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Effects of viscous dissipation on MHD natural convection flow along a vertical wavy surface

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Abstract

An analysis is presented to investigate the influences of viscous dissipation on MHD natural convection flow along a uniformly heated vertical wavy surface. The governing equations are transformed into dimensionless non-similar equations using a set of suitable transformations and solved numerically by the implicit finite difference method known as the Keller box scheme. Numerical results for the velocity profiles, temperature profiles, skin friction coefficient, and the rate of heat transfers are shown graphically and in tabular form for different values of the selective set of parameters.

Keywords: Natural convection; Uniform surface temperature; Wavy surface; Magnetic parameter; Prandtl number

Background

The viscous dissipation effect plays an important role in natural convection in various devices which are subjected to large deceleration or which operate at high rotational speeds and also in strong gravitational field processes on large scales (on large planets) and in geological processes. The natural convection along a vertical wavy surface was first studied by Yao [1] and using an extended Prantdl's transposition theorem and a finite difference scheme. He proposed a simple transformation to study the natural convection heat transfer from isothermal vertical wavy surfaces, such as the sinusoidal surface. Moulic and Yao [2] also investigated mixed convection heat transfer along a vertical wavy surface. Alam et al. [3] have also studied the problem of free convection from a wavy vertical surface in the presence of a transverse magnetic field. Combined effects of thermal and mass diffusion on the natural convection flow of a viscous incompressible fluid along a vertical wavy surface have been investigated by Hossain and Rees [4]. Wang and Chen [5] investigated transient force and free convection along a vertical wavy surface in micropolar fluid. Hossain [6] studied the problem of natural convection of fluid with temperature-dependent viscosity along a heated vertical wavy surface. Natural and mixed convection heat and mass transfer along a vertical wavy surface have been investigated by Jang [7,8]. Recently, Molla et al. [9] have studied natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation/absorption. Tashtoush and Al-Odat [10] investigated magnetic field effect on heat and fluid flow over a wavy surface with a variable heat flux. Hossain [11] investigated the natural convection flow past a permeable wedge for the fluid having temperature-dependent viscosity and thermal conductivity. Very recently, Parveen and Alim [12] investigated Joule heating effect on magnetohydrodynamic natural convection flow along a vertical wavy surface with viscosity dependent on temperature. The thermal conductivity of the fluid had been assumed to be constant in all the above studies. However, it is known that this physical property may be changed significantly with temperature.

The present study aims to incorporate the idea of the effects of viscous dissipation on MHD natural convection flow along a uniformly heated vertical wavy surface. Numerical results of the velocity profiles, temperature profiles, local skin friction coefficient, and rate of heat transfer are shown graphically. Some selected results of skin friction coefficient and rate of heat transfer for different values of magnetic parameter M have been shown in tabular form and then discussed.

Formulation of the problem

Steady two-dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically

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conducting fluid along a vertical wavy surface in the presence of magnetic field is considered. It is assumed that the wavy surface is electrically insulated and is maintained at a uniform temperature T_w. The fluid is stationary above the wavy plate and is kept at a temperature T_∞. The surface temperature T_w is greater than the ambient temperature T_∞, that is, $T_w > T_\infty$. The flow configuration of the wavy surface and the two-dimensional Cartesian coordinate system are shown in Figure 1.

The boundary layer analysis outlined below allows $\bar{\sigma}(X)$ being arbitrary, but our detailed numerical work assumed that the surface exhibits sinusoidal deformations. The wavy surface may be defined by:

$$Y_w = \bar{\sigma}(X) = \alpha \sin\left(\frac{n\pi X}{L}\right), \tag{1}$$

where α is the amplitude and L is the wavelength associated with the wavy surface.

The governing equations of such flow of magnetic field along a vertical wavy surface under the usual Boussinesq approximations can be written in a dimensional form as:

Continuity equation

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{2}$$

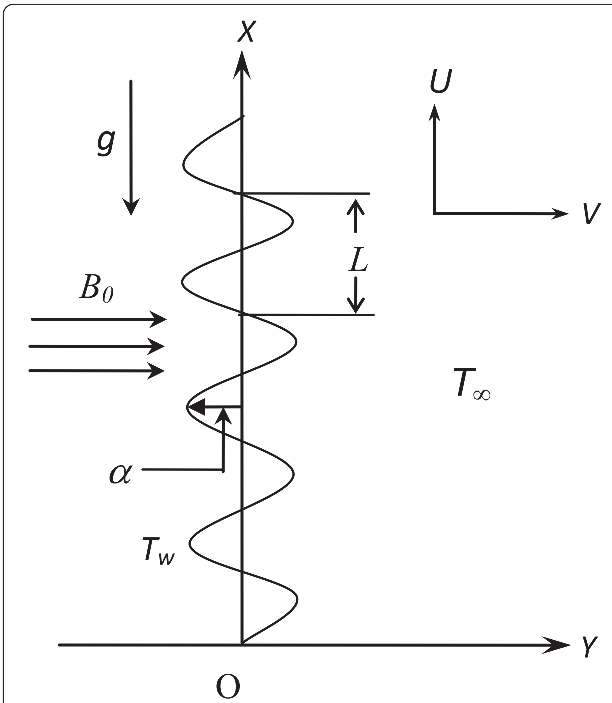

Figure 1 Physical model and coordinate system.

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{1}{\rho}\frac{\partial P}{\partial X} + \frac{1}{\rho}\nabla.(\mu\nabla U) + g\beta(T - T_\infty) - \frac{\sigma_0 B_0^2}{\rho}U, \tag{3}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{1}{\rho}\frac{\partial P}{\partial Y} + \frac{1}{\rho}\nabla.(\mu\nabla V), \tag{4}$$

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{k}{\rho C_P}\nabla^2 T + \frac{\mu}{\rho C_P}\left(\frac{\partial U}{\partial Y}\right)^2, \tag{5}$$

where (X, Y) are the dimensional coordinates along and normal to the tangent of the surface, (U, V) are the velocity components parallel to (X, Y), g is the acceleration due to the Earth's gravity, P is the dimensional pressure of the fluid, T is the temperature of the fluid in the boundary layer, C_p is the specific heat at constant pressure, μ is the viscosity of the fluid, ρ is the density, ν is the kinematic viscosity, where $\nu = \mu/\rho$, k is the thermal conductivity of the fluid, β is the volumetric coefficient of thermal expansion, B_0 is the strength of the magnetic field, σ_0 is the electrical conductivity of the fluid, and ∇^2 is the Laplacian operator, where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

The boundary conditions for the present problem are:

$$\begin{array}{l} U=0, V=0, T=T_w \quad at \quad Y=Y_w=\bar{\sigma}(X) \\ U=0, T=T_\infty \quad as \quad Y\rightarrow\infty \end{array}. \tag{6}$$

Using Prandtl's transposition theorem to transform the irregular wavy surface into a flat surface as extended by Yao [1] and the boundary layer approximation, the following dimensionless variables are introduced for non-dimensionalizing the governing equations:

$$\begin{array}{l} x = \frac{X}{L}, y = \frac{Y-\bar{\sigma}}{L}Gr^{\frac{1}{4}}, p = \frac{L^2}{\rho\nu^2}Gr^{-1}P, \\ u = \frac{U}{u_0} = \frac{\rho L}{\mu}Gr^{-1/2}U, v = \frac{\rho L}{\mu}Gr^{-1/4}(V - \sigma_x U), \\ \theta = \frac{T-T_\infty}{T_w - T_\infty}, \sigma_x = \frac{d\bar{\sigma}}{dX} = \frac{d\sigma}{dx}, Gr = \frac{g\beta(T_w - T_\infty)}{\nu^2}L^3 \end{array} \tag{7}$$

where $u_0 = \frac{\mu}{\rho L}Gr^{1/2}$ is the characteristic velocity, θ is the dimensionless temperature function, and (u, v) are the dimensionless velocity components parallel to (x, y). Here, p is the dimensionless pressure of the fluid, L is the wavelength associated with the wavy surface, and Gr is the Grashof number. Introducing the above dimensionless dependent and independent variables into Equations 2 to 5, the following dimensionless form of the governing equations are obtained after ignoring terms of smaller orders of magnitude in the Grashof number Gr,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{8}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+Gr^{1/4}\sigma_x\frac{\partial p}{\partial y}+\left(1+\sigma_x^2\right)\frac{\partial^2 u}{\partial y^2}-Mu+\theta, \tag{9}$$

$$\sigma_x\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-Gr^{1/4}\frac{\partial p}{\partial y}+\sigma_x\left(1+\sigma_x^2\right)\frac{\partial^2 u}{\partial y^2}-\sigma_{xx}u^2, \tag{10}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{Pr}\left(1+\sigma_x^2\right)\frac{\partial^2 \theta}{\partial y^2}+Ec\left(\frac{\partial u}{\partial y}\right)^2. \tag{11}$$

It is worth noting that the σ_x and σ_{xx} indicate the first and second derivatives of σ with respect to x; therefore, $\sigma_x = d\bar{\sigma}/dX = d\sigma/dx$ and $\sigma_{xx} = d\sigma_x/dx$.

In the above equations, Pr, M, and Ec are respectively known as the Prandtl number, magnetic parameter, and viscous dissipation parameter (characterized by the Eckert number), which are defined as:

$$Pr=\frac{C_p\mu}{k}, M=\frac{\sigma_0 B_0^2 L^2}{\mu Gr^{1/2}}, Ec=\frac{u_0^2}{C_p(T_w-T_\infty)}. \tag{12}$$

For the present problem, this pressure gradient $(\partial p/\partial x = 0)$ is 0. Thus, the elimination of $\partial p/\partial y$ from Equations 9 and 10 leads to:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(1+\sigma_x^2\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_x\sigma_{xx}}{1+\sigma_x^2}u^2-\frac{M}{1+\sigma_x^2}u+\frac{1}{1+\sigma_x^2}\theta. \tag{13}$$

The corresponding boundary conditions for the present problem then turn into:

$$\left.\begin{array}{l} u=v=0, \theta=1 \quad \text{at} \quad y=0 \\ u=0, \theta=0 \quad \text{as} \quad y\to\infty \end{array}\right\}. \tag{14}$$

Now, we introduce the following transformations to reduce the governing equations to a convenient form:

$$\psi=x^{3/4}f(x,\eta), \quad \eta=yx^{-1/4}, \quad \theta=\theta(x,\eta), \tag{15}$$

where $f(x, \eta)$ is the dimensionless stream function, η is the dimensionless similarity variable, and ψ is the stream function that satisfies the continuity Equation 8 and is related to the velocity components in the usual way as:

$$u=\frac{\partial \psi}{\partial y}, \quad v=-\frac{\partial \psi}{\partial x}. \tag{16}$$

Introducing the transformations given in Equation 15 and using (16) into Equations 13 and 11 are transformed into the new coordinate system. Thus, the resulting equations are:

$$\begin{aligned}\left(1+\sigma_x^2\right)f''' + \frac{3}{4}ff'' - \left(\frac{1}{2}+\frac{x\sigma_x\sigma_{xx}}{1+\sigma_x^2}\right)f'^2+\frac{1}{1+\sigma_x{}^2}\theta \\ -\frac{Mx^{1/2}}{1+\sigma_x^2}f' = x\left(f'\frac{\partial f'}{\partial x}-f''\frac{\partial f}{\partial x}\right)\end{aligned}, \tag{17}$$

$$\begin{aligned}\frac{1}{Pr}\left(1+\sigma_x^2\right)\theta''+\frac{3}{4}f\theta'+Ecx\left(f''\right)^2 \\ = x\left(f'\frac{\partial \theta}{\partial x}-\theta'\frac{\partial f}{\partial x}\right).\end{aligned} \tag{18}$$

The boundary conditions (14) now take the following form:

$$\left.\begin{array}{ll} f(x,o)=f'(x,o)=0, & \theta(x,o)=1 \\ f'(x,\infty)=0, & \theta(x,\infty)=0 \end{array}\right\}. \tag{19}$$

Here, the prime denotes the differentiation with respect to η.

However, once we know the values of the functions f and θ and their derivatives, it is important to calculate the values of the rate of heat transfer in terms of local Nusselt number Nu_x and the shearing stress τ_w in terms of the local skin friction coefficient C_{fx} from the following relations:

$$C_{fx}=\frac{2\tau_w}{\rho U^2}, Nu_x=\frac{q_w X}{k(T_w-T_\infty)}, \tag{20}$$

$$\text{where} \quad \tau_w=(\mu\bar{n}.\nabla\bar{u})_{y=0} \quad \text{and} \quad q_w=-k(\bar{n}.\nabla T)_{y=0}. \tag{21}$$

Also, $U=\mu_\infty \mathrm{Gr}^{1/2}/\rho L$.

Here, $\bar{n}=\frac{\bar{i}f_x+\bar{j}f_y}{\sqrt{f_x^2+f_y^2}}$ is the unit normal to the surface. Using the transformations (15) and (21) into Equation 20, the local skin friction coefficient C_{fx} and the rate of heat transfer in terms of the local Nusselt number Nu_x take the following forms:

$$\frac{1}{2}(Gr/x)^{1/4}C_{fx}=\sqrt{1+\sigma_x^2}\quad f''(x,0), \tag{22}$$

$$Gr^{-1/4}x^{-3/4}Nu_x=-\sqrt{1+\sigma_x^2}\theta'(x,0). \tag{23}$$

For computational purpose, the period of oscillations in the waviness of this surface has been considered to be π.

Table 1 Comparison of the values of skin friction coefficient and heat transfer coefficient

Pr	$f''(x, 0)$		$-\theta'(x, 0)$	
	Hossain et al. [11]	Present work	Hossain et al. [11]	Present work
1.0	0.908	0.90813	0.401	0.40102
10.0	0.591	0.59270	0.825	0.82662
25.0	0.485	0.48732	1.066	1.06848
50.0	0.485	0.41728	1.066	1.28878
100.0	0.352	0.35558	1.542	1.54828

Numerical results of Hossain et al. [11] and the present work for different values of Prandtl number *Pr* while $M = 0.0$ and $Ec = 0.0$ with $\alpha = 0.2$.

Method

The governing partial differential equations are reduced to dimensionless local non-similar equations by adopting appropriate transformations. The transformed boundary layer equations are solved numerically using the Keller box method described by Keller [13] and Cebeci and Bradshaw [14] and used by Hossain et al. [11] and many other authors.

Results and discussion

The effects of viscous dissipation on a magnetohydrodynamic natural convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface have been investigated. Although there are four parameters of interest in the present problem, the effects of viscous dissipation parameter which is characterized by the Eckert number *Ec*, magnetic parameter *M*, and Prandtl number *Pr* on the surface shear stress, rate of heat transfer, velocity, and temperature are focused.

A comparison of the present numerical results of skin friction coefficient and rate of heat transfer coefficient with that of Hossain et al. [11] has been shown in Table 1. To make the numerical data comparable with [11] for different values of Prandtl number Pr, the magnetic parameter *M* and the Eckert number *Ec* are ignored. It is evident from the comparison that the present results agreed well with the results of Hossain et al. [11].

Numerical values of local shearing stress and the rate of heat transfer are calculated from Equations 22 and 23 in terms of the skin friction coefficients C_{fx} and Nusselt number Nu_x, respectively, for a wide range of the axial distance variable x starting from the leading edge for different values of the parameters *Pr*, *M*, *Ec*, and α.

The velocity and temperature of the flow field is found to change more or less with the variation of the flow parameters. The effects of the flow parameters on the velocity and temperature fields are analyzed with the help of graphs.

The effects of Prandtl number *Pr* on velocity and temperature are illustrated in Figure 2a,b. For the higher values of Prandtl number *Pr*, both the velocity and temperature decreases such that there exists a local maximum of the velocity within the boundary layer. The maximum values of velocities are recorded as 0.53860, 0.49030, 0.43351, 0.39544, and 0.28892 for Prandtl number $Pr = 0.72$, 1.0, 1.5, 2.0, and 5.0 at the position of $\eta = 1.36929$, 1.30254, 1.23788, 1.23788, and 1.05539, respectively, and the maximum velocity decreases by 46.36%. The values of temperature are recorded as 0.50125, 0.44706, 0.37840, 0.32953, and 0.18365 for Prandtl number $Pr = 0.72$, 1.0, 1.5, 2.0, and 5.0 at the position of $\eta = 1.50946$, and the temperature decreases by 63.36%. Figure 2b displays the results that the change of

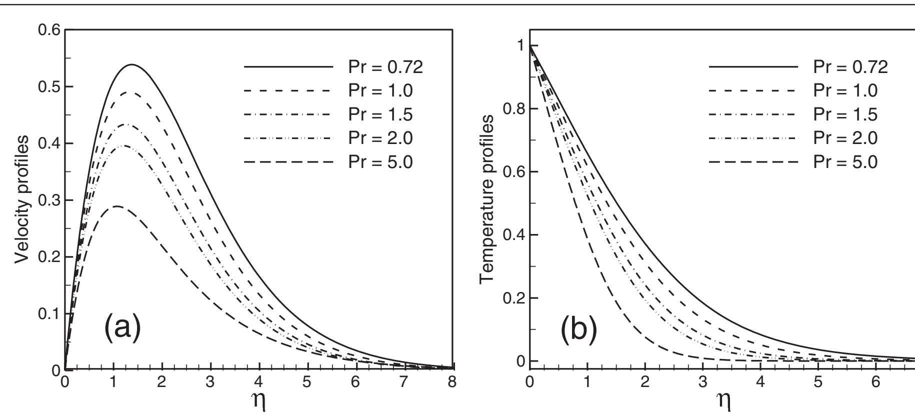

Figure 2 Velocity (a) and temperature (b) profiles against η **for different values of Pr.** With $\alpha = 0.1$, $M = 0.1$, and Ec =10.0.

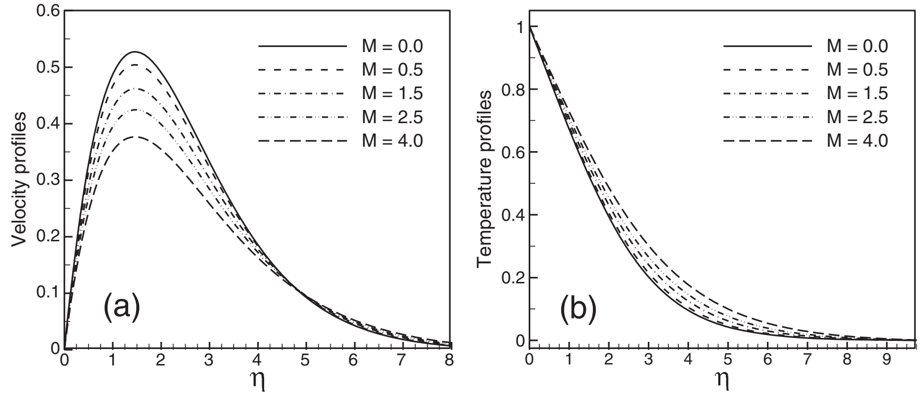

Figure 3 Velocity (a) and temperature (b) profiles against *η* for different values of *M*. With $Pr = 0.72$, $\alpha = 0.1$, and $Ec = 10.0$.

temperature profiles in the η direction reveals the typical temperature profiles for natural convection boundary layer flow, i.e., the temperature is 0 at the boundary wall. It is observed that the velocity as well as the boundary layer thickness decreases, and the temperature as well as the thermal boundary layer thickness decreases for the increasing values of Prandtl number.

The effects for different values of the magnetic parameter M on the velocity and temperature profiles have been presented graphically in Figure 3a,b. It is seen from Figure 3a that for the higher values of magnetic parameter M, the velocity decreases along the η direction. At the position of η = 4.75, the velocity becomes constant, that is, velocity profiles meet at a point and then cross the side, increasing with the magnetic parameter M. This is because the velocity profiles having lower peak values for higher values of magnetic parameter M tend to decrease comparatively slower along the η direction. The maximum values of velocities are found as 0.52703, 0.50401, 0.46183, 0.42449, and 0.37662 for magnetic parameter M = 0.0, 0.5, 1.5, 2.5, and 4.0, respectively, which occur at the position η = 1.43822. Here, it is observed that at η = 1.43822, the maximum velocity decreases by 28.54% as the magnetic parameter M changes from 0.0 to 4.0. The values of dimensionless temperature are observed as 0.14214, 0.15361, 0.17704, 0.20063, and 0.23522 for the magnetic parameter M = 0.0, 0.5, 1.5, 2.5, and 4.0 occurring at the position of η = 3.4792, and the temperature increases by 65.49%. The change of temperature profiles in the η direction also shows the typical temperature profiles for natural convection boundary layer flow, that is, the value of temperature is 1.0 at the boundary wall then the temperature decreases gradually along the η direction to the asymptotic value 0.

Figure 4a,b demonstrate the velocity and temperature distribution for different values of the Eckert number Ec. It has been seen from Figure 4a that as the Eckert number Ec increases, the velocity profiles rise until the position of η = 1.43822 for the selective values of the Eckert number Ec, and from that position of η, velocities fall slowly and finally approaches to 0. It is also observed from Figure 4b that the temperature profiles increases with the Eckert number Ec. The maximum values of velocities are recorded as 0.52232, 0.52570, 0.52909,

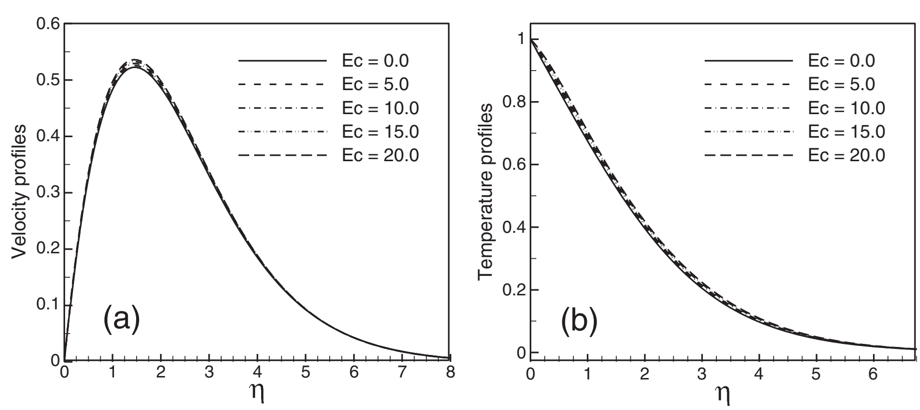

Figure 4 Velocity (a) and temperature (b) profiles against *η* for different values of *Ec*. With $Pr = 0.72$, $\alpha = 0.1$, and $M = 0.1$.

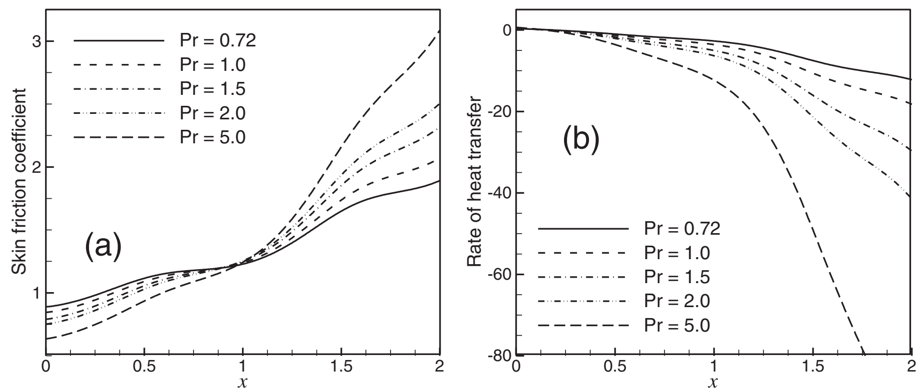

Figure 5 Skin friction coefficient (a) and rate of heat transfer (b) against *x* for different *Pr* values. With α = 0.1, *Ec* = 10.0, and *M* = 0.1.

0.53251, and 0.53595 for the Eckert number Ec = 0.0, 5.0, 10.0, 15.0, and 20.0, respectively, which occur at the same position η = 1.43822, and the maximum velocity increases by 2.61%. Temperature is recorded as 0.17726, 0.18163, 0.18604, 0.19050, and 0.19501 for the Eckert number Ec = 0.0, 5.0, 10.0, 15.0, and 20.0, respectively, at the same position of η = 3.20, and the temperature profiles increase by 10.01%, that is, velocity boundary layer thickness and thermal boundary layer thickness are unchanged.

In Figure 5a,b, the skin friction coefficient C_{fx} and local rate of heat transfer Nu_x for different values of Prandtl number Pr have been displayed. It is observed from Figure 5a that for higher values of Prandtl number, skin friction decreases to the axial position of x = 1.0 and then becomes constant for all values of Prandtl number Pr, that is, the values of skin friction coefficient meet together at the position of x = 1.0 and cross the sides that means after the axial position of x = 1.0, skin friction is increasing with Prandtl number but frictional force at the wall always rising towards downstream. It is seen from Figure 5b that for higher values of Prandtl number, the rate of heat transfer decreases, that is, heat transfer slows down for higher Prandtl number.

In Figure 6a,b, the effects of magnetic parameter M on skin friction and the rate of heat transfer have been presented. From Figure 6a, it is found that skin friction decreases significantly for greater magnetic field strength. This is physically realizable as the magnetic field retards the velocity and consequently reduces the frictional force at the wall. However, the rate of heat transfer shows opposite pattern due to the change of magnetic parameter M to higher values as depicted in Figure 6b.

The effect of different values of the Eckert number Ec on the skin friction coefficients and the rate of heat transfer are shown graphically in Figures 7a,b, respectively. In this case, the values of local skin friction coefficient C_{fx} are recorded to be 0.91799, 1.19362, 1.65790, 2.41420, and 3.60650 for Ec = 0.0, 5.0, 10, 15.0, and 20.0, respectively, which occur at the same point x = 1.51. From Figure 7a, it is observed that at x = 1.51, the skin friction coefficient increases by 292.87% due to the higher value of the Eckert number Ec. However, the values of the rate of heat transfer are found to be 0.34382, −1.91932, −7.63071, −21.73558, and −55.25804 for Ec = 0.0, 5.0, 10.0, 15.0, and 20.0, respectively, which occur at the same point x = 1.51. It is seen from

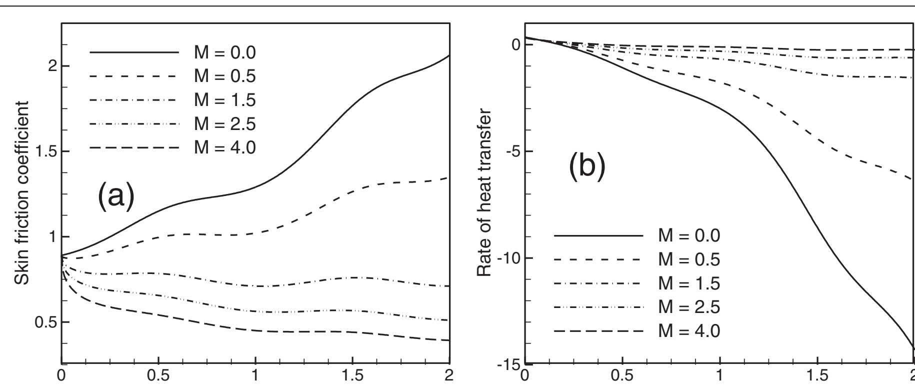

Figure 6 Skin friction coefficient (a) and rate of heat transfer (b) against *x* for different *M* values. With *Pr* = 0.72, α = 0.1, and *Ec* = 10.0.

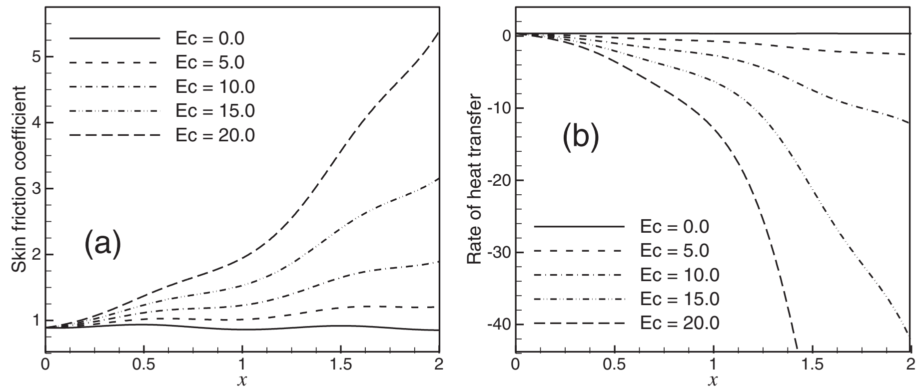

Figure 7 Skin friction (a) coefficient and rate of heat transfer (b) against *x* for different *Ec* values. With $Pr = 0.72$, $\alpha = 0.1$, and $M = 0.1$.

Figure 7b that for higher values of the Eckert number *Ec*, the rate of heat transfer decreases, that is, heat transfer slows down significantly for higher Eckert number *Ec*.

Some numerical values of skin friction coefficient C_{fx} and rate of heat transfer Nu_x calculated from Equations 22 and 23 for the wavy surface from lower stagnation point at $x = 0.0$ to $x = 2.0$ are presented in Table 2.

Conclusion

The effects of the Prandtl number *Pr*, magnetic parameter *M*, Eckert number *Ec*, and amplitude of wavy surface *α* on the magnetohydrodynamic natural convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface have been studied. From the present investigations, the following conclusions may be drawn:

Table 2 Skin friction coefficient and rate of heat transfer against *x*

x	*Ec* = 0.0		*Ec* = 5.0		*Ec* = 10.0		*Ec* = 15.0	
	C_{fx}	Nu_x	C_{fx}	Nu_x	C_{fx}	Nu_x	C_{fx}	Nu_x
0.00000	0.89054	0.34850	0.89054	0.34850	0.89054	0.34850	0.89054	0.34850
0.10500	0.88777	0.34643	0.90137	0.25694	0.91545	0.16195	0.93002	0.06107
0.20500	0.90018	0.34674	0.92809	0.15979	0.95797	−0.05077	0.98993	−0.28799
0.30500	0.91755	0.34769	0.96211	0.04413	1.01153	−0.32014	1.06630	−0.75746
0.40500	0.93184	0.34861	0.99526	−0.08888	1.06826	−0.65060	1.15213	−1.37193
0.50500	0.93585	0.34897	1.01881	−0.22687	1.11802	−1.01939	1.23628	−2.10935
0.60500	0.92673	0.34849	1.02769	−0.35198	1.15306	−1.38341	1.30794	−2.89932
0.70500	0.90767	0.34722	1.02364	−0.45407	1.17295	−1.71082	1.36371	−3.67511
0.80500	0.88584	0.34551	1.01415	−0.53885	1.18507	−2.00802	1.41053	−4.43932
0.90500	0.86868	0.34385	1.00842	−0.62386	1.20076	−2.32048	1.46226	−5.28221
1.00500	0.86118	0.34262	1.01374	−0.73030	1.23054	−2.71288	1.53413	−6.36013
1.10500	0.86505	0.34206	1.03386	−0.87763	1.28163	−3.25681	1.63899	−7.87489
1.20500	0.87854	0.34218	1.06846	−1.07963	1.35664	−4.01868	1.78493	−10.05539
1.30500	0.89666	0.34277	1.11274	−1.33723	1.45201	−5.03515	1.97182	−13.09772
1.40500	0.91213	0.34344	1.15762	−1.62842	1.55664	−6.26727	2.18686	−17.02235
1.50500	0.91802	0.34381	1.19231	−1.90671	1.65357	−7.56767	2.40385	−21.50929
1.60500	0.91131	0.34362	1.20956	−2.12335	1.72759	−8.73511	2.59395	−25.96354
1.70500	0.89466	0.34285	1.21020	−2.26197	1.77502	−9.65738	2.74427	−29.93858
1.80500	0.87475	0.34171	1.20265	−2.35113	1.80640	−10.40497	2.86776	−33.54552
1.90500	0.85873	0.34054	1.19810	−2.44416	1.84028	−11.18956	2.99585	−37.45445
2.00000	0.85170	0.33970	1.20512	−2.58533	1.89162	−12.20614	3.15484	−42.34764

Different values of Eckert number *Ec* with other controlling parameters $Pr = 0.72$, $\alpha = 0.1$, and $M = 0.1$.

- Velocity, temperature, and the frictional force at the wall enhance due the higher values of the Eckert number *Ec*, while the rate of heat transfer reduces for the greater values of Eckert number. The effects of *Ec* on skin friction and on the rate of heat transfer are more significant than that on the velocity and temperature.
- As the Prandtl number *Pr* increases, the velocity, temperature, and rate of heat transfer decrease while the skin friction initially decreases and becomes constant near $x = 1.0$ after that position skin friction increases with Prandtl number *Pr*.
- Magnetic field strength enhancement causes the temperature and rate of heat transfer to rise and the velocity and skin friction coefficient to fall within the boundary layer. At the position of $\eta = 4.75$, the velocity becomes constant and then crosses the side, increasing with the magnetic parameter.

Nomenclature

B_0, applied magnetic field strength
C_{fx}, local skin friction coefficient
C_p, specific heat at constant pressure (J kg^{-1} K^{-1})
f, dimensionless stream function
g, acceleration due to gravity (m s^{-2})
Gr, Grashof number
k, thermal conductivity (W m^{-1} K^{-1})
k_∞, thermal conductivity of the ambient fluid (W m^{-1} K^{-1})
L, characteristic length associated with the wavy surface (m)
$\bar{n}$, unit normal to the surface
Nu_x, local Nusselt number
P, pressure of the fluid (N m^{-2})
Pr, Prandtl number
q_w, heat flux at the surface (W m^{-2})
T, temperature of the fluid in the boundary layer (K)
T_w, temperature at the surface (K)
T_∞, temperature of the ambient fluid (K)
u, v, dimensionless velocity components along the (x, y) axes (m s^{-1})
x, y, axis in the direction along and normal to the tangent of the surface.

Greek symbols

α, amplitude of the surface waves
β, volumetric coefficient of thermal expansion (K^{-1})
η, dimensionless similarity variable
θ, dimensionless temperature function
ψ, stream function (m^2 s^{-1})
μ, viscosity of the fluid (kg m^{-1} s^{-1})
μ_∞, viscosity of the ambient fluid
ν, kinematic viscosity (m^2 s^{-1})
ρ, density of the fluid (kg m^{-3})
σ_0, electrical conductivity
τ_w, shearing stress

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

KHK carried out the analytical calculations, prepared the data and figures, and drafted the manuscript. MAA prepared the programming code, did the numerical implementation, helped prepare the data and figures, and provided guidance at various stages of the study. LSA also provided guidance and advice to improve the quality of the manuscript. All authors read and approved the final manuscript.

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