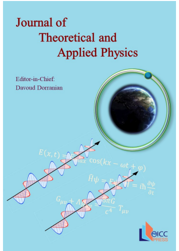

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Three-Body force effects on breakup and formation of ^{6}Li nuclei

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Abstract:
During helium transforms into heavier elements, both of ^{6}Li radiative capture reaction and its breakup occur in the stars. ^{6}Li radiative capture reaction and its inverse have been studied using Effective Field Theory (EFT), up to next to leading order (NLO). The deuteron-alpha reaction and the photodisintegration rates of the $^{6}\text{Li}(\gamma,\alpha)d$ reaction have been calculated. Alpha particle was assumed to be structureless and coulomb effects considered between the charged particles. The inverse reaction rate has been estimated for E_1 and E_2 transitions by adding the three-body forces, up to NLO. The scattering amplitude are calculated at the initial P-wave states of deuteron-alpha for the sum of both E_1 and E_2 multipole transitions. The obtained results are in good agreement with the available experimental data and those of other theoretical models, at the energies relevant to the Big-Bang Nucleosynthesis (BBN). The $^{6}\text{Li}(\gamma,\alpha)d$ reaction rate is also found to be acceptable in comparison with the other theoretical results.

Keywords: Deuteron-alpha radiative capture; Effective field theory; Three-body force; The astrophysical reaction rate

1. Introduction

The alpha process is one of the two major categories of nuclear fusion processes that occur in stars formation. The radiative capture of deuteron-alpha and the $^{6}\text{Li}(\gamma,\alpha)d$ reactions have formerly been investigated both experimentally and theoretically, for the last few decades. The experimental studies of the deuteron-alpha radiative capture reaction have been done for the resonance 0^{+}, $T = 1$, 3.56 MeV state of ^{6}Li, at the astrophysical energies [1]. The values of S-wave cross section and the astrophysical S-factors are also reported in the ref. [2]. The cross section for the $d(\alpha,\gamma)^{6}\text{Li}$ reaction was obtained, in the energy of $E_{\alpha,lab}$= 2 MeV, corresponds to the 3^{+} resonance in ^{6}Li at E_x= 2186 KeV [3]. Recently, a compilation of charged-particle induced thermonuclear reaction rates, are reported at temperatures in the range from 10^{6}-10^{10} K, for $1\leq Z\leq 14$ [4]. More recently, an experiment was performed at the LUNA 400 KV accelerators, and the results were reported as the first measurement of the $d(\alpha,\gamma)^{6}\text{Li}$ cross section at BBN energies by Anders et al. [5]. They reported the direct experimental cross section, astrophysical S-factor and the reaction rate data of the $d(\alpha,\gamma)^{6}\text{Li}$ reaction for the relevant BBN energies. The asymptotic normalization coefficient of the $\alpha+d\rightarrow{}^{6}\text{Li}$ process and for the direct astrophysical S-factor are estimated by Tursunmakhatov et al. [6].

In the other hand, some theoretical researches have been done by utilizing the symmetry properties of the wave functions of the alpha particle and of various produced nuclei in the usual first order perturbation theory, for studding of the deuteron-alpha radiative capture [7–11]. The authors used corrections to E_1 multipole in long wave approximation, which resulted in the dominance of E_2 multipole.

The information on the nuclear vertex constant and the asymptotic normalization coefficient have extracted on the states of ^{6}Li nucleus by Blokhintsev et al. [12]. They used the well-known coulomb modified effective-range theory expression for synthesis of $^{6}\text{Li}\rightarrow{}^{4}\text{He}+d$.

Ryzhikh et al. studied the low-energy fusion reaction $d(\alpha,\gamma)^{6}\text{Li}$ in the framework of the multi cluster dynamic model with Pauli projection (MDMP). A larger basis is used for the radial wave function of $\alpha-2N$ relative motion in the ground state of ^{6}Li to construct a high precision wave function [13]. Additionally, Nollet et al. have computed the cross section for the $d(\alpha,\gamma)^{6}\text{Li}$ reaction at the energies relevant to the primordial nucleosynthesis [14]. They show that the E_2 cross section is dominant and is within reasonable agreement with the experimental data. More

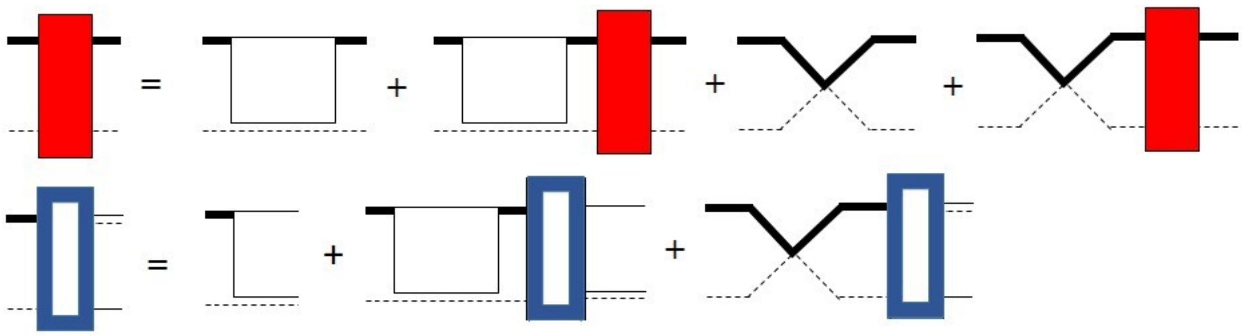

Figure 1. Faddeev's equations of $d-\alpha$ scattering in different channels up to LO with three-body force. The filled rectangle shows the amplitude F_α of channel $d+\alpha \to d+\alpha$. The $d+\alpha \to N+N_\alpha$ channel also shows by empty rectangle.

recently, Tursunov et al. studied the astrophysical capture process of $d(\alpha,\gamma)^6$Li in a three-body model and found that the contribution of the E_1 transition operator from the initial iso-singlet states to the iso-triplet components of the final state is negligible [15]. They found the reaction rate and primordial abundance of the $d(\alpha,\gamma)^6$Li astrophysical capture process in a three-body model. The final nucleus ^{6}Li is considered as a three-body bound state $\alpha+n+p$ in the hyper spherical Lagrange-mesh method. further, Tursunov et al. found that the contribution of the E_1 transition operator from the initial iso-singlet states to the iso-triplet components of the final state is dominant. And they found the contribution of the E_1 transition is larger than the E_2 contribution at energies below 100 keV [16].

The calculation of isospin forbidden E_1 capture has been performed in the two-body model based on the cluster idea by Baye et al. [17]. The authors also performed the E_2 radiative capture contribution to the $\alpha+d$ elastic scattering with a realistic nucleon-nucleon (NN) force.

Recently, we also studied the deuteron-alpha radiative capture reaction using EFT up to NLO without coulomb effect [18] and with coulomb effect [19]. We calculated the E_1 and E_2 multipole transitions in framework of the pionless EFT, by only considering of dibaryon and alpha fields. We also considered the relative momentum of dibaryon and alpha less than m_π. The $(^3P_2)$ $n-p$ and $(^2P_{3/2})\alpha-N$ resonances considered. In this paper, we calculated the photodisintegration rates of the ^{6}Li$(\gamma,\alpha)d$ reaction and its reaction rate with considering of the three body forces.

This paper is organized as follows: a brief description of the relevant Lagrangian and Faddeev integral equations are given in the next section. We present the calculation of the deuteron-alpha radiative capture reaction, total cross section and the ^{6}Li$(\gamma,\alpha)d$ reaction rate, by considering of three body force, in this section II. Section III, is devoted to the discussions of the results of the photodisintegration rates of the ^{6}Li$(\gamma,\alpha)d$ reaction and comparison with the available experimental data and the recent results of different theoretical approaches. The summary and conclusions are also given in section IV.

2. Theoretical framework

The deuteron-alpha scattering process at low-energies leads to three possible processes with a pure hadronic signature: (a) the elastic channel ($d+\alpha \to d+\alpha$), (b) the neutron-transfer channel ($d+\alpha \to p+n\alpha$) and (c) the three-body final state ($d+\alpha \to p+n+\alpha$). The two following Faddeev equations get mixed for the two-body and three-body calculation of the deuteron-alpha scattering, a) $d_s+A_\alpha \to d_s+A_\alpha$, b) $(NA_\alpha)+N \to d_s+A_\alpha$ and c) $N+N+A_\alpha \to d_s+A_\alpha$.

The following Lagrangian of $d(\alpha,\gamma)^6$Li reaction includes one-, two-, and three-body terms [20],

$$\begin{aligned}\mathfrak{L} &= N^\dagger\left[i\partial_0+\frac{\nabla^2}{2m_N}\right]N + A_\alpha^\dagger\left[i\partial_0+\frac{\nabla^2}{2m_\alpha}\right]A_\alpha \\ &+\eta_0 d_s^\dagger\left[-\Delta_0+i\partial_0+\frac{\nabla^2}{2(m_n+m_p)}\right]d_s \\ &+\eta_1(NA_\alpha)^\dagger\left[-\Delta_1+i\partial_0+\frac{\nabla^2}{2(m_N+m_\alpha)}\right](NA_\alpha) \\ &+g_0(d_s^\dagger C_0^{\sigma i}N_\sigma N_\delta+h.c.)+g_1\left((NA_\alpha)^\dagger C_a^{\sigma i}N_\sigma\overline{\partial}_i A_\alpha+h.c\right) \\ &-h\left(C_0^{ab}C_a^{i\sigma}(NA_\alpha)_b\overline{\partial}_i N_\sigma\right)^\dagger\left(C_0^{cd}C_c^{j\sigma}(NA_\alpha)_d\overline{\partial}_j N_\delta\right) \quad (1)\end{aligned}$$

where m_N, N, A_α and m_α, are the nucleon mass, nucleon field, alpha field and alpha mass, respectively. Furthermore, d_s is an auxiliary field corresponding to deuteron and the spin-singlet virtual bound state in S-wave nucleon-nucleon scattering, and (NA_α) is the four component field for the $^2P_{3/2}$ resonance in the N_α system. ∂_i indicates the Galilean invariant derivative, where $\eta_0=\eta_1=1$, for P-wave. g_0 is nucleon-nucleon coupling constant for $^3P_2$, $^3P_0$ channels and g_1 is nucleon-alpha coupling constant for $^2P_{3/2}$ channel. $g_{0(1)}$ and $\Delta_{0(1)}$ are given by,

$$g_0^2=\frac{8\pi}{m_N^2 r^{(^3P_2)}},\quad g_1^2=\frac{8\pi}{\mu_{N\alpha}^2 r^{(^2P_{3/2})}}, \quad (2)$$

$$\Delta_0=\frac{2}{m_N r^{(^3P_2)}}\left(\frac{1}{a^{(^3P_2)}}-\mu\right),\ \Delta_1=\frac{2}{\mu_{N\alpha} r^{(^2P_{3/2})}}\left(\frac{1}{a^{(^2P_{3/2})}}-\mu\right), \quad (3)$$

where μ is the renormalization scale and h is coupling constant of $NN\alpha$ contact term. This coupling constant is derived via three-body renormalization as $h= m_n/\Lambda^2$ where Λ is the cut-off. The Clebsch-Gordan coefficients are denoted by $C_a^{\sigma i}$ in the above Lagrangian ($C_a^{\sigma i}\equiv C(LSJ\mid\sigma ia)$. The

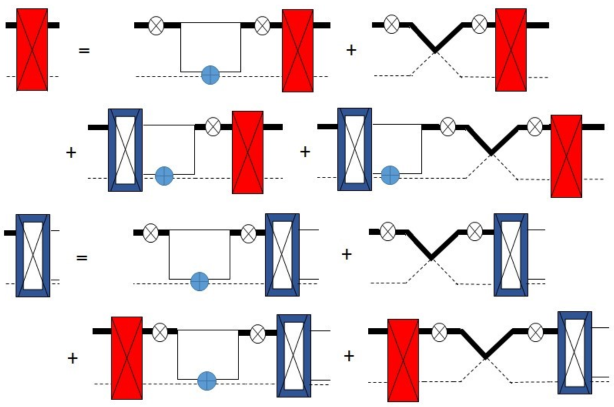

Figure 2. Faddeev's integral equations of $d-\alpha$ process with three-body force in different channels, up to NLO. The filled rectangle represents the F_α amplitude for channel $d+\alpha \to d+\alpha$ and the empty rectangle represents channel $d+\alpha \to N+N_\alpha$.

spin projections of the fields denoted by $\sigma, \delta, ... = \pm 1/2$, and Clebsch-Gordan coefficients indices $a, b, ...$ takes the values of $\pm 1/2, 3/2$. The operator indices are given by $i, j, ...= 0, \pm 1$.
The neutron-proton (subscript 0) and nucleon-nucleon (subscript 1) propagators are given by,

$$iD_{0,1}\left(E-\frac{3q^2}{4m_N}\right)=\frac{4\pi}{m_N g_{0,1}^2}$$
$$\times \frac{-i}{-\mu-\frac{4\pi\Delta_{0,1}}{m_N g_{0,1}^2}+\sqrt{\frac{3}{4}q^2-m_N E-i\varepsilon}+i\varepsilon}, \quad (4)$$

where

$$\mu+\frac{4\pi\Delta_0}{m_N g_0^2}=\frac{1}{a_0}, \;\; \mu+\frac{4\pi\Delta_1}{m_N g_1^2}=\frac{1}{a_1}. \quad (5)$$

The a_0 and a_1 are the scattering lengths for the $^3P_2$ and $^2P_{3/2}$ channels, respectively.
In the proton-proton (pp) interaction, the coulomb interaction dresses the two-nucleon Greens function at the bubble diagram for the dressed dibaryon propagator. The two-nucleon propagator for the pp channel is given by,

$$iD_{s(pp)}\left(E-\frac{3q^2}{4m_p}\right)=\frac{4\pi}{m_p g_0^2}\frac{-i}{-\frac{4\pi\Delta_0^R}{m_p g_0^2}-2KH(\eta)}, \quad (6)$$

where

$$H(\eta)=\psi(i\eta)+\frac{1}{2i\eta}-\ln(i\eta), \quad (7)$$

ψ is the logarithmic derivative of the Gamma function. The relation between the coulomb scattering length a_C and the renormalized constant Δ_0^R in the pp channel is given by,

$$\frac{1}{a_C}=\frac{4\pi\Delta_{0(pp)}}{m_N g_0^2}+\mu-2K[1-\gamma_E+\ln(\frac{\mu}{4K})]. \quad (8)$$

where $K=\eta\acute{p}$, $\acute{p}=\sqrt{(3/4)q^2-m_N E-i\varepsilon}$ and γ_E= 0.577215 is Eulers constant.
The numerical calculations for the three-body system, with inserting of the three-body force are performed using the following Faddeev equation,

$$F_i(q)=\sum_{j\neq i}4\pi\int_0^\Lambda \acute{q}^2 d\acute{q} D_{s(pp)}\left(X_{ij}(q,\acute{q},E)-\frac{m_n q\acute{q}}{\Lambda^2}\right)\tau_j(\acute{q},E)F_j(\acute{q}), \quad (9)$$

where the ultraviolet cut-off, Λ, is introduced for regularization, and $\tau_i(q,E)\equiv\tau_{j,k}(E-q^2/2\mu_{i(jk)})$ where τ_{jk} is two body t-matrix.
The kernel function X_{ij} is defined by [20],

$$X_{ij}(q,\acute{q},E)=\int\int\left\{\begin{array}{c} p^2 dp\acute{p}^2 d\acute{p} g_{l_i}(p)G_0^{(i)}(p,q,E)\times \\ g_{l_i}(\acute{p})_i\langle p,q,\Omega_i|\acute{p},\acute{q},\Omega_j\rangle_j \end{array}\right\}, \quad (10)$$

where, the g_{l_i} and g_{l_j} are the two-body form factors. $G_0^{(i)}(p,q,E)=(E-p^2/2\mu_{jk}-q^2/2\mu_{i(jk)})^{-1}$ is the three-body Green function. ${}_i\langle p,q,\Omega_i|\acute{p},\acute{q},\Omega_j\rangle_j$ is the free Hamiltonian eigenstate spectator projection i to j. The detailed expression for $|p,q,\Omega_i\rangle_i$ is given in ref. [20]. Photons are included in the Lagrangian (1) via minimal coupling,

$$\partial_\mu \to D_\mu = \partial_\mu + ie\hat{Q}A_\mu. \quad (11)$$

The charge operator $\hat{Q}$ takes different values, depending on whether it is acting on a deuteron dibaryon field or an alpha field. The eigenvalues of the operator $\hat{Q}$ for the deuteron dibaryon and alpha fields denoted by Q_d and Q_α. e^2= $4\pi\alpha_{em}$ defines the electric charge unit in terms of the fine structure constant, α_{em}= 1/137.036. For P-wave the dominant terms of the electric response can be computed using only the Lagrangian of (12) and the minimal coupling Eq. (11).
We concentrate on the transition from the initial deuteron-alpha system with J^π= 1^+ to the final ^{6}Li bound state with J^π= 1^+. The outgoing photon interacts with the electric charges of the alpha particle and the electric charge of the deuteron at LO. The terms including the first and second order derivative operators acting on photon field which contribute to P-wave P-wave transitions are [21],

$$\mathfrak{L}=\frac{eQ_\alpha}{2m_\alpha}[\vec{p}_\alpha+\vec{p}_\alpha]\cdot\vec{\varepsilon}_\gamma^{\,*}+\frac{eQ_d}{2m_d}[\vec{p}_d+\vec{p}_d]\cdot\vec{\varepsilon}_\gamma^{\,*}$$
$$-L_{E_1}^{(d_s)}\sum_i d_{si}^{(j)\dagger}(\nabla^2 A_0-\partial_0(\nabla\cdot A))d_{si}^{(j)}$$
$$-L_{E_2}^{(NA_\alpha)}\sum_i (NA_\alpha)_i^{(j)\dagger}(\nabla^2 A_0-\partial_0(\nabla\cdot A))(NA_\alpha)_i^{(j)}, \quad (12)$$

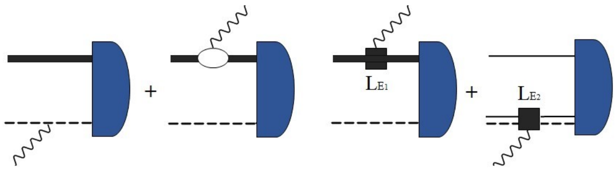

Figure 3. The photon folded diagrams up to NLO. The first two diagrams are the interaction of photons with alpha and dibaryon at LO and the last two diagrams are the interaction of photons with dibaryon and nucleon-alpha clusters at the NLO.

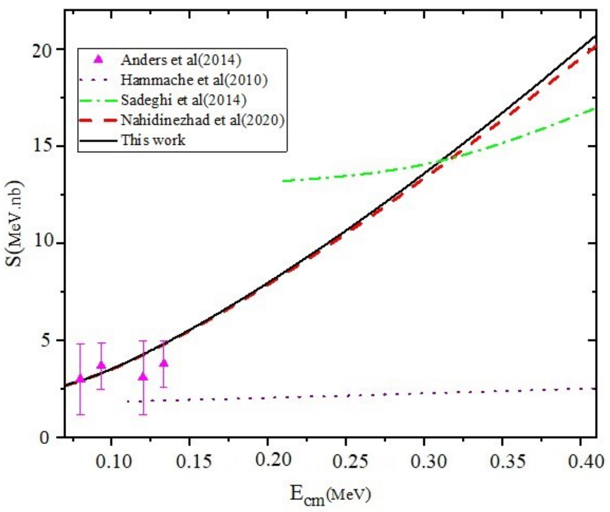

Figure 4. The results of the astrophysical S-factors for $d(\alpha,\gamma)^6$Li reaction with three body force as a function of center of mass energy of alpha E (MeV), up to NLO in comparison with the available experimental data [5] and the other theoretical works [19, 22, 23].

where, Q_d= 1 and Q_α= 2 are electric charge numbers of the deuteron (or dibaryon) and the alpha, respectively. $\vec{p}_\alpha(\vec{p}_\alpha)$ is the incoming (outgoing) momentum of the alpha in alpha-alpha-photon vertex.
$\vec{p}_\alpha(\vec{p}_\alpha)$ represents the incoming (outgoing) dibaryon momentum in dibaryon-dibaryon photon vertex, and $\vec{\varepsilon}^*_\gamma$ is polarization of the outgoing photon. L_{E_1} and L_{E_2} are phenomenological next-to leading order for deuteron and alpha nucleon cluster coupling constants to real photons. The P-wave operators contribute only to the channels with low-energy resonance or shallow bound state; thus j could be either 1/2 or 3/2.
For the calculation of the deuteron-alpha scattering amplitude, we used the Faddeev-type integral equations in momentum space for three-body transition operators. The coupled Faddeev equations for $d-\alpha$ scattering in the two body and three body channels up to LO, are schematically shown in Fig. 1. The NLO contribution can then be obtained by perturbing around the LO solution with the one deuteron kinetic energy operator insertion [19] and by inserting of three body force. The coupled Faddeev equations for $d-\alpha$ scattering in the two body and three body channels up to NLO, are schematically shown in Fig. 2. Fig. 3 show LO and NLO order of photon interaction with the undetermined phenomenological L_{E_1} and L_{E_2} vertices. The transition amplitudes can be written as,

$$|E_l^{L,S,j_i,j_f}|^2 = \left(\frac{1}{2}F_\alpha\right)^2 + \left(\frac{1}{2}F_p\right)^2, \qquad (13)$$

where E_l^{L,S,j_i,j_f} is the matrix element of electric transition relating the scattering state LSJ_iJ_f to the ^{6}Li bound state with angular momentum J_f.
The cross section for $d(\alpha,\gamma)^6$Li reaction is related to the multipole transition amplitude E_1 and E_2 as,

$$\sigma = \left(\frac{1}{f}\right)\left(\frac{q}{1+\frac{q}{\mu}}\right)\sum_{L,S,j_i,j_f}\frac{1}{2J_f+1}|E_l^{L,S,j_i,j_f}|^2, \qquad (14)$$

where $f = v_{rel}/8\pi\alpha_{em}$, α_{em} is fine structure constant, v_{rel} is the deuteron and alpha relative velocity, and μ is the reduced mass of the deuteron-alpha system.
The photodisintegration rates (in s^{-1}) for the inverse reaction ^{6}Li+$\gamma \to d+\alpha$ is written as [4],

$$\lambda_{^6\mathrm{Li}+\gamma\to d+\alpha} = 9.8686\times10^9 T_9^{3/2}\frac{(2I_d+1)(2I_\alpha+1)}{(2I_{^6\mathrm{Li}}+1)}\times$$

$$\left(\frac{G_dG_\alpha}{G_{^6\mathrm{Li}}}\right)\left(\frac{A_dA_\alpha}{A^6\mathrm{Li}}\right)^{\frac{3}{2}} N_A(\sigma v)_{d+\alpha\to^6\mathrm{Li}+\gamma}e^{(-11.605\frac{Q}{T_9})}$$

$$= 1.531\times10^{10}T_9^{3/2}e^{(-17.108\frac{1}{T_9})}\ N_A(\sigma v)_{d+\alpha\to^6\mathrm{Li}+\gamma} \qquad (15)$$

where T_9 is a variable for the temperature in units of 109 K. $I_d(A_d)$, $I_\alpha(A_\alpha)$ and $I_{^6\mathrm{Li}}$ $(A_{^6\mathrm{Li}})$ are the spins (mass of nucleus) of deuteron, alpha and ^{6}Li in units of $\hbar$ (amu). Q is the Q-value for reaction $d+\alpha\to^6$Li+γ and G_i ($i=d,\alpha$ and ^{6}Li) is the temperature dependent normalized partition function for nucleus i and it is defined as,

$$G_i(T) = \frac{1}{2J_i^0+1}\sum_\mu(2J_i^\mu+1)e^{\left(\frac{-\varepsilon_i^\mu}{KT}\right)}, \qquad (16)$$

where μ is the excited state of the nucleus i and the summation on the excited states μ of the nucleus i is performed. ε_i^μ is the excitation energy of the nucleus i. For deuteron and alpha nuclei, excitation energy is zero. J_i^0 and J_i^μ are total angular momentum of the nucleus i for bound and excitation states respectively. $N_A(\sigma v)_{d+\alpha\to^6\mathrm{Li}+\gamma}$ is the reaction rate for deuteron-alpha radiative capture reaction [4, 16]. $N_A(\sigma v)_{d+\alpha\to^6\mathrm{Li}+\gamma}$ is defined as,

$$N_A(\sigma v)_{d+\alpha\to^6\mathrm{Li}+\gamma} = 3.7313\times10^{10}A^{1/2}T_9^{-3/2}\times$$

$$\int_0^\infty \sigma(E_{cm})E_{cm}e^{(-11.605\frac{E_{cm}}{T_9})}dE_{cm}, \qquad (17)$$

where A= 4/3 is reduced mass number for the $\alpha+d$ system.

Table 1. Theoretical values of the calculated total astrophysical S-factors (in MeV.nb) at the NLO at energies in the range 0.07 MeV$\leq E \leq$0.14 MeV, in comparison with the available experimental data.

E (MeV)	S-factor Anders et al.	S-factor Nahidinezhad et al.	S-factor This work
0.07-0.08	3.0±1.8	2.88	3.15
0.09-0.10	3.7±1.2	3.28	3.42
0.12	3.1±1.9	4.12	4.25
0.13-0.14	3.8±1.2	4.53	4.79

Table 2. The results of our theoretical model for the photodisintegration rates (in s^{-1}) for the $^6\mathrm{Li}(\gamma,\alpha)d$ at 0.70$\leq T_9 \leq$2.0, along with the available experimental data [24] and the two sets of results of other theoretical calculations (model A and model B) [16].

T_9	λ Trezzi et al.	λ (model A) Tursunov et al.	λ (model B) Tursunov et al.	λ Nahidinezhad et al.	λ This work
0.70	4.0×10^{-3}	3.2×10^{-3}	7.4×10^{-3}	5.0×10^{-3}	7.440×10^{-3}
1.0	0.37×10^{2}	0.32×10^{2}	0.53×10^{2}	0.52×10^{2}	0.536×10^{2}
2.0	0.50×10^{7}	0.46×10^{7}	0.60×10^{7}	0.80×10^{7}	0.602×10^{7}

3. Results and discussion

We used a neutron mass of m_n= 939.57 MeV, proton mass of m_p= 938.27 MeV, an alpha mass of m_α= 3728.42 MeV and the ^{6}Li binding energy of B_3= 28.29 MeV. The two-coupled Faddeev equations derived for the deuteron-alpha scattering amplitude with coulomb effect [18]. These Faddeev integral equations have been solved for the deuteron-alpha scattering state by folding photon diagrams, up to NLO. We also calculated the deuteron-alpha radiative capture reaction rate and the $^6\mathrm{Li}(\gamma,\alpha)d$ photodisintegration rates, at low energies by using EFT, see [25, 26] for more details. We considered the three-body force in the present calculation. The numerical values have been used for the scattering lengths a_0= 23.71 fm, a_1= 5.423 fm and effective range r_1= 1.76 fm. The phenomenological coupling constants L_{E_1}= L_{E_2}= 3 have been determined by fitting to the well-known astrophysical S-factor experimental data.
In Fig. 4, the results of the astrophysical S-factors for deuteron-alpha radiative capture reaction have been shown, as a function of center of mass energy of alpha E (MeV) up to NLO. Our present calculation by considering of the three-body force have been compared with the available experimental data [5] and the other theoretical works [19, 22, 23], at the 0.07 MeV $\leq E \leq$ 0.41 MeV range of energies.
We show in Fig. 4, results with coulomb as well as three-body force effects. Inclusion of the three-body force moves the results of the astrophysical S-factors to the better situation respect to the other experimental and the theoretical calculations. It is found the results is nearly better in comparison with the available experimental data.
Our calculated results for the astrophysical S-factors for the $d-\alpha$ radiative capture reaction by considering of the three-body force have been also compared with the experimental data [5] as well as the other theoretical results [19, 22, 23], at the low-energies in Table 1.
The results of Table 1 show that by including the three-body force in the calculations, the results are in good agreement with the Anders et al. [5] obtained data. It is found that S (E= 0.07 - 0.08)= 3.15 MeV.nb in comparison with the S (E= 0.07 - 0.08)= 3.0$\pm$1.8 MeV.nb experimental data [5]. All the performed comparisons, show that the results for the $d(\alpha,\gamma)^6\mathrm{Li}$ reaction are better described in order of calculation by considering of the three-body forces. Three-body forces improves few percent agreement between the experimental data and the calculated results. We calculated the deuteron-alpha radiative capture reaction rate and the $^6\mathrm{Li}(\gamma,\alpha)d$ photodisintegration rates at low energies [25, 26]. The $^6\mathrm{Li}(\gamma,\alpha)d$ photodisintegration rates at low energies are dominated by the P-waves radiative captures in the two and three body channels resulting the J= 1^+ ground state of ^{6}Li. To compare of the obtained results with other experimental and theoretical results, the values of the inverse reaction rate of corresponding to several temperatures are given in Table 2 at $0.70 \leq T_9 \leq 2.0$.
Table 2 compares the obtained results with the available experimental data from [24] and the two sets of the other theoretical calculations (model A and model B) [16] and theoretical calculations of [26]. It is worthy to mention that by insertion of three body force, we achieved to a better agreement between calculation and the experimental data.

4. conclusion

We applied an EFT theory approach with the two- and three body forces to compute the deuteron-alpha radiative capture reaction, at the astrophysical energies. In this work, alpha particle was assumed to be structureless and coulomb effects considered between the charged particles. The inverse reaction rate has been estimated for E_1 and E_2 transitions by adding the three-body force, up to NLO. The $^6\mathrm{Li}(\gamma,\alpha)d$ photodisintegration rates at low energies is dominated by the P-waves radiative captures in the $d+\alpha \to d+\alpha$ and $d+\alpha \to N+N\alpha$ channels resulting the J= 1^+ ground state of ^{6}Li. Our results are cut off independent at the order of the calculation. The $^6\mathrm{Li}(\gamma,\alpha)d$ photodisintegration rates are found as a function of temperature corresponding with the recent data of the LUNA Collaboration. The obtained $^6\mathrm{Li}(\gamma,\alpha)d$ reaction rate is found to be in good agreement with theoretical results. The used method provides new possibilities for investigation of few-alpha radiative capture reactions, at low energies. Our model can be also improved to achieve greater accuracy by performing higher-order computations.

Conflict of interest statement
The authors declare that they have no conflict of interest.

References

[1] R. G. H. Robertson, P. Dyer, R. C. Melin, T. J. Bowles, A. B. McDonald, G. C. Ball, W. G. Davies, and E. D. Earle. *Physical Review C*, **29**:755, 1984.

[2] J. Kiener, H. J. Gils, H. Rebel, S. Zagromski, G. Gsottschneider, N. Heide, H. Jelitto, J. Wentz, and G. Baur. *Physical Review C*, **44**:2195, 1991.

[3] P. Mohr, V. K¨ölle, S. Wilmes, U. Atzrott, G. Staudt, J. W. Hammer, H. Krauss, and H. Oberhummer. *Physical Review C*, **50**:1543, 1994.

[4] C. Angulo, M. Arnould, M. Rayet, P. Descouvemont, D. Baye, C. Leclercq-Willain, A. Coc, S. Barhoumi, P. Aguer, and C. Rolfs et al. *Nuclear Physics A*, **656**:3, 1999.

[5] M. Anders, D. Trezzi, R. Menegazzo, M. Aliotta, A. Bellini, D. Bemmerer, C. Broggini, and A. Caciolli et al. *Physical Review Letters*, **113**:042501, 2014.

[6] K. I. Tursunmakhatov, R. Yarmukhamedov, and S. B. Igamov. *EPJ Web of Conferences*, **227**:02016, 2020.

[7] B. H. Flowers and F. Mandl. *Advances in High Energy Physics*, **0060**:131, 1950.

[8] P. E. Shanley. *Physical Review Letters*, **21**:627, 1968.

[9] H. M. Hofmann and D. Fick. *Zeitschrift für Physik A Hadrons and nuclei*, **240**:420, 1970.

[10] D. R. Lehman and M. Rajan. *Physical Review C*, **25**:2743, 1982.

[11] N. A. Burkova, K. A. Zhaksibekova, M. A. Zhusupov, and R. A. Eramzhyan. *Physics Letters B*, **248**:15, 1990.

[12] L. D. Blokhintsev, V. I. Kukulin, A. A. Sakharuk, D. A. Savin, and E. V. Kuznetsova. *Physical Review C*, **48**:2390, 1993.

[13] G. G. Ryzhikh, R. A. Eramzhyan, and S. Shlomo. *Physical Review C*, **51**:3240, 1995.

[14] K. M. Nollett, R. B. Wiringa, and R. Schiavilla. *Physical Review C*, **63**:024003, 2001.

[15] E. M. Tursunov, A. S. Kadyrov, S. A. Turakulov, and I. Bray. *Physical Review C*, **94**:015801, 2016.

[16] E. M. Tursunov, S. A. Turakulov, A. S. Kadyrov, and I. Bray. *Physical Review C*, **98**:055803, 2018.

[17] D. Baye and E. M. Tursunov. *Journal of Physics G: Nuclear and Physics*, **45**:085102, 2018.

[18] S. Nahidinezhad, H. Sadeghi, and H. Khalili. *New Astronomy*, **80**:101424, 2020.

[19] S. Nahidinezhad, H. Sadeghi, and H. Khalili. *Astrophysics and Space Science*, **365**:74, 2020.

[20] C. Ji, C. Elster, and D. R. Phillips. *Physical Review C*, **90**:044004, 2014.

[21] H. W. Hammer, C. Ji, and D. R. Phillips. *Journal of physics G, Nuclear and particle physics*, **44**:103002, 2017.

[22] H. Sadeghi, A. Moghadasi, and M. Ghamary. *Journal of Astrophysics and Astronomy*, **35**:675, 2014.

[23] F. Hammache, M. Heil, S. Typel, D. Galaviz, and K. Sümmerer et al. *Physical Review C*, **82**:065803, 2010.

[24] D. Trezzi, M. Anders, M. Aliotta, A. Bellini, D. Bemmerer, A. Boeltzig, C. Broggini, C. G. Bruno, A. Caciolli, and F. Cavanna et al. *Astroparticle Physics*, **89**:57, 2017.

[25] S. Nahidinezhad, H. Sadeghi, and M. Khoddam. *New Astronomy*, **82**:101461, 2020.

[26] H. Sadeghi and S. Nahidinezhad. *New Astronomy*, **86**:101584, 2021.