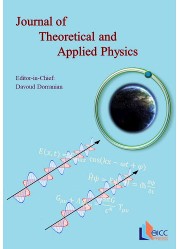

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Low-frequency electrostatic waves and chaotic motions in collisional superthermal plasmas

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Abstract:
In this work, the dynamics and chaotic behavior of ion-acoustic (IA) traveling waves in a collisional plasma consisting of cold ions, superthermal electrons, and immobile neutral particles are studied. The effect of ion-neutral collisions is also considered here. Using the reductive perturbation technique, a forced modified Korteweg-de Vries (FMK-dV) equation is obtained in the presence of an externally applied force. The periodic, quasi-periodic, and chaotic motions of IA waves are investigated by considering three-dimensional phase portraits and time-series analysis. It is noted that ion-neutral collisional frequency (ν_0), strength (f_0), and frequency (ω) of the external periodic force play an important role in controlling the dynamic motion of ion-acoustic waves. Moreover, it is found that the strength of the external perturbation can be considered as the main parameter for the transition from quasi-periodic motion to chaotic motion.

Keywords: Ion-acoustic waves; Forced modified $K-dV$ equation; Collisional plasmas; Haotic motions; Traveling waves

1. Introduction

The existence of highly energetic particles is well known in space [1–6] and laboratory plasmas [7–12]. External forces or interactions between particles can produce these particles in different types of plasma. These particles generally result in long-tailed distributions. In such cases, other non-Maxwellian distributions, such as the kappa distribution [13, 14] or Tsallis distribution [15], may be appropriate. In this study, we consider a non-Maxwellian plasma model that is given by the kappa distribution. The three-dimensional (3D) isotropic kappa-velocity distribution function (DF) is given as follows [16–18]

$$f_{k,j}(v) = \frac{n_{0j}}{(\pi k\theta_{kj}^2)^{3/2}} \frac{\Gamma(k+1)}{\Gamma(k-\frac{1}{2})} (1+\frac{v^2}{k\theta_{kj}^2})^{-(k+1)}, \quad (1)$$

where $\theta_{kj} = [(1-3/(2k))(2\mathrm{k}_B T_j/m_j)]^{1/2}$ is the effective thermal velocity, modified by the spectral index $k(>3/2)$, m_j is the mass of species j, n_{0j} is their number density and T_j is their equivalent temperature [19]. Here, $\Gamma(x)$ is the usual gamma function and $v^2 = v_x^2 + v_y^2 + v_z^2$ clearly shows the square velocity norm of the velocity v. The spectral index k is a measure of the slope of the energy spectrum of the superthermal particles forming the tail of the velocity distribution function. The smaller value of k corresponds to more superthermal particles in the DF tail. The kappa DF decreases in the limit of the Maxwellian distribution as $k \to \infty$.

It should be noted that the temperature definition, although suitable for a uniform Maxwellian distribution, is not valid for the kappa distribution, but there are practical advantages to using such an equivalent kinetic temperature, which can be a useful concept already accepted in practice for non-Maxwellian distributions. According to the paper by Hellberg et al. [20] and references therein, it is known that the equivalent temperature T can be calculated from mean particle energy as $T = (< mv^2 >)/(3\mathrm{k}_B) = (mv_0^2)/(\mathrm{k}_B).k/(2k-3)$, (see relation (8) in Ref. [20]) where they used v_0 in place of the notation θ_{kj} that is given in Eq. (1). This relation shows the relationship between the temperature T and the most probable speed v_0 (θ_{kj} in the present study). Therefore, in plasmas with a kappa distribution, as seen, the characteristic speed (the most probable speed $\theta_{kj}$is proportional to the thermal speed of the equivalent Maxwellian and this relation is clearly dependent on k. Therefore, from the definition of temperature, one can calculate the temperature of superthermal electrons when the most probable speed of electrons is clear.

Many researchers have studied the characteristics of solitary waves with superthermal electrons [19–30]. Most of these studies have been conducted on the propagation of IA solitary (soliton) waves. Recently, great interest has been shown in the study of nonlinear traveling waves. In fact, the bifurcation theory [31] is usually used for planer dynamical systems to investigate traveling nonlinear plasma waves. Several authors have studied the dynamical structures of IA waves in plasmas via the direct approach method [32–35]. It should be remembered that these works are effective for simple plasma systems where collisional effects and external perturbations have been neglected.

However, some authors have investigated the structure of solitary waves in the presence of damping and external force conditions. For example, Chatterjee et al. [36] studied the effects of dust ion-acoustic (DIA) solitary waves in the framework of damped forced Korteweg-de Vries (K-dV) equation in the superthermal plasmas with k-distributed electrons. Chowdhury et al. [37] investigated the properties of IA solitary waves in the framework of forced K-dV like the Schamel equation in the presence of the trapped electron inside the superthermal plasmas. Paul et al. [38] studied the effects of damping and external periodic force on a solitary wave solution in the framework of the damped forced modified K-dV equation. Recently, the properties of the DIA solitary waves were investigated by a damped forced K-dV Burgers (K-dVB) equation in an unmagnetized collisional dusty plasma with q-nonextensive distributed electrons in Ref. [39]. Very recently, some nonlinear structures like solitary waves, periodic and quasi-periodic oscillations, and chaotic motions were studied in a magnetized plasma with trapped electrons [40].

There is special attention to the study of nonlinear traveling waves in plasmas and in this work, we will focus on the traveling wave. It is noted that the traveling wave properties can be affected by damping and external periodic perturbations [41–46]. We should note that researchers in Refs. [41], [45] and [46] investigated various aspects of DIA waves using planar dynamical systems theory. They observed that the frequency of dust-ion collisions significantly changes the dynamics of DIA waves and plays an important role in a plasma model. Additionally, in several articles, nonlinear Zakharov-Kuznetsov (ZK) equation for IA traveling waves has been developed using the reductive perturbation method. It should be noted that this general equation is not an extension of the K-dV equation derived in the present study. In other words, the K-dV equation derived from the ZK equation is valid only when the traveling waves are parallel to the external magnetic field. However, to the best of our knowledge, ion-neutral collisional effects on the dynamical structure of IA traveling waves in the presence of external force have not been recorded in non-Maxwellian plasmas. In this paper, our aim is to investigate the traveling IA waves in an unmagnetized plasma with kappa-distributed electrons in the presence of collisional effects and external periodic forces. The layout of the manuscript is as follows. The governing equations are given in Sec.2. The forced modified K-dV equation is derived in Sec. 3. The periodic behavior of IAWs is studied in Sec. 4. Quasiperiodic and chaotic motions are investigated in Sec. 5 and finally, the conclusions of this study are presented in Sec. 6.

2. Model equations

We consider an unmagnetized collisional plasma containing cold ions and non-Maxwellian (kappa) distributed electrons. We also assume that the electron inertia can be neglected in the present model because the thermal speed of the electrons is much larger than the phase velocity of the IA waves. Furthermore, we assume that the temperature of the electrons is much greater than the temperature of the ions ($T_e \gg T_i$). This condition satisfies the minimum Landau damping in our plasma model.

On the other hand, to study the nonlinear dynamics of ion-acoustic waves, ions are assumed to be inertial. Furthermore, we assume that the ion particles interact with neutral atoms through mutual ion-neutral collisions with frequency v_{in}, where the ion-neutral collision frequency is much lower than the ion plasma frequency, i.e., $v_{in} \ll \omega_{pi}$. This interaction is described by the friction force $F_c = -m_i n_i v_{in}(U_i - U_n)$ (Krook approximation) [47] in the momentum equation for ions. Here $U_{i(n)}$ is the velocity of ion (neutral) particles, n_i and m_i are the ion number density and the ion mass, respectively. Using the assumption $U_n \cong 0$, the collision term can be rewritten as $F_c = -m_i n_i v_{in} U_i$. Therefore, the dynamics of the nonlinear IAWs are given by the following normalized hydrodynamic equations:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0, \tag{2}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{\partial \phi}{\partial x} - vu \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n + S(x,t), \tag{4}$$

$S(x,t)$ is the variable charge density source which results in different physical situations. The ‘space debris’ like dead satellites or parts of smashed spacecraft can be considered as a factor for the production of this term. When these space debris are moving in the astrophysical plasmas, they become charged interacting with ions and electrons. Movement of this charged debris may produce periodic perturbations. This external force typically models a localized topography or a moving source. In this paper, we will conduct a comprehensive investigation into the interaction between nonlinear waves and external forces within the framework of the K-dV equation. By incorporating an external force, we will investigate how the presence of this force affects the dynamics and behavior of the ion-acoustic waves. Moreover, we considered a slowly varying source term here. Therefore, we can ignore electromagnetic field effects.

In addition, we should note that in Eq. (3), the ions are treated as cold species (i.e., the ion pressure is ignored). Therefore, the effect of ion temperature on the right-hand side of Eq. (3). (i.e., $-\nabla P_i$ where $P_i = Cn^{\gamma}$ is the pressure of the ionic fluid and γ, is the ratio of specific heats) is neglected. This assumption is relevant for a typical space plasma with an electron temperature of $10 - 100$ eV so,

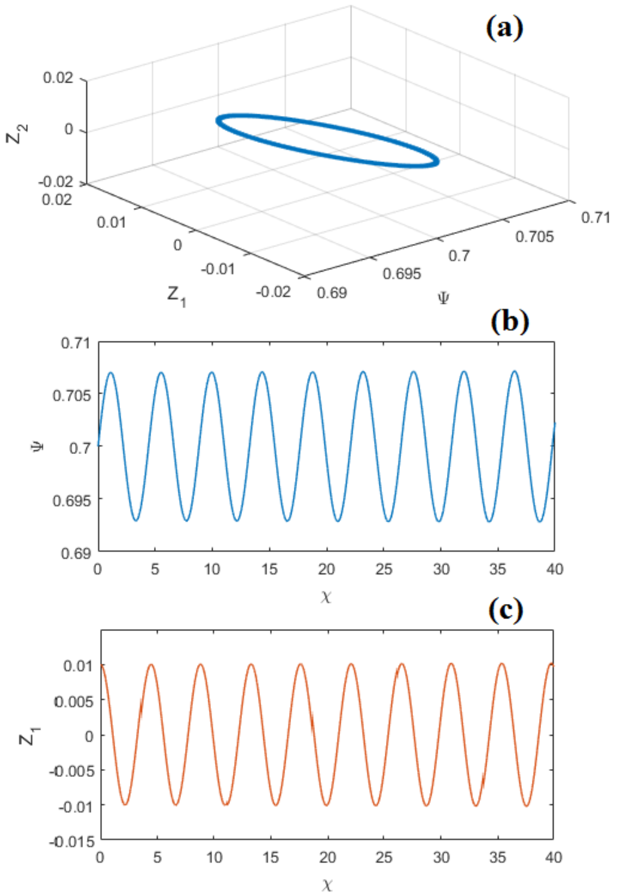

Figure 1. (a) 3D plot of the phase orbits and (b)-(c) nonlinear IA periodic waves Ψ and electric field structure Z_1 for the case $\nu_0 = 0$. The other parameters are as $k = 4$ and $U_0 = 0.1$. The initial value is $(\Psi, Z_1, Z_2) = (0.7, 0.01, 0)$.

$\sigma = T_i/T_e \cong 0.01 - 0.1$. Indeed, the addition of the ion temperature only gives a correction term and is therefore ignored here.

The normalized kappa distributed electron number density n_e is given by the relation

$$n_e = \left(1 - \frac{\phi}{k - \frac{3}{2}}\right)^{-k+\frac{1}{2}} \tag{5}$$

In the above equations, $\nu = \nu_{in}/\omega_{pi}$ is the ion-neutral collision frequency where ν_{in} is the unnormalized ion collisional frequency and $\omega_{pi} = (e^2 n_0/\varepsilon_0 m_i)^{1/2}$ is the ion plasma frequency. We have included the following normalization schemes in the model equations: the number density of ions (electrons) n (n_e) is normalized to n_0 (i.e., the unperturbed ion and electron number density in the equilibrium state), the ion fluid velocity u is normalized by ion sound speed $C_s = (\mathrm{k}_B T_e/m_i)^{1/2}$ and the electrostatic potential ϕ is normalized by $\mathrm{k}_B T_e/e$. The time t and space variable x are normalized by the ion plasma frequency $\omega_{pi} = (e^2 n_0/\varepsilon_0 m_i)^{1/2}$ and the plasma Debye length $\lambda_{Di} = (\varepsilon_0 \mathrm{k}_B T_e/e^2 n_0)^{1/2}$, respectively. Here, e represents the elementary charge of the electron, m_i denotes the mass of the ion, T_e is the electron temperature, and k_B is the Boltzmann constant.

3. Derivation of the forced modified K-dV equation

To investigate the nonlinear IA waves in a collisional superthermal plasma in the presence of the external force, the standard reductive perturbation technique (RPT) [48] is applied to derive the forced modified Korteweg-de-Vries (FMK-dV) equation. We rescale independent variables [48]

$$\xi = \varepsilon^{\frac{1}{2}}(x - Vt) \text{ and } \tau = \varepsilon^{\frac{3}{2}} t. \tag{6}$$

The quantity V is the phase speed of waves in the considered plasma model (normalized by the ion sound speed C_s). Furthermore, the parameter ε is a measure of the smallness of the perturbed amplitude to the corresponding equilibrium quantity (e.g., $n^{(1)}/n_0 \ll 1$ where $n^{(1)}$ is the perturbed density and n_0 is the equilibrium density). Generally, this expansion parameter ε is defined by $\varepsilon = |V_0 - C_s|/C_s = |M - 1|$ [49], where V_0 is the pulse speed and M is the Mach number $M = V_0/C_s$. According to the results given in Ref. [49], it was found theoretically that the accessible range of the Mach number is [1, 1.58] in an electron-ion plasma with the Maxwellian distribution and this range of accessible Mach numbers will be reduced in the presence of superthermal electrons [19]. Therefore, the real value of the parameter ε can be of the order $\sim 10^{-2} - 10^{-1}$. The dependent plasma variables n, u, and ϕ are expanded about their equilibrium values as power series of ε as

$$n = 1 + \varepsilon n^{(1)} + \varepsilon^2 n^{(2)} + ..., \tag{7}$$

$$u = 0 + \varepsilon u^{(1)} + \varepsilon^2 u^{(2)} + ..., \tag{8}$$

$$\phi = 0 + \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + ..., \tag{9}$$

$$S = \varepsilon^2 S_2(x,t) + ... \tag{10}$$

Here, we assume a weak collision and can set $\nu = \varepsilon^{3/2}\nu_0$, where ν_0 is a finite quantity. Therefore, substituting Eqs. (6) – (10) into basic Equations (2) - (4) and equating the coefficients of similar powers of ε, one may obtain the lowest order of ε as

$$n^{(1)} = \frac{u^{(1)}}{V} = \frac{\phi^{(1)}}{V^2}, \tag{11}$$

and the phase velocity is given by $V = \sqrt{(k-3/2)/(k-1/2)}$.

For the next order of ε, the following coupled partial differential equations for the second-order perturbed quantities are obtained

$$-V\frac{\partial n^{(2)}}{\partial \xi} + \frac{\partial u^{(2)}}{\partial \xi} + \frac{\partial n^{(1)}}{\partial \tau} + \frac{\partial (n^{(1)} u^{(1)})}{\partial \xi} = 0, \tag{12}$$

$$-V\frac{\partial u^{(2)}}{\partial \xi} + \frac{\partial \phi^{(2)}}{\partial \xi} + \frac{\partial u^{(1)}}{\partial \tau} + u^{(1)}\frac{\partial u^{(1)}}{\partial \xi} + \nu_0 u^{(1)} = 0, \tag{13}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} - C_1 \phi^{(2)} - C_2 (\phi^{(1)})^2 + n^{(2)} - S_2(\xi, \tau) = 0, \tag{14}$$

where $C_1 = (k-1/2)/(k-3/2)$ and $C_2 = [(k-1/2)(k+1/2)]/[2(k-3/2)^2]$.

By eliminating the second-order perturbed quantities ($n^{(2)}$,

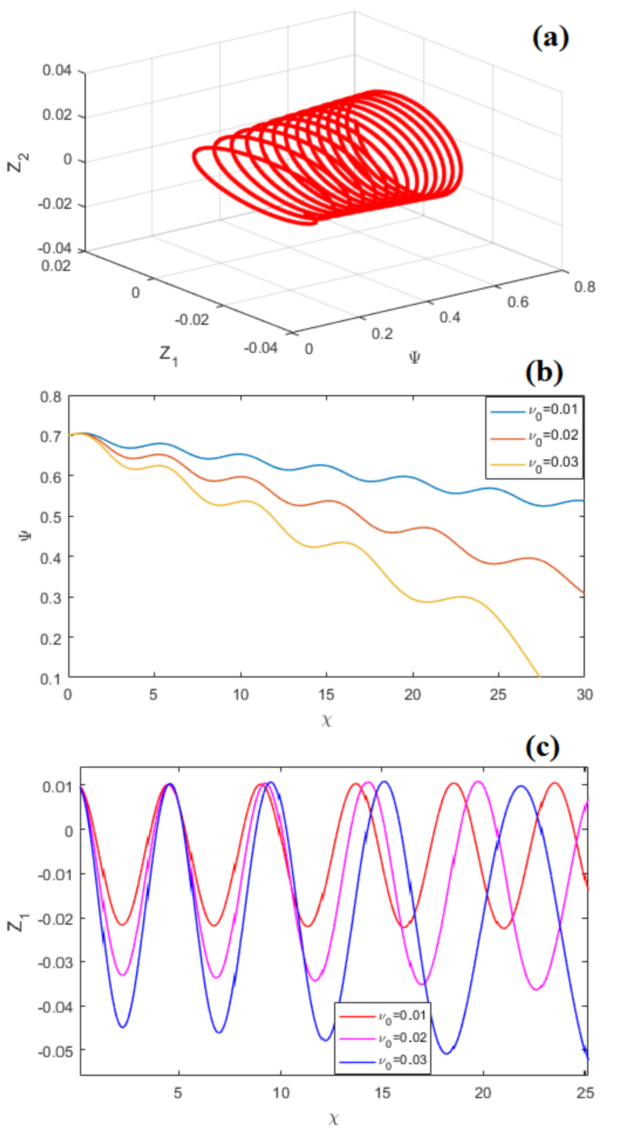

Figure 2. (a) Phase portrait of the dynamical system (22) for IA waves in the presence of the collisional effects in three dimensions. The collisional effects on (b) the profile of IA periodic waves and (c) the electric field in a collisional superthermal plasma. The other parameters are the same as Fig. 1.

$u^{(2)}$ and $\phi^{(2)}$), and with the help of Eq. (11), we finally get an evolution equation for IAWs as

$$\frac{\partial\phi^{(1)}}{\partial\tau}+A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi}+B\frac{\partial^3\phi^{(1)}}{\partial\xi^3}+C\phi^{(1)}=B\frac{\partial S_2}{\partial\xi} \quad (15)$$

where A is the nonlinear and B is the dispersive coefficient. The term $C\phi^{(1)}$ arises due to the collisional effects and the term $B(\partial S_2/\partial\xi)$ indicates the external force effects. These coefficients are given as

$$A=(-2C_2+\frac{2C_1}{V^2}+\frac{1}{V^4})B,\ \ B=\frac{V^3}{2}\ \text{ and }\ C=\frac{\nu_0}{2}. \quad (16)$$

The exact analytical solution of FMK-dV Equation (15), is not possible. Therefore, we will solve it numerically in the following. However, in the absence of collisional effects ($C=0$) and external force (i.e., $S_2=0$), the solitary wave solution is expressed as

$$\phi^{(1)}(\xi,\tau)=\phi_m\text{sech}^2[W^{-1}(\xi-M\tau)], \quad (17)$$

where M is the solitary wave velocity in the co-moving frame. The maximum amplitude ϕ_m and the width W are given by $\phi_m=3M/A$ and $W=2\sqrt{B/M}$, respectively. Furthermore, for the case $C\neq 0$ and $S_2=0$, the analytical solution of Eq. (15) in a collisional plasma model is given by [50–52]

$$\phi^{(1)}(\xi,\tau)=\phi_0(\tau)\text{sech}^2\sqrt{\frac{A\phi_0(\tau)}{12B}}\left(\xi-\frac{A}{3}\int_0^\tau\phi_0(\bar\tau)d\bar\tau\right), \quad (18)$$

where the amplitude, width, and velocity of the pulse are functions of time given, respectively, by

$$\begin{aligned}\phi_0(\tau)&=\phi_m\exp(-\frac{2\nu_0}{3}\tau),\\ L(\tau)&=\sqrt{\frac{12B}{A\phi_m}}\exp(\frac{\nu_0}{3}\tau),\\ \text{v}(\tau)&=\frac{A\phi_m}{3}\exp(-\frac{2\nu_0}{3}\tau),\end{aligned} \quad (19)$$

ϕ_m is the initial pulse amplitude at the time $\tau=0$. It should be noted that Eqs. (17) and (18) describe the solitary wave structure in a collisionless/collisional plasma, respectively. However, in this study, we will investigate other nonlinear waves (i.e., periodic and quasi-periodic) and their chaotic behaviors in a collisional superthermal plasma. Therefore, to evaluate the effects of the plasma parameters in Eq. (15), we performed the numerical solution Eq. (15) in the following.

4. Periodic behavior of IAWs

In this section, we study the structure of the periodic waves based on Eq. (15) in a collisional superthermal plasma. Here, for simplicity, we consider $S_2=0$ in our approach. This means that we transform the forced modified K-dV Equation (15) to the damped K-dV equation taking into account $S_2=0$. Therefore, in the absence of an external force we have

$$\frac{\partial\phi^{(1)}}{\partial\tau}+A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi}+B\frac{\partial^3\phi^{(1)}}{\partial\xi^3}+C\phi^{(1)}=0, \quad (20)$$

taking $\phi^{(1)}=\Psi(\chi)$ as the traveling wave solution of Eq. (20) and by introducing a new variable $\chi=\xi-U_0\tau$ (where U_0 is the speed of the wave in the moving reference frame and normalized by C_s), one can rewrite Eq. (20) as

$$-\frac{U_0}{B}\frac{d\Psi}{d\chi}+\frac{A}{B}\Psi\frac{d\Psi}{d\chi}+\frac{d^3\Psi}{d\chi^3}+\frac{\nu_0}{2B}\Psi=0 \quad (21)$$

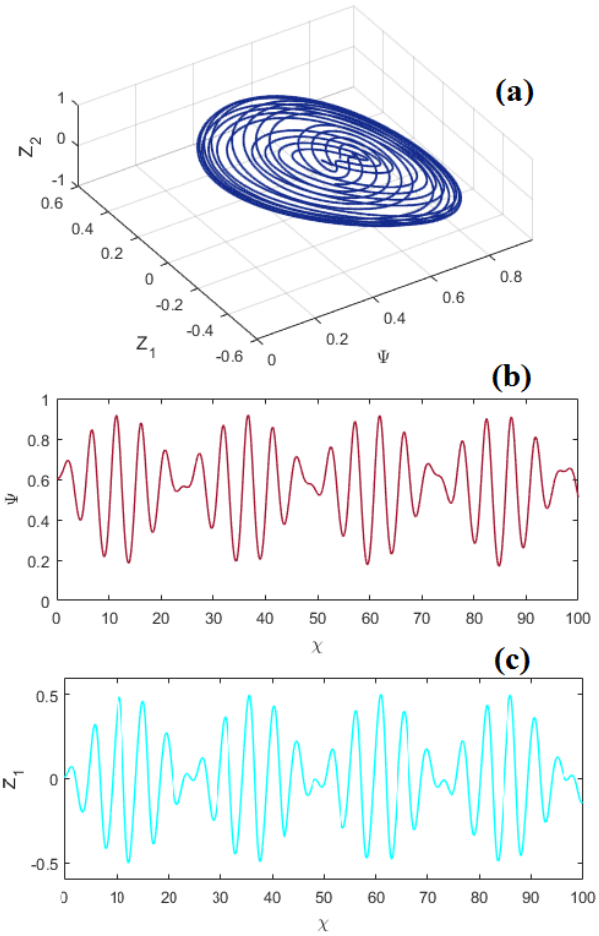

Figure 3. (a) 3D plot of the phase orbits and (b)-(c) time-series analysis of Ψ and Z_1 vs. χ of the system (24) for initial conditions: $(\Psi, Z_1, Z_2) = (0.6, 0.01, 0)$ with $k = 4$, $U_0 = 0.1$, $v_0 = 0$, $f_0 = 0.05$ and $\omega = 1.5$.

The planar dynamical system (21) can be cast into a more convenient form, which is given as

$$\begin{aligned} \frac{d\Psi}{d\chi} &= Z_1 \\ \frac{dZ_1}{d\chi} &= Z_2 \\ \frac{dZ_2}{d\chi} &= \frac{U_0}{B} Z_1 - \frac{A}{B}\Psi Z_1 - \frac{v_0}{2B}\Psi, \end{aligned} \tag{22}$$

For the numerical study of periodic motions corresponding to the dynamical system (22), different techniques, such as (i) phase portrait and (ii) time-series analysis can be used [53–55]. A geometric structure of the trajectories of a three-dimensional dynamical system is shown by analyzing a 3D phase portrait in phase space. In the phase portrait, each set of initial conditions is represented by a different curve or point. It consists of a plot of the trajectories in the state space. This gives information about whether there is an attractor, a repellor, or a limit cycle for a set of parameter values. On the other hand, a time-series analysis shows a series of data points indexed in time series. In fact, nonlinear time-series analysis allows one to extract from the measured time series the physical properties of the system that generated them.

The dynamical system (22) contains three independent parameters k, U_0, and v_0. Here, we have considered the periodicity structure of the system Eq. (22) using phase plan analysis for two cases a) without and b) with collisional effects. Figures 1(a)-(c), present the phase portrait, the periodic behavior of potential Ψ, and electric field Z_1 in a collisionless superthermal plasma, respectively (i.e., $v_0 = 0$). Fig 1(a), shows that the periodic orbit of Equation (22) corresponds to the periodic traveling wave solution of Equation (15). Figures 1(b) and 1(c) present a graph for the electrostatic potential Ψ and electric field Z_1 of the periodic traveling waves for $U_0 = 0.1$ and $k = 4$ with the initial value (Ψ, Z_1, Z_2)=$(0.7, 0.01, 0)$. It is clear that the structure (22) exhibits a periodic behavior (with a fixed amplitude) when the ion-neutral collision effect is not considered.

In Fig. 2, we investigated the structure of nonlinear IA periodic waves in the presence of collisional effects (i.e., $v_0 \neq 0$). Fig. 2(a) indicates the periodic behavior of nonlinear waves with $v_0 = 0.01$. Other parametric data are the same as Fig. 1. On the other hand, it is seen that with an increase v_0, the amplitude of the periodic wave gradually decreases. This is because as the collision frequency increases, the interaction between the plasma particles in a narrower region of the plasma space enhance, and hence the internal potential energy of the nonlinear wave decreases and as a result, the wave amplitude decreases. See Fig. 2(a) for more details. The variations of the electric field vs. χ for different values of ion-neutral collision frequency are identified too in Fig. 2(c). The electric field amplitude increases when ion-neutral collision frequency increases for periodic ion-acoustic waves.

5. Chaotic structurer of the system

Here, we carry out the investigation of the chaotic behavior of a perturbed system. In a nonlinear dispersive media, turbulence is strongly associated with the configurations of the system. In other words, the chaotic behavior of the dynamical system is influenced by initial conditions such that small changes lead to different results as discussed in Refs. [56, 57]. However, we report how the external force affects the interplay of quasiperiodic motions and chaotic structures. The external periodic force or the source term can exist in a variety of forms in the plasma system [58–62]. Some theoretical works have investigated the excitation of nonlinear waves in plasma models by considering the source term in the Poisson's equation [37, 39, 60]. It is important to note that the source term may be of different kinds because of the presence of space debris in plasmas. For example, the Gaussian forcing term [58], hyperbolic forcing term [58], and trigonometric forcing term [62]. We consider the source term as $S_2 = f_0/(B\omega)\sin(\omega\chi)$ [62], where f_0 is the strength of the source, ω is the source frequency, and $\chi = \xi - U_0\tau$ is a new variable. Therefore, Eq. (15) can be rewritten as:

$$-\frac{U_0}{B}\frac{d\Psi}{d\chi} + \frac{A}{B}\Psi\frac{d\Psi}{d\chi} + \frac{d^3\Psi}{d\chi^3} + \frac{v_0}{2B}\Psi = f_1\cos(\omega\chi) \tag{23}$$

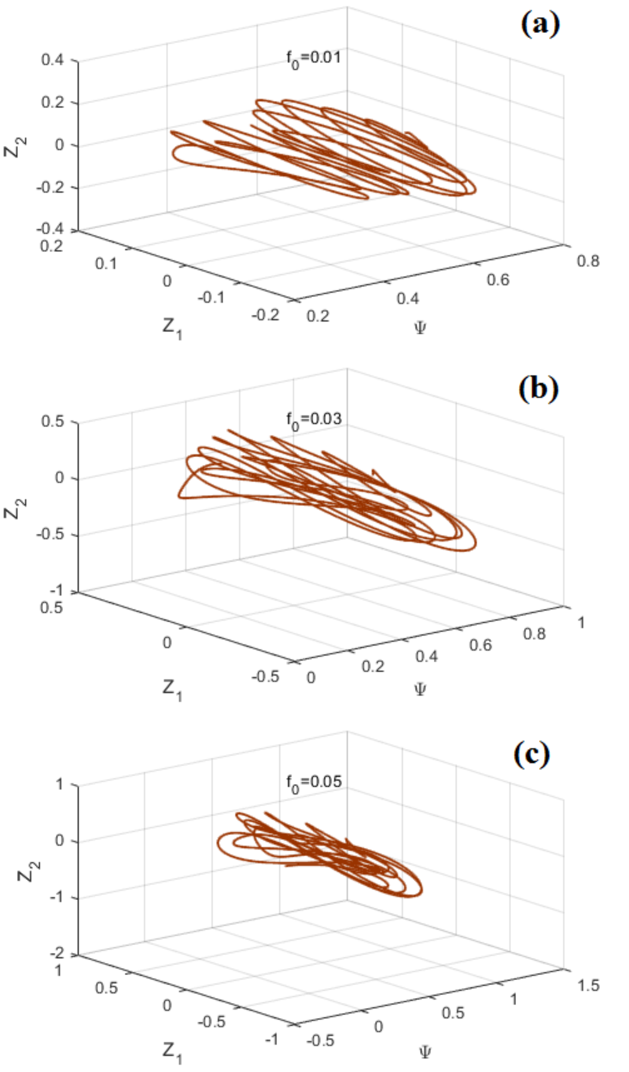

Figure 4. 3D plot of the phase orbits of the system (24) for (a) $f_0 = 0.01$, (b) $f_0 = 0.03$, and (c) $f_0 = 0.05$. Other parameters are as $k = 4$, $U_0 = 0.1$, $\omega = 1.5$ and $\nu_0 = 0.01$.

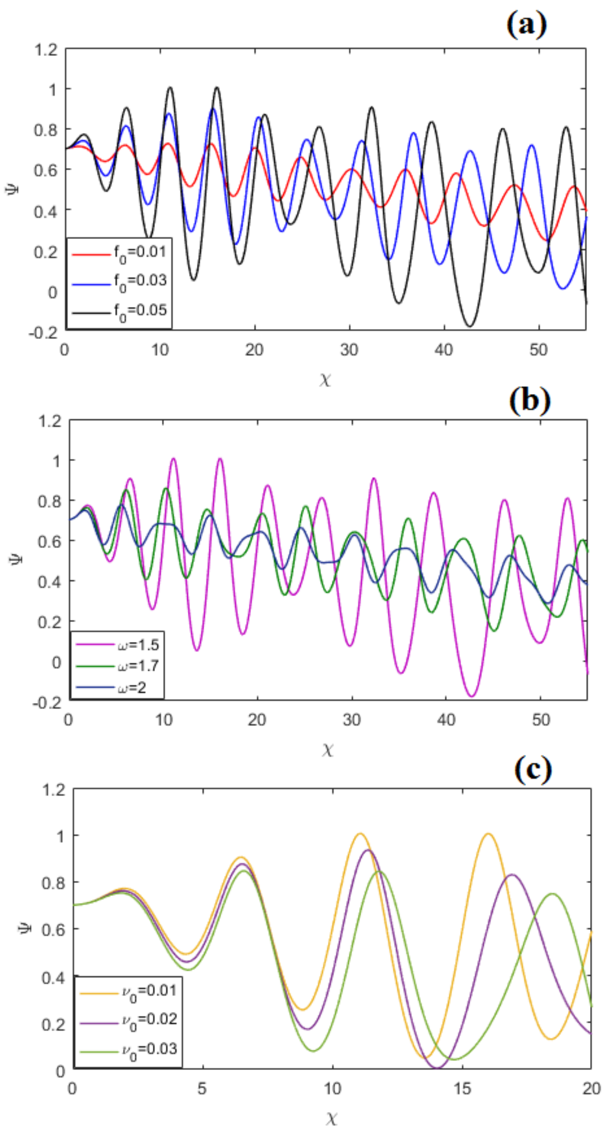

Figure 5. Variations of the potential of the nonlinear waves vs. χ for different values of (a) the amplitude, (b) the frequency of external force and (c) ion-neutral collision frequency.

where $f_1 = f_0/B$.
Following the procedure mentioned for Eq. (22), the above equation can be written as

$$\frac{d\Psi}{d\chi} = Z_1$$
$$\frac{dZ_1}{d\chi} = Z_2 \qquad (24)$$
$$\frac{dZ_2}{d\chi} = \frac{U_0}{B}Z_1 - \frac{A}{B}\Psi Z_1 - \frac{\nu_0}{2B}\Psi + f_1\cos(\omega\chi)$$

Now we investigate the IAW structures in the presence of an external periodic force governed by the system (24) when ion-neutral collision frequency is ignored (i.e., $\nu_0 = 0$). The corresponding 3D phase orbits are presented in Fig. 3(a). Here, $k = 4$, $U_0 = 0.1$, $f_0 = 0.05$ and $\omega = 1.5$. The initial condition is indicated by $(0.6, 0.01, 0)$. Figure 3(a) displays that the ion trajectories lie on the surface of the torus which is a sign of the quasiperiodic solution. They confirm that the plasma system produces quasi-periodic oscillation for the IAWs. In Figs. 3(b) - (c), time-series analysis of Ψ and Z_1 versus χ are presented for the dynamical system (24) when $\nu_0 = 0$. Other parameters are the same as Fig. 3(a). In this condition multi-periodic oscillations for ion-acoustic waves in superthermal plasmas are identified.
In continue, taking into account collisional effects between ions and neutral particles, we will investigate the dynamical features of nonlinear waves in the presence of an external force. To study the structure of nonlinear waves in the presence of external periodic force with collisional effects on the system (24), we have depicted a 3D plot of the phase orbit in Fig. 4 for different values of the source power. Here, the plasma parameters are considered as $k = 4$, $U_0 = 0.1$, $\omega = 1.5$ and $\nu_0 = 0.01$. In this case, conservative chaotic oscillation of the ion-acoustic waves is observed. In other words, chaotic structures appear when the solution demon-

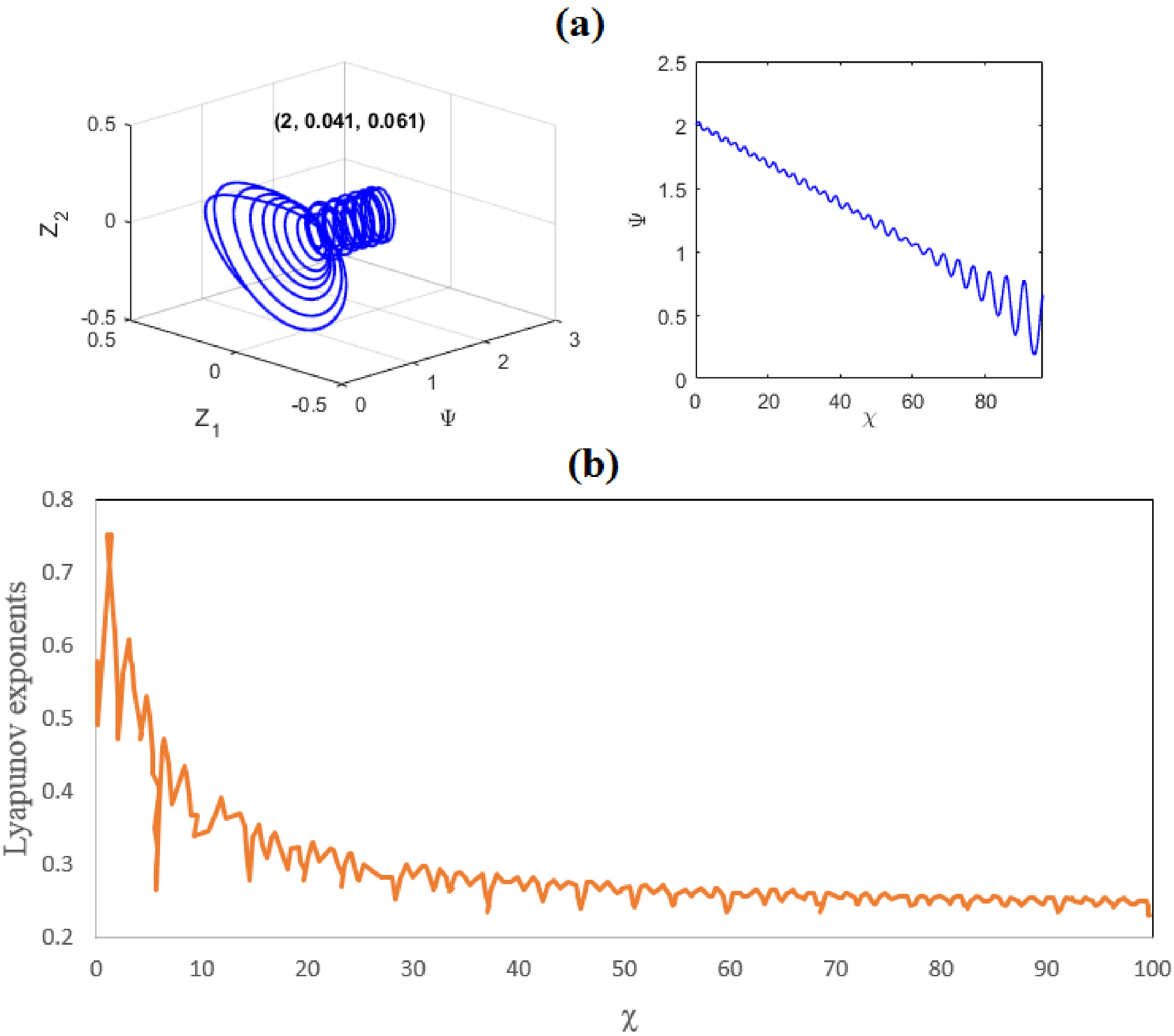

Figure 6. (a) Profile of the phase portrait (left) and time-series analysis of Ψ vs. χ (right) for initial conditions $(\Psi, Z_1, Z_2) = (2, 0.041, 0.061)$. Other parameters are the same as Fig. 4(c), (b) Lyapunov exponent for the chaotic behavior of the dynamical system (24) for Figure 4(c).

strates a random oscillation instead of periodic behavior. It is seen that increasing f_0, a dynamical transition from quasi-periodic motion to chaotic motion occurs. Therefore, the dynamical structure (24) supports the chaotic nature of the ion-acoustic waves in the presence of the external force with collisional effects.

As mentioned above, the external force plays an important role in the dynamics of chaotic structures which is associated with ion-acoustic waves. We have examined the influence of f_0, ω, and v_0 on the structure of nonlinear traveling waves in the following.

In Fig 5(a) - (c), we presented a time-series analysis of electric potential Ψ versus χ for different values of f_0, ω, and v_0 respectively. From Fig. 5(a), it is found that if the strength of the external force gradually increases, the amplitude of the periodic waves will be increased. It happens because, by increasing of the strength of external force, the IAW potential energy enhancements thus, the amplitude of nonlinear wave will be increased. Here, the plasma parameters are the same as Fig. 4. In continue, we represent the variation of the amplitude of nonlinear waves for different frequencies (ω) of the external periodic force with $f_0 = 0.05$, in Fig. 5(b). Other parameters are the same as in Fig. 5(a). The amplitude of periodic waves decreases as the frequency ω of the external periodic force increases. This is because, the frequency of the external force affects the internal potential energy of IAWs and consequently, significant changes are observed in the amplitude of the traveling waves. Moreover, from Fig. 5(c), it is seen that in the presence of an external force with $f_0 = 0.05$ and $\omega = 1.5$, the amplitude of the oscillations declines as the collision frequency increases.

A plot of the phase portrait and time-series analysis of Ψ vs. χ of Eq. (24) for the sensibility of the perturbed dynamical system (24) with initial conditions is depicted in Fig. 6(a). We find that the structure (24) displays a random oscillation. It means that the present system can also show the chaotic feature of the ion-acoustic waves with these initial values. Furthermore, in Fig. 6(b), the graph of the Lyapunov expo-

nent of the perturbed system (24) is presented for the same values of parameters as Fig. 4(c). We observed that the Lyapunov exponent is positive and this confirms the chaotic behavior of the perturbed system (24).
To complete the discussion, we should note that the results of this work differ from other works. See Refs. [36–40], for example. The characteristics of dust-ion-acoustic (DIA) waves in a collisional dusty plasma with superthermal electrons were investigated by Chatterjee et al., [36]. They described both the rarefactive and compressive solitary waves. Furthermore, the propagation of IA waves in a plasma with cold ion fluid, trapped electrons, and in the presence of an external force was studied by Chowdhury et al., [37]. They were considered an extension of Schimel's distribution for electrons. It was observed that the effect of external force parameters (i.e., the amplitude f_0 and the frequency ω) on IA solitons is similar to Ref. [36]. However, we should note that the present study is focused on traveling waves instead solitons and our numerical analysis showed that external periodic force plays a different role on the traveling waves in comparison to solitons. In other words, The external force may produce chaotic motions associated with IA waves. On the other hand, the collisional effects were not investigated in Refs. [36, 37]. However, this study demonstrates that collisional effects and external periodic force have significant effects on ion-acoustic traveling waves and caused our results to be distinct from the other studies.
Recently, the effects of the strength and frequency of the external periodic force and also the collision frequency between dust and ions on dust-ion-acoustic (DIA) solitons were investigated in the framework of damped forced K-dV Burger's equation in a collisional dusty plasma with q-nonextensive distributed electrons in Refs. [38, 39]. Very recently, nonlinear dynamics of ion-acoustic waves in a magnetized plasma with vortex-like distributed electrons were studied by Bellahsene et al., [40]. They found that a transition from a quasi-periodic to a chaotic behavior can occur when the magnetic field strength increases [40]. However, as far as we know, IA traveling wave excitations are not recorded in non-Maxwellian collisional plasmas. Therefore, our theoretical results demonstrate that ion-neutral collision frequency and external periodic force parameters (i.e., f_0 and ω) have remarkable effects on IAW dynamics.

6. Conclusion

The characteristics and chaotic motions of the nonlinear ion-acoustic waves were investigated in a non-Maxwellian collisional plasma containing cold ions and kappa-distributed electrons, in the presence of an external periodic force. Using RPT, the modified K-dV equation for ion-acoustic waves is derived. The effects of ion-neutral collision frequency ν_0, the amplitude f_0, and the frequency ω of the periodic force on IA wave structures were discussed through numerical simulations. It is observed that these parameters have remarkable effects on the nonlinear structure of the IA waves in collisional non-Maxwellian plasmas. In other words, they play a crucial role in the control of the dynamic motions of the system (24) from quasiperiodic to chaotic behavior. The results of this study may be useful in laboratory plasmas as well as in space environments (such as mercury, solar wind and Saturn) with space debris where kappa-distributed electrons are present.

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Conflict of interest statement:
The authors declare that they have no conflict of interest.

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