Rajabi and Hamzavi *Journal of Theoretical and Applied Physics* 2013, **7**:17
http://www.jtaphys.com/content/7/1/17

Journal of Theoretical and Applied Physics
a SpringerOpen Journal

RESEARCH Open Access

A new Coulomb ring-shaped potential via generalized parametric Nikiforov-Uvarov method

Ali Akbar Rajabi and Majid Hamzavi*

Abstract

In this paper, the Schrödinger equation is analytically solved for the Coulomb potential with a novel angle-dependent part. The generalized parametric Nikiforov-Uvarov method is used to obtain energy eigenvalues and corresponding eigenfunctions. We presented the effect of the angle-dependent part on radial solutions and some special cases are also discussed.

Keywords: Schrödinger equation, Coulomb potential, Novel angle-dependent potential, Generalized parametric Nikiforov-Uvarov method

Introduction

Noncentral potentials have been studied in various fields of nuclear physics and quantum chemistry, which may be used to the interactions between the deformed pair of nuclei and ring-shaped molecules such as benzene [1-12]. There has been continuous interest in the solutions of the Schrödinger, Klein-Gordon, and Dirac equations for some central and noncentral potentials [13-41]. Yasuk et al. presented an alternative and simple method for the exact solution of the Klein-Gordon equation in the presence of noncentral equal scalar and vector potentials using the Nikiforov-Uvarov method [42]. A spherically harmonic oscillatory ring-shaped potential is proposed, and its exactly complete solutions are presented via the Nikiforov-Uvarov method by Zhang et al. [43]. Bayrak et al. [44] and Chen et al. [45] presented exact solutions of the Schrödinger equation with the Makarov potential using asymptotic iteration method and partial wave method. Souza Dutra and Hott solved the Dirac equation by constructing the exact bound state solutions for a mixing of vector and scalar generalized Hartmann potentials [46]. Kandirmaz et al., using path integral method, investigated the coherent states for a particle in the noncentral Hartmann potential [47]. Chen studied the Dirac equation with the Hartmann potential [48]. A kind of novel angle-dependent (NAD) potential is introduced by Berkdemir [49,50]:

$$V_{1\theta}(\theta) = \frac{\gamma + \beta\sin^2\theta + \eta\sin^4\theta}{\sin^2\theta\cos^2\theta}. \quad (1)$$

Hamzavi and Rajabi also solved the Dirac equation for Coulomb plus above the NAD potential when the scalar potential is equal to the vector potential [51]. Very recently, another kind of NAD potential is introduced by Zhang and Huang-Fu [52]:

$$V_{2\theta}(\theta) = \frac{\gamma + \beta\cos^2\theta + \eta\cos^4\theta}{\cos^2\theta\sin^2\theta}. \quad (2)$$

They solved the Dirac equation for oscillatory potential under a pseudospin symmetry limit. Therefore, the motivation of the present work is to solve the Schrödinger equation with the NAD Coulomb potential:

$$\begin{aligned} V(r,\theta) &= -\frac{A}{r} - \frac{\hbar^2}{2\mu}\frac{V_{2\theta}(\theta)}{r^2} \\ &= -\frac{A}{r} - \frac{\hbar^2}{2\mu}\frac{\gamma + \beta\cos^2\theta + \eta\cos^4\theta}{r^2\cos^2\theta\sin^2\theta}, \end{aligned} \quad (3)$$

where $A = Z\alpha$, $\alpha = \frac{e^2}{\hbar c}$ is the fine structure constant, and μ is the reduced mass. In this article, we solve Schrödinger equation with the NAD Coulomb potential (Equation 3)

* Correspondence: majid.hamzavi@gmail.com
Physics Department, Shahrood University of Technology, Shahrood, Iran

using the generalized parametric Nikiforov-Uvarov method, and we present the effect of the angle-dependent part on radial solutions.

Nikiforov-Uvarov method

To solve second-order differential equations, the Nikiforov-Uvarov method can be used with an appropriate coordinate transformation $s = s(r)$ [53,54]:

$$\psi_n''(s) + \frac{\widetilde{\tau}(s)}{\sigma(s)}\psi_n'(s) + \frac{\widetilde{\sigma}(s)}{\sigma^2(s)}\psi_n(s) = 0, \tag{4}$$

where $\sigma(s)$ and $\widetilde{\sigma}(s)$ are polynomials, at most, of second-degree, and $\widetilde{\tau}(s)$ is a first-degree polynomial. The following equation is a general form of the Schrödinger-like equation written for any potential [55,56]:

$$\left[\frac{d^2}{ds^2} + \frac{\alpha_1 - \alpha_2 s}{s(1-\alpha_3 s)}\frac{d}{ds} + \frac{-\xi_1 s^2 + \xi_2 s - \xi_3}{[s(1-\alpha_3 s)]^2}\right]\psi_n(s) = 0. \tag{5}$$

According to the Nikiforov-Uvarov method, the eigenfunctions and eigenenergy function become the following equations, respectively:

$$\psi(s) = s^{\alpha_{12}}(1-\alpha_3 s)^{-\alpha_{12}-\frac{\alpha_{13}}{\alpha_3}} P_n^{\left(\alpha_{10}-1,\frac{\alpha_{11}}{\alpha_3}-\alpha_{10}-1\right)}(1-2\alpha_3 s), \tag{6}$$

$$\begin{aligned}&\alpha_2 n - (2n+1)\alpha_5 + (2n+1)\left(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}\right)\\ &\quad + n(n-1)\alpha_3 + \alpha_7 + 2\alpha_3\alpha_8 + 2\sqrt{\alpha_8\alpha_9} = 0,\end{aligned} \tag{7}$$

where

$$\begin{aligned}&\alpha_4 = \frac{1}{2}(1-\alpha_1), && \alpha_5 = \frac{1}{2}(\alpha_2 - 2\alpha_3),\\ &\alpha_6 = \alpha_5^2 + \xi_1, && \alpha_7 = 2\alpha_4\alpha_5 - \xi_2,\\ &\alpha_8 = \alpha_4^2 + \xi_3 && \alpha_9 = \alpha_3\alpha_7 + \alpha_3^2\alpha_8 + \alpha_6,\end{aligned} \tag{8}$$

and

$$\begin{aligned}&\alpha_{10} = \alpha_1 + 2\alpha_4 + 2\sqrt{\alpha_8}, && \alpha_{11} = \alpha_2 - 2\alpha_5 + 2\left(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}\right),\\ &\alpha_{12} = \alpha_4 + \sqrt{\alpha_8}, && \alpha_{13} = \alpha_5 - \left(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}\right).\end{aligned} \tag{9}$$

In some problems $\alpha_3 = 0$. For this type of problems when

$$\lim_{\alpha_3\to 0} P_n^{\left(\alpha_{10}-1,\frac{\alpha_{11}}{\alpha_3}-\alpha_{10}-1\right)}(1-\alpha_3)s = L_n^{\alpha_{10}-1}(\alpha_{11}s), \tag{10}$$

and

$$\lim_{\alpha_3\to 0}(1-\alpha_3 s)^{-\alpha_{12}-\frac{\alpha_{13}}{\alpha_3}} = e^{\alpha_{13}s}, \tag{11}$$

the solution given in Equation 6 becomes as follows [55,56]:

$$\psi(s) = s^{\alpha_{12}} e^{\alpha_{13}s} L_n^{\alpha_{10}-1}(\alpha_{11}s). \tag{12}$$

Separating variables of the Schrödinger equation with the noncentral potential

In the spherical coordinates, the Schrödinger equation with noncentral potential can be written as follows [57]:

$$\begin{aligned}&-\frac{\hbar^2}{2\mu}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]\psi(r,\theta,\varphi)\\ &+V(r,\theta,\varphi)\psi(r,\theta,\varphi) = E\psi(r,\theta,\varphi).\end{aligned} \tag{13}$$

Let us decompose the spherical wave function as follows:

$$\psi(r,\theta,\varphi) = \frac{u(r)}{r}H(\theta)\phi(\varphi), \tag{14}$$

and also, substituting Equation 3 into Equation 13, we obtain the following equations:

$$\frac{d^2u(r)}{dr^2} + \left[\frac{2\mu}{\hbar^2}\left(E + \frac{A}{r}\right) - \frac{\lambda}{r^2}\right]u(r) = 0, \tag{15a}$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}\right)H(\theta) + \left[\lambda - \frac{m^2}{\sin^2\theta} - \frac{\gamma + \beta\cos^2\theta + \eta\cos^4\theta}{\cos^2\theta\sin^2\theta}\right]H(\theta) = 0, \tag{15b}$$

$$\frac{d^2\phi(\varphi)}{d\varphi^2} + m^2\phi(\varphi) = 0, \tag{15c}$$

where λ and m^2 are separation constants. It is well known that the solution of Equation 15c is as follows:

$$\Phi_m(\phi) = \frac{1}{\sqrt{2\pi}}e^{im\phi}m = 0, \pm1, \pm2, \ldots. \tag{16}$$

Solution of polar angle part

We are now going to derive eigenvalues and eigenfunctions of the polar part of the Schrödinger equation, i.e., Equation 15b, with generalized parametric Nikiforov-

Uvarov method. Using transformation $s = \cos^2\theta$, Equation 15b becomes the following:

$$\frac{d^2H(s)}{ds^2}+\frac{1-3s}{2s(1-s)}\frac{dH(s)}{ds}+\frac{1}{4s^2(1-s)^2}\left(\lambda s(1-s)-m^2s-\gamma-\delta s-\eta s^2\right)H(s)=0. \quad (17)$$

Comparing Equations 17 and 5, one obtains the following:

$$\begin{array}{ll}\alpha_1=1/2, & \xi_1=1/4(\lambda+\eta),\\ \alpha_2=3/2, & \xi_2=1/4(\lambda-m^2-\delta),\\ \alpha_3=1, & \xi_3=\gamma/4,\end{array} \quad (18)$$

and

$$\begin{array}{ll}\alpha_4=1/4, & \alpha_5=-1/4,\\ \alpha_6=1/16+1/4(\lambda+\eta), & \alpha_7=-1/8-1/4(\lambda-m^2-\delta),\\ \alpha_8=1/16+\frac{q}{4}, & \alpha_9=1/4(m^2+\gamma+\beta+\eta),\\ \alpha_{10}=1+\sqrt{\gamma+1/4}, & \alpha_{11}=2+\left(\sqrt{m^2+\gamma+\beta+\eta}+\sqrt{\gamma+1/4}\right),\\ \alpha_{12}=1/4+1/2\sqrt{\gamma+1/4}, & \alpha_{13}=-1/4-1/2\left(\sqrt{m^2+\gamma+\beta+\eta}+\sqrt{\gamma+1/4}\right).\end{array} \quad (19)$$

From Equations 18 and 19 and Equation 7, we obtain the following:

$$\lambda=4(\tilde{n}+1/2)^2+2(2\tilde{n}+1)\left[\sqrt{m^2+\gamma+\beta+\eta}+\sqrt{\gamma+1/4}\right]+2\sqrt{(m^2+\gamma+\beta+\eta)(\gamma+1/4)}+m^2+2\gamma+\beta, \quad (20)$$

where $\tilde{n}$ is a nonnegative integer. For the corresponding wave functions of the polar part, from non-negative Equations 6 and 19, we obtain the following:

$$H(s)=s^{\alpha_{12}}(1-\alpha_3 s)^{-\alpha_{12}-\frac{\alpha_{13}}{\alpha_3}}P_{\tilde{n}}^{\left(\alpha_{10}-1,\frac{\alpha_{11}}{\alpha_3}-\alpha_{10}-1\right)}(1-2\alpha_3 s)=s^{1/4+1/2\sqrt{\gamma+1/4}}(1-s)^{1/2\sqrt{m^2+\gamma+\beta+\eta}}P_{\tilde{n}}^{\left(\sqrt{\gamma+1/4},\sqrt{m^2+\gamma+\beta+\eta}\right)}(1-2s), \quad (21)$$

or equivalently

$$H(\theta)=C_{\tilde{n}m}(\cos\theta)^{1/2+\sqrt{\gamma+1/4}}(\sin\theta)^{\sqrt{m^2+\gamma+\beta+\eta}}P_{\tilde{n}}^{\left(\sqrt{\gamma+1/4},\sqrt{m^2+\gamma+\beta+\eta}\right)}(-\cos 2\theta), \quad (22)$$

where $C_{\tilde{n}m}$ is the normalization constant.

Solution of the radial equation

For eigenvalues and corresponding eigenfunctions of the radial part, i.e., solution of Equation 15a, we rewrite it as follows:

$$\frac{d^2u(r)}{dr^2}+\frac{1}{r^2}\left(-\varepsilon r^2+A'r-\lambda\right)u(r)=0 \quad (23)$$

where $\varepsilon=-\frac{2\mu}{\hbar^2}E$ and $A'=\frac{2\mu}{\hbar^2}A$.

Again, comparing Equations 23 and 5 leads to the following:

$$\begin{array}{ll}\alpha_1=0, & \xi_1=\varepsilon,\\ \alpha_2=0, & \xi_2=A',\\ \alpha_3=0, & \xi_3=\lambda,\end{array} \quad (24)$$

and

$$\begin{array}{ll}\alpha_4=1/2, & \alpha_5=0,\\ \alpha_6=\varepsilon, & \alpha_7=-A',\\ \alpha_8=\lambda+1/4, & \alpha_9=\varepsilon,\\ \alpha_{10}=1+\sqrt{\lambda+1/4}, & \alpha_{11}=2\sqrt{\varepsilon},\\ \alpha_{12}=1/2+\sqrt{\lambda+1/4}, & \alpha_{13}=-\sqrt{\varepsilon}.\end{array} \quad (25)$$

The energy eigenvalues of the radial part can be obtained from Equations 24 and 25 and Equation 7 as follows:

$$E=-\frac{2\mu A^2}{\hbar^2\left(2n+1+2\sqrt{\lambda+1/4}\right)^2} \quad (26)$$

where n is the nonnegative integer. Although one can immediately obtain energy eigenvalues of Equations 15a or 23 from hydrogen problem, here, we have tried the Nikiforov-Uvarov method to show the simplicity of usage of the mentioned method. To find the corresponding radial eigenfunctions, we refer to Equations 11 and 25, and then we obtain the following:

$$u(r)=s^{\alpha_{12}}e^{\alpha_{13}s}L_n^{\alpha_{10}-1}(\alpha_{11}s)=r^{1/2+\sqrt{\lambda+1/4}}e^{-\sqrt{\varepsilon}r}L_n^{\sqrt{\lambda+1/4}}\left(2\sqrt{\varepsilon}r\right). \quad (27)$$

For effect of the angle-dependent part on radial solutions, we substitute Equation 20 into Equation 27, and we obtain the following:

$$E_{n,\tilde{n},m}=-\frac{2\mu(Z\alpha)^2}{\hbar^2\left(2n+1+2\sqrt{\begin{array}{c}\{4(\tilde{n}+1/2)^2+2(2\tilde{n}+1)\left[\sqrt{m^2+\gamma+\beta+\eta}+\sqrt{\gamma+1/4}\right]\\+2\sqrt{(m^2+\gamma+\beta+\eta)(\gamma+1/4)}+m^2+2\gamma+\beta+1/4\}\end{array}}\right)^2}. \quad (28)$$

When $\gamma = \eta = 0$, the potential (Equation 3) reduces to the Hartmann potential, and the energy eigenvalues can be obtained as follows [58]:

$$E_{n,\tilde{n},m} == -\frac{\mu(Z\alpha)^2}{2\hbar^2\left(n+\sqrt{m^2+\beta}+\tilde{n}+1\right)^2}. \tag{29}$$

Also, when $\gamma = \beta = \eta = 0$, the potential (Equation 3) reduces to the Coulomb potential, and the energy eigenvalues in Equation 28 reduces to the following [57]:

$$E_{(\text{Coulomb})} == -\frac{\mu(Z\alpha)^2}{2\hbar^2 n'^2} = -\frac{1}{2}\frac{\mu}{c^2}\frac{Z^2e^4}{n'^2}, \tag{30}$$

where $n\prime = n + l + 1$ and $l = 2\tilde{n} + m + 1$.

Finally, we can write $\psi(r, \theta, \varphi)$ as follows:

$$\begin{aligned}\psi(r,\theta,\varphi) &= \frac{u(r)}{r}H(\theta)\Phi(\phi)\\ &= \frac{C_{n\tilde{n}m}}{\sqrt{2\pi}}r^{-\frac{1}{2}+\sqrt{\lambda+\frac{1}{4}}}e^{-\sqrt{\varepsilon^2}r}L_n^{2\sqrt{\lambda+\frac{1}{4}}}\left(2\sqrt{\varepsilon^2}r\right)\\ &\times(\cos\theta)^{1/2+\sqrt{\gamma+1/4}}(\sin\theta)^{\sqrt{m^2+\gamma+\beta+\eta}}\\ &\quad P_{\tilde{n}}^{\left(\sqrt{\gamma+1/4},\sqrt{m^2+\gamma+\beta+\eta}\right)}(-\cos2\theta)e^{im\phi}.\end{aligned} \tag{31}$$

where $C_{\tilde{n}m}$ is the normalization constant.

Conclusions

We have studied the exact solutions of the Schrödinger equation with the Coulomb plus, a novel angle-dependent potential, using the generalized parametric Nikiforov-Uvarov method. It can be found that this method is a powerful mathematical tool for solving second-order differential equations. The bound-state energy eigenvalues and the corresponding wave functions are obtained. We point that these results may have interesting applications in the study of different quantum mechanical systems and atomic physics [1,2,59,60] and, two special cases, i.e., Hartmann potential and pure Coulomb potential, were also discussed.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

AAR and MH carried out all the analysis, designed the study, and drafted the manuscript together. Both authors read and approved the final manuscript.

Acknowledgments

The authors thank the kind referees for their positive and invaluable suggestions, which improved this article greatly.

Received: 1 January 2013 Accepted: 12 February 2013
Published: 4 April 2013

References

1. Hartmann, H: Die Bewegung eines Korpers in einem ringformigen Potentialfeld. Theor. Chim. Acta **24**, 201 (1972)
2. Hartmann, H, Schuck, R, Radtke, J: Die diamagnetische Suszeptibilität eines nicht kugel- symmetrischen systems. Theor. Chim. Acta **46**, 1 (1976)
3. A systematic search for nonrelativistic systems with dynamical symmetries. Nuovo Cimento A **52**, 1061 (1967)
4. Khare, A, Bhaduri, RK: Exactly solvable noncentral potentials in two and three dimensions. Am. J. Phys. **62**, 1008 (1994)
5. Zhang, XA, Chen, K, Duan, ZL: Bound states of Klein-Gordon equation and Dirac equation for ring-shaped non-spherical oscillator scalar and vector potentials. Chin. Phys. **14**, 42 (2005)
6. Dong, SH, Sun, GH, Lozada-Cassou, M: An algebraic approach to the ring-shaped non-spherical oscillator. Phys. Lett. A **328**, 299 (2005)
7. Chen, CY, Dong, SH: Exactly complete solutions of the Coulomb potential plus a new ring-shaped potential. Phys. Lett. A **335**, 374 (2005)
8. Alhaidari, AD: Scattering and bound states for a class of non-central potentials. J. Phys. A: Math. Gen. **38**, 3409 (2005)
9. Hamzavi, M, Amirfakhrian, M: Dirac equation for a spherically pseudoharmonic oscillatory ring-shaped potential. Int. J. Phys. Sci. **6**, 3803 (2011)
10. Dong, SH, Chen, CY, Lozada-Cassou, M: Quantum properties of complete solutions for a new non-central ring-shaped potential. Int. J. Quant. Chem. **105**, 453 (2005)
11. Dong, SH, Lozada-Cassou, M: Exact solutions of the Klein-Gordon equation with scalar and vector ring-shaped potentials. Phys. Scr. **74**, 285 (2006)
12. Kibler, M, Mardoyan, LG, Pogosyan, GS: On a generalized Kepler-Coulomb system: interbasis expansions. Int. J. Quantum Chem. **52**, 1301 (1994)
13. Gereiner, W: Relativistic Quantum Mechanics. Wave Equations, Springer, Third Edition (2000)
14. Quense, C: Supersymmetry and the Dirac oscillator. Int. J. Mod. Phys. A **6**, 1567 (1991)
15. Sutherland, B: Exact coherent states of a one-dimensional quantum fluid in a time-dependent trapping potential. Phys. Rev. Lett. **80**, 3678 (1998)
16. Chen, CY, Lu, FL, Sun, DS: Relativistic scattering states of coulomb potential plus a new ring-shaped potential. Commun. Theor. Phys. **45**, 889 (2006)
17. Quesne, C: A new ring-shaped potential and its dynamical invariance algebra. J. Phys. A: Math. Gen. **21**, 3093 (1988)
18. Zhang, MC, Wang, ZB: Exact solutions of the Klein-Gordon equation with a new anharmonic oscillator potential. Chin. Phys. Lett. **22**, 2994 (2005)
19. Yasuk, F, Berkdemir, C, Berkdemir, A: Exact solutions of the Schrödinger equation with non-central potential by the Nikiforov-Uvarov method. J. Phys. A: Math. Gen. **38**, 6579 (2005)
20. Jia, CS, Guo, P, Peng, XL: Exact solution of the Dirac-Eckart problem with spin and pseudospin symmetry. J. Phys. A: Math. Gen. **39**, 7737 (2006)
21. Qiang, WC, Zhou, RS, Gao, Y: Application of the exact quantization rule to the relativistic solution of the rotational Morse potential with pseudospin symmetry. J. Phys. A: Math. Theor. **40**, 1677 (2007)
22. Arda, A, Sever, R, Tezcan, C: Approximate pseudospin and spin solutions of the Dirac equation for a class of exponential potentials. Chinese J. Phys. **48**, 27 (2010)
23. Soylu, A, Bayrak, O, Boztosun, I: An approximate solution of Dirac-Hulthén problem with pseudospin and spin symmetry for any κ state. J. Math. Phys. **48**, 082302 (2007)
24. Dong, SH, Gu, XY: Arbitrary *l* state solutions of the Schrödinger equation with the Deng-Fan molecular potential. J. Phys.: Conf. Ser. **96**, 012109 (2008)
25. Wei, GF, Dong, SH: Approximately analytical solutions of the Manning-Rosen potential with the spin-orbit coupling term and spin symmetry. Phys. Lett. A **373**, 49 (2008)
26. Soylu, A, Bayrak, O, Boztosun, I: κ State solutions of the Dirac equation for the Eckart potential with pseudospin and spin symmetry. J. Phys. A: Math. Theor. **41**, 065308 (2008)
27. Xu, Y, He, S, Jia, CS: Reply to 'Comment on 'Approximate analytical solutions of the Dirac equation with the Pöschl-Teller potential including spin-orbit coupling'. J. Phys. A: Math. Theor. **42**, 198002 (2009)
28. Hamzavi, M, Hassanabadi, H, Rajabi, AA: Exact solutions of Dirac equation with Hartmann potential by Nikiforov-Uvarov method. Int. J. Mod. Phys. E **19**, 2189 (2010)
29. Liu, XY, Wei, GF, Long, CY: Arbitrary wave relativistic bound state solutions for the Eckart potential. Int. J. Theor. Phys. **48**, 463 (2009)

30. Chen, T, Diao, YF, Jia, CS: Bound state solutions of the Klein-Gordon equation with the generalized Pöschl-Teller potential. Phys. Scr. **79**, 065014 (2009)
31. Aydoğdu, O, Sever, R: Approximate analytical solutions of the Klein-Gordon equation for the Hulthén potential with the position-dependent mass. Phys. Scr. **79**, 015006 (2009)
32. Ikhdair, SM: Approximate solutions of the Dirac equation for the Rosen-Morse potential including the spin-orbit centrifugal term J. Math. Phys. **51**, 023525 (2010)
33. Hamzavi, M, Rajabi, AA, Hassanabadi, H: Exact spin and pseudospin symmetry solutions of the Dirac equation for Mie-type potential including a coulomb-like tensor potential. Few-Body Syst. **48**, 171 (2010)
34. Berkdemir, C, Sever, R: Pseudospin symmetry solution of the Dirac equation with an angle-dependent potential. J. Phys. A: Math. Theor. **41**, 045302 (2008)
35. Hamzavi, M, Hassanabadi, H, Rajabi, AA: Exact solution of Dirac equation for Mie-type potential by using the Nikiforov-Uvarov method under the pseudospin and spin symmetry limit. Mod. Phys. Lett. A **25**, 2447 (2010)
36. Kandirmaz, N, Sever, R: Coherent states for PT-/non-PT-symmetric and non-Hermitian Morse potentials via the path integral method. Phys. Scr. **81**, 035302 (2010)
37. Hu, XQ, Luo, G, Wu, ZM, Niu, LB, Ma, Y: Solving Dirac equation with new ring-shaped non-spherical harmonic oscillator potential. Commun. Theor. Phys. **53**, 242 (2010)
38. Sun, GH, Dong, SH: New type shift operators for three-dimensional infinite well potential. Mod. Phys. Lett A **26**, 351 (2011)
39. Qiang, WC, Dong, SH: The rotation-vibration spectrum for Scarf II potential. Int. J. Quan. Chem. **110**, 2342 (2010)
40. Dong, SH, Garcia-Ravelo, J: Exact solutions of the Schrodinger equation with the Pöschl-Teller like potential. Mod. Phys. Lett. B **23**, 603 (2009)
41. Dong, SH, Gonzalez-Cisneros, A: Energy spectra of the hyperbolic and second Pöschl-Teller like potentials solved by new exact quantization rule. Ann. Phys. **323**, 1136 (2008)
42. Yasuk, F, Durmus, A, Boztosun, I: Exact analytical solution to the relativistic Klein-Gordon equation with noncentral equal scalar and vector potentials. J. Math. Phys. **47**, 082302 (2006)
43. Zhang, MC, Sun, GH, Dong, SH: Exactly complete solutions of the Schrodinger equation with a spherically harmonic oscillatory ring-shaped potential. Phys. Lett. A **374**, 704 (2010)
44. Bayrak, O, Karakoc, M, Boztosun, I, Sever, R: Analytical solution of the Schrödinger equation for Makarov potential with any ℓ angular momentum. Int. J. Theor. Phys. **47**, 3005 (2008)
45. Chen, CY, Liu, CL, Lu, FL: Exact solutions of Schrödinger equation for the Makarov potential. Phys. Lett. A **374**, 1346 (2010)
46. de Souza Dutra, A, Hott, M: Dirac equation exact solutions for generalized asymmetrical Hartmann potentials. Phys. Lett. A **356**, 215 (2006)
47. Kandirmaz, N, Ünal, N: Coherent states for the Hartmann potential. Theor. Math. Phys. **155**, 884 (2008)
48. Chen, CY: Exact solutions of the Dirac equation with scalar and vector Hartmann potentials. Phys. Lett. A **339**, 283 (2005)
49. Berkdemir, C: A novel angle-dependent potential and its exact solution. J. Math. Chem. **46**, 139 (2009)
50. Berkdemir, C, Cheng, YF: On the exact solutions of the Dirac equation with a novel angle-dependent potential. Phys. Scr. **79**, 034003 (2009)
51. Hamzavi, M, Rajabi, AA: Exact solutions of the Dirac equation with Coulomb plus a novel angle-dependent potential. Z. Naturforsch **66a**, 533 (2011)
52. Zhang, MC, Huang-Fu, GQ: Pseudospin symmetry for a new oscillatory ring-shaped noncentral potential. J. Math. Phys. **52**, 053518 (2011)
53. Nikiforov, AF, Uvarov, VB: Special Functions of Mathematical Physic. Birkhausr, Berlin (1988)
54. Miranda, MG, Sun, GH, Dong, SH: The solution of the second Pöschl-Teller like potential by Nikiforov-Uvarov method. Int. J. Mod. Phys. E. **19**, 123 (2010)
55. Tezcan, C, Sever, R: A general approach for the exact solution of the Schrödinger equation. Int. J. Theor. Phys. **48**, 337 (2009)
56. Ikhdair, SM: Bound states of the Klein-Gordon equation for vector and scalar general Hulthén-type potentials in D-dimension. Int. J. Mod. Phys. C **20**(25), 25 (2009)
57. Dong, SH: Factorization Method in Quantum Mechanics. Springer, Netherlands (2007)
58. Yasuk, F, Berkdemir, C, Sever, R: Exact solutions of the Schrödinger equation via Laplace transform approach: pseudoharmonic potential and Mie-type potentials. J. Math. Chem. **50**, 971 (2012)
59. Carpio-Bernido, MV, Bernido, CC: An exact solution of a ring-shaped oscillator plus a potential. Phys. Lett. A **134**, 315 (1989)
60. Ramos Jr, RC, Bernido, CC, Carpio-Bernido, MV: Path-integral treatment of ring-shaped topological defects. J. Phys. A: Math. Gen. **27**, 8251 (1994)

doi:10.1186/2251-7235-7-17
Cite this article as: Rajabi and Hamzavi: **A new Coulomb ring-shaped potential via generalized parametric Nikiforov-Uvarov method.** *Journal of Theoretical and Applied Physics* 2013 **7**:17.