Journal of Theoretical and Applied Physics
a SpringerOpen Journal

RESEARCH Open Access

New approach to nuclear binding energy in integrated nuclear model

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Abstract

In this paper, the integrated nuclear model is introduced, and a binding energy formula based on this model is presented. The binding energies of all nuclides in this model are compared with available experimental values and also with values from liquid drop model.

Keywords: Nuclear model, Binding energy, Nuclides

Background

One of the purposes of the nuclear physics is to introduce the proper mathematical models from which the properties and the behavior of nuclides can be explained. One of the outstanding features of the nuclides is the fact that their nuclear density is approximately constant. Therefore, the volume of nuclide is proportional to the mass number A. The same proportionality holds for liquids, and one of the early fundamental nuclear models presented by Carl Friedrich Von Weizsäcker [1] and developed by Niels Bohr and John Archibald Wheeler [2] was based upon liquid drops. Nuclides are considered as incompressible liquid drops with enormous density. Based upon the fact that the average binding energy per nucleon and the nuclear density are constant, Weizsäcker was able to present his liquid drop model with the following basic assumptions:

(1) The nuclides are made of incompressible matter so that $R \propto A^{1/3}$ (R is the mean nuclear radius).
(2) The nuclear force is the same for each nucleon.
(3) The nuclear force saturates.

The liquid drop model led to the famous semi-empirical mass formula from which the dependency of nuclear mass upon A and Z is given [3]. First, the nucleus is considered as a collection of interacting particles like a liquid drop. Then, the Coulomb force, the Pauli exclusion principle effect, and other details are added to the model as corrections, and finally, the following formula is derived for nuclear binding energy:

$$B(A,Z) = a_v A - a_S A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_a (N-Z)^2 A^{-1} \pm \delta + \eta \tag{1}$$

In the liquid drop model, nucleons are not described individually; they are considered as averaged values. Therefore, this model has been successful in describing some properties of nuclei such as average binding energy per nucleon, whereas for other nuclear properties such as nuclear excited states, magic numbers and nuclear magnetic moments have not so much to present.

The nuclear properties can be described simply in terms of free particle behavior instead of strongly interacting particles as viewed in the liquid drop model. If nuclide is considered as a degenerate Fermi gas of nucleons (Fermi gas model), then a nuclear free particle model is obtained. In this model, it is assumed that nucleons are freely (except under the Pauli exclusion constraint) moving within a nuclide with radius $R = R_0 A^{1/3}$. Using the quantum tunneling theory and Pauli exclusion principle, one can find the average kinetic energy of the nucleons within the nuclide as follows:

$$E(Z,N) = \frac{3}{10m}\frac{\hbar^2}{R_0^2}\left(\frac{9\pi}{8}\right)^{2/3}\left\{A + \frac{5}{9}\left(\frac{Z-N}{A}\right)^2 + \cdots\right\}. \tag{2}$$

In contrast to the liquid drop model and Fermi gas model in which the macroscopic properties of nuclei are

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presented, the nuclear shell model [4-6] deals with the microscope properties of nuclei. The nucleons as free particles moving in a spherical potential and also the Pauli exclusion principle intensively limit the interaction between the nucleons. Such consideration in the shell model provides orbits with approximate stability and defined energy levels. The fundamental assumption in a nuclear shell model is the independence of nucleon motions (free particles) regardless of the existence of strong attractive force between the nucleons. With these assumptions, it is predictable that such model is able to describe nuclear microscopic properties such as excited state energy, magic number, and nuclear magnetic moments, but it is important to provide a nuclear binding energy formula.

In this paper, it is attempted to present an integrated new clear model and a new formula for binding energy of all nuclides based upon intuitive assumptions that will be presented in the next section.

Nuclear binding energy in integrated nuclear model

In general, the total mass of nuclei (Z,N) is less than the sum of the masses of its constituent particles namely protons and neutrons. This mass difference is defined as nuclear binding energy. In 1966, Garvey and Kelson presented a formula for the nuclear binding energy [7,8]. Since the nuclear energy possesses saturation property, therefore, it is possible for the mass difference between two neighboring nuclides of (Z_0,N_0) and (Z,N), namely $M(Z,N) - M(Z_0,N_0)$, to be expanded as power series in terms of $\Delta Z = Z - Z_0$ and $\Delta N = N - N_0$. Consequently, it is possible to write the following formula for the nuclear binding energy [9,10]:

$$B(Z,N) = B(Z_0,N_0) + B_{10}\Delta Z + B_{01}\Delta N + B_{20}(\Delta Z)^2 + B_{02}(\Delta N)^2 + B_{11}(\Delta Z)(\Delta N) + \cdots, \quad (3)$$

where the coefficients B_{10}, B_{01}, ... are the partial derivatives of $B(Z,N)$ with respect to $(Z,N) = (Z_0,N_0)$. A good approximation is to neglect the second and higher order derivatives in series (Equation 3). Then, considering a linear relation for the binding energy, we will have the following two formulas [11,12]:

$$B(Z,N) = g_1(Z) + g_2(N) + g_3(N+Z), \quad (4)$$

$$B(Z,N) = f_1(Z) + f_2(N) + f_3(N-Z). \quad (5)$$

Now, we may use the mentioned models, namely liquid drop, Fermi gas, and shell models in addition to relations 1, 2, 4, and 5 to express our fundamental assumptions in order to present a new formula for the nuclear binding energy:

(1) The nuclear binding energy is of the order of 1% of the energy of the total rest mass of the constituent nucleons [11].
(2) The nuclear binding energy is proportional to the volume of the nuclide ($B \propto A$).
(3) The nuclear binding energy depends upon the asymmetry between the number of protons and neutrons (specially in heavy nuclides) and also depends upon the coulomb repulsion force between protons.

From the conditions of relations 4 and 5, it is noticed that the binding energy is proportional to both $(N+Z)$ and $(N-Z)$. Therefore, a term $\frac{N^2-Z^2}{Z}$ appears for the nucleon asymmetry and coulomb correction in the third assumption.

Based upon the above assumptions, the following formula is presented for the nuclear binding energy of all elements:

$$B(Z,N) = \left\{\left(\frac{3^2}{3^2+1}\right)\left[A - \left(\frac{(N^2-Z^2)+\delta(N-Z)}{Z} + 3\right)\right] x \frac{m_N C^2}{100}\ A > 5 \quad (6)$$

where δ stands for nuclear beta-stability line condition and is defined as follows:

$$\delta(N-Z) = \left\{\begin{matrix} 0 & for\ N \neq Z \\ 1 & for\ N = Z \end{matrix}\right\}. \quad (7)$$

The factor 0.9 in front of Equation 6 will be explained in next the section.

In Table 1, the nuclear binding energy for all nuclides is given using Equation 6 and has been compared with the results of liquid drop models (LDMs) and with experimental results. The nuclear binding energies per nucleon obtained using Equation 6 are in good agreement with the existing experimental data and also with LDM for all mass numbers as shown in Figures 1, 2, 3.

Discussion and conclusion

The constant factor $\left(\frac{3^2}{3^2+1}\right)$ in binding energy (Equation 6) may be explained in two different contexts. One has to do with the defined nuclear region [12] in which the density remains constant. In other words, it is assumed that in about 10% of outer nuclear region, the density is no longer constant and falls rapidly and is ignored in the integrated model. The other context has to do with the 3^n law for $n = 2$ as stated in the quark plasma nuclear model [13,14] due to the fact that each nucleon is made of 3 quarks and due to the existence of a new threefold symmetry in this

Table 1 Nuclear binding energy per nucleon

Nucleus	Z	A	B/A (our model, MeV)	B/A (LDM, MeV)	B/A (EXP, MeV)
H	1	3	3.29123	0.6105	2.827
He	2	4	7.40526	5.4863	7.074
Li	3	6	4.93684	4.60667	5.33233
Li	3	7	4.54503	5.48336	5.60629
Be	4	9	5.75965	6.2924	6.46278
B	5	10	6.91158	6.30939	6.4751
B	5	11	6.52262	6.82388	6.92773
C	6	12	7.40526	7.31242	7.68017
C	6	13	7.0466	7.20223	7.46985
N	7	14	7.75789	7.11861	7.47564
N	7	15	7.42877	7.48535	7.69947
O	8	16	8.02237	7.73211	7.97619
O	8	17	7.71987	7.70438	7.75076
F	9	19	7.94899	7.87776	7.779
Ne	10	21	8.13404	8.01728	7.97171
Na	11	23	8.28661	8.17431	8.11148
Mg	12	25	8.41457	8.22397	8.22352
Mg	12	26	8.18587	8.35642	8.33388
Al	13	27	8.52344	8.30071	8.33156
Si	14	29	8.61718	8.36406	8.44866
Si	14	30	8.41614	8.48917	8.52067
P	15	31	8.69875	8.4163	8.48119
S	16	34	8.59107	8.57741	8.5835
Cl	17	37	8.49231	8.58581	8.5703
Ar	18	38	8.72849	8.6344	8.61429
K	19	41	8.63155	8.64551	8.57607
Ca	20	43	8.69114	8.66599	8.60067
Sc	21	45	8.74526	8.68134	8.61884
Ti	22	47	8.79465	8.69223	8.66113
Ti	22	48	8.65817	8.74848	8.72292
V[a]	23	50	8.70888	8.69335	8.69588
Cr	24	52	8.75551	8.76065	8.77594
Mn	25	55	8.67687	8.74901	8.765
Fe	26	56	8.83839	8.75985	8.79032
Fe	26	57	8.72109	8.75142	8.77026
Co	27	59	8.76215	8.75084	8.76802
Ni	28	61	8.80037	8.7476	8.76502
Cu	29	63	8.83606	8.74199	8.75214
Cu	29	65	8.62354	8.76012	8.75711
Zn	30	66	8.76663	8.77127	8.75964
Ga	31	69	8.70121	8.75196	8.72458
Ge	32	70	8.83342	8.75222	8.72173
As	33	75	8.58113	8.73985	8.70085
Se	34	76	8.70953	8.74887	8.71149
Br	35	79	8.65242	8.72461	8.68761
Kr	36	80	8.77204	8.72447	8.69293
Rb	37	85	8.54673	8.70027	8.69745
Sr	38	84	8.82816	8.69437	8.67745
Sr	38	86	8.66314	8.70634	8.70847
Sr	38	88	8.49775	8.69804	8.7326
Y	39	89	8.61257	8.67965	8.71391
Zr	40	90	8.72175	8.67799	8.70992
Nb	41	93	8.67217	8.65307	8.66414
Mo	42	94	8.77494	8.64476	8.6623
Mo	42	95	8.6999	8.63786	8.64868
Ru	44	100	8.67987	8.61943	8.61928
Ru	44	101	8.608	8.60868	8.60129
Rh	45	103	8.6353	8.59314	8.58411
Pd	46	105	8.66145	8.57653	8.57061
Pd	46	106	8.59256	8.58669	8.57994
Ag	47	107	8.68651	8.55892	8.55386
Cd	48	110	8.64446	8.55271	8.55133
Cd	48	111	8.57832	8.53359	8.53714
In	49	113	8.60403	8.52577	8.52296
Sn	50	115	8.62874	8.50802	8.5141
Sn	50	116	8.56514	8.51632	8.52314
Sb	51	121	8.40274	8.4768	8.48202
Te	52	122	8.49162	8.47611	8.47814
I	53	127	8.33638	8.4321	8.44549
Xe	54	126	8.54152	8.44149	8.44372
Cs	55	133	8.27464	8.38604	8.40998
Ba	56	132	8.47384	8.39985	8.40938
Ba	56	134	8.35965	8.38827	8.40818
La[a]	57	138	8.27326	8.3425	8.37517
La	57	139	8.21706	8.33888	8.37806
Ce	58	138	8.41064	8.35613	8.37707
Pr	59	141	8.38058	8.32701	8.35404
Nd	60	143	8.40491	8.31014	8.33053
Nd[a]	60	144	8.35149	8.31093	8.32697
Sm	62	149	8.34778	8.26527	8.26351
Sm	62	150	8.29602	8.2647	8.26167
Eu	63	153	8.26956	8.2354	8.22875
Gd	64	155	8.29409	8.21945	8.2133
Gd	64	156	8.24389	8.21778	8.21537
Tb	65	159	8.21899	8.18856	8.18885
Dy	66	160	8.29227	8.18534	8.18409
Dy	66	161	8.24356	8.17299	8.17335
Ho	67	165	8.17136	8.14135	8.14701
Er	68	167	8.1959	8.12611	8.13178
Tm	69	169	8.21974	8.11017	8.11451
Yb	70	173	8.15089	8.079	8.08746
Lu[a]	71	176	8.1293	8.05062	8.05906
Hf	72	179	8.1083	8.0318	8.0386
Hf	72	180	8.06351	8.02743	8.03498

Table 1 Nuclear binding energy per nucleon *(Continued)*

Ta	73	181	8.13205	8.01643	8.02343
W	74	186	8.02434	7.97987	7.98861
Re[a]	75	187	8.09161	7.96947	7.97795
Os	76	192	7.98718	7.93248	7.94852
Ir	77	193	8.05322	7.92257	7.93812
Pt	78	198	7.95188	7.88531	7.91418
Au	79	197	8.09854	7.89091	7.91566
Hg	80	204	7.91831	7.83841	7.88555
Tl	81	205	7.982	7.82921	7.8784
Pb	82	208	7.96525	7.80973	7.86746
Bi[a]	83	209	8.02686	7.79827	7.84799
Th[a]	90	232	7.84441	7.62579	7.61503
U[a]	92	234	7.95839	7.60908	7.60071
U[a]	92	235	7.92315	7.59957	7.59091
U[a]	92	238	7.81742	7.58055	7.57013

Nuclear binding energy for most of the known nuclei in our model, LDM, and experimental values. LDM, liquid drop models; EXP, experimental values.

model. Attention should be paid to the fact that for $A < 5$ such as ${}^{4}_{2}He$ and ${}^{3}_{2}H$, the factor 3 in the second term of binding energy of Equation 6 changes to 1, and for other light nuclei, our given formula needs minor correction due to the facts that for these light nuclides at least two nucleons should participate, and the spherical distribution of the nucleons inside the nuclide changes, the problem that exists in other models too.

The semi-empirical Equation 1, based upon only liquid drop model, contains at least five terms to be calculated, whereas in our Equation 6, only two terms are calculated. Careful consideration of Table 1 and Figures 2 and 3 reveals the meaningful accuracy of our integrated model compared to liquid drop model with respect to experimental data (Figure 1). Special features of the experimental diagram such as having maximum value for Fe and its local extrema coincide with the calculated values from Equation 6. The binding energy in Equation 6 is extracted from various existing models and that is why it is called integrated model. In this model, the constituent nuclear particles are considered 'free' in a dense plasma-type media. It is interesting that in such plasma model of nuclei, based on a statistical view, all the magic numbers and the new magic number, namely 184, are also obtained with no spherical potential and spin-orbit coupling assumptions [13,14]. Here, attempts are made to conceptualize an integrated nuclear model capable of

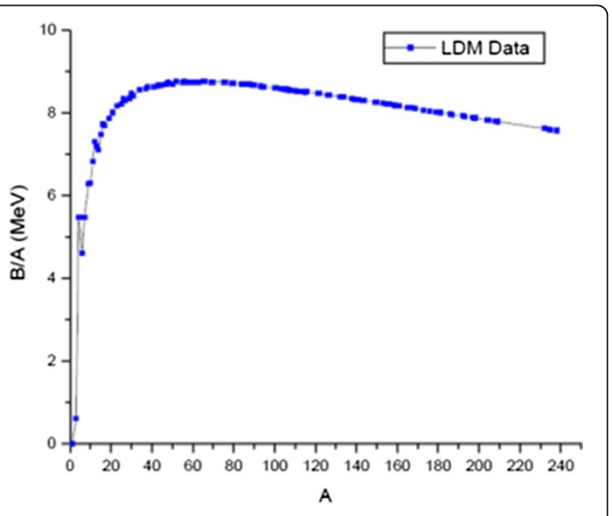

Figure 2 LDM data. Data of nuclear binding energy per nucleon in terms of mass number for most of the known stable nuclei.

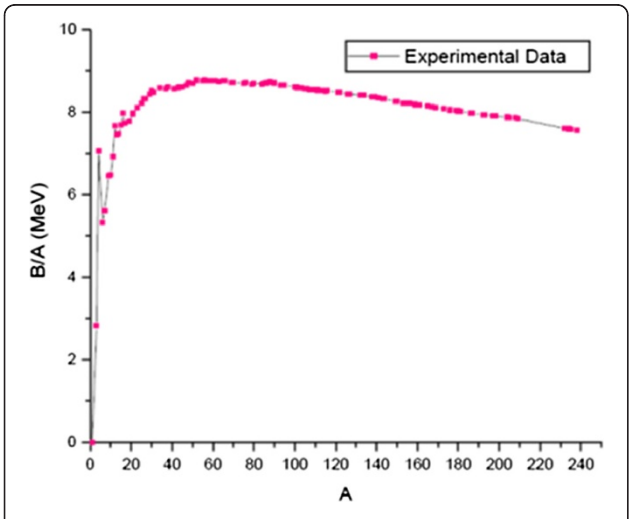

Figure 1 Experimental data. Data of nuclear binding energy per nucleon in terms of mass number for most of the known stable nuclei.

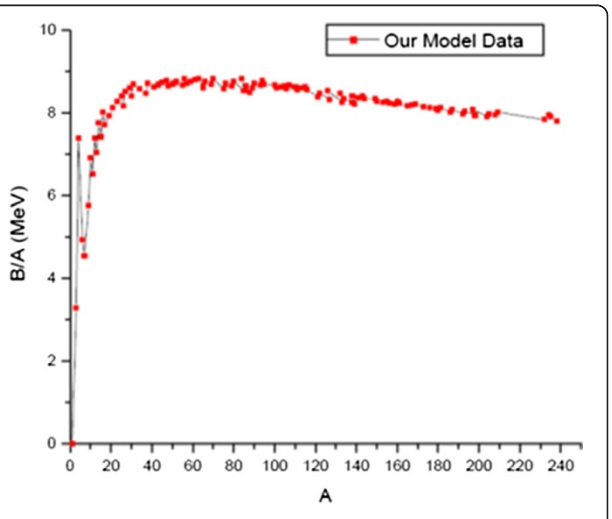

Figure 3 Our integrated model data. Data of nuclear binding energy per nucleon in terms of mass number for most of the known stable nuclei.

providing all nuclear characteristics such as binding energy per nucleon, magic number, excited states, and magnetic moments. Such concepts may lead us to understand a realistic picture of nuclei.

We believe the results obtained from the integrated model is not only simple to understand but also more physical and relatively closer to the experimental data than other models. Other characteristics of nuclei are being studied in the framework of the integrated model in our group.

Competing interests
The authors did not provide this information.

Authors' contributions
The authors did not provide this information.

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Received: 28 March 2012 Accepted: 28 March 2012
Published: 2 July 2012

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doi:10.1186/2251-7235-6-3
Cite this article as: Ghahramany *et al.*: New approach to nuclear binding energy in integrated nuclear model. *Journal of Theoretical and Applied Physics* 2012 **6**:3.