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The generalized second law of thermodynamics in $f(\mathcal{R})$ gravity for various choices of scale factor

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Abstract

The present study is aimed at investigating the validity of the generalized second law (GSL) of thermodynamics in $f(\mathcal{R})$ gravity. Choosing $f(\mathcal{R}) = \mathcal{R} + \xi\mathcal{R}^{\mu} + \zeta\mathcal{R}^{-\nu}$ with $\xi,\ \mu,\ \zeta,\ \nu > 0$ (following the study of Nojiri and Odintsov in 2003), we have computed the time derivatives of total entropy for various choices of scale factor pertaining to emergent, intermediate, and logamediate scenarios of the universe. We have taken into account the radii of Hubble, apparent, particle, and event horizons while computing the time derivatives of entropy under various situations being considered. After analyzing through the plots of time derivative of total entropy against cosmic time, it is observed that the derivative always stays at positive level. This indicates the validity of GSL of thermodynamics in the $f(\mathcal{R})$ gravity irrespective of the choices of scale factor and enveloping horizon.

Keywords: $f(\mathcal{R})$ gravity, GSL of thermodynamics, Scale factor

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Background

Accelerated expansion of the universe is well documented in literature (detailed discussion is available in [1] and references therein). Approximately 76% of the energy content of the universe is not dark or luminous matter, but it is instead a mysterious form of dark energy that is exotic, invisible, and unclustered [2]. In order to explain the accelerated expansion of the universe, three main classes of model exist [3]: (1) a cosmological constant Λ, (2) dark energy, and (3) modified gravity. The last class of models known as extended theories of gravity corresponds to the modification of the action of the gravitational fields [3]. These theories are based on the idea of an extension of the Einstein-Hilbert action by adding higher order curvature invariants. Modified gravity theories have been reviewed in [4-6]. Nojiri and Odintsov [7] suggested $f(\mathcal{R})$ gravity characterized by the presence of effective cosmological constant epochs in such a way that early time inflation and late time cosmic acceleration are mutually unified within a single model. In another work, Nojiri and Odintsov [8] proposed another class of modified $f(\mathcal{R})$ to unify $\mathcal{R}^m$ inflation with Λ-cold dark matter era. Chattopadhyay and Debnath [9] considered $f(\mathcal{R})$ gravity in an universe characterized by a special form scale factor known as emergent scenario and observe and concluded that the EoS parameter behaves like quintessence in this situation.

The relevance of studying the thermodynamics of the universe was precisely mentioned in [10] that investigated the validity of generalized second law (GSL) in $f(\mathcal{R})$ gravity. In the present work are going to investigate the GSL of thermodynamics for various choices of the scale factor a. The choices are named as 'emergent', 'intermediate' and 'logamediate'. The physical aspects behind such choices are well documented in the literature [11-13]. However, for the sake of convenience we shall give a brief overview in a subsequent section. In this work the thermodynamic consequences of the universe for the said choices of scale factor would be examined in a modified gravity theory named as $f(\mathcal{R})$ gravity that has gained immense interest in recent times. In the present work we have extended the study of [9] to the investigation of the generalized second law (GSL) of thermodynamics in emergent scenario with the universes enveloped by Hubble, apparent, particle, and event horizons, respectively. In the remaining part of the paper, the radii of the said horizons are denoted by $R_{\rm H}$, $R_{\rm A}$, $R_{\rm P}$, and $R_{\rm E}$, respectively. The validity

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of the GSL implies that the sum of the time derivatives of the internal entropy and entropy on the horizon is non-negative. Hence, the primary objective of this work is to discern whether $\dot{S}_{\text{total}} = \dot{S}_{\text{internal}} + \dot{S}_{\text{horizon}} \geq 0$ holds for the situations under consideration. The GSL would be investigated based on the first law of thermodynamics. Relevance of the laws of thermodynamics in cosmology was discussed by [14,15]. The validity of GSL in various DE candidates and their interactions has been discussed in several papers [16-22]. The works on the validity of the GSL in modified gravity theories include [23-27]. Bamba and Geng in [28] studied thermodynamics of the apparent horizon in $f(\mathcal{R})$ gravity, and it was demonstrated that an $f(\mathcal{R})$ gravity can realize a crossing of the phantom divide and can satisfy the second law of thermodynamics in the effective phantom phase as well as non-phantom one. Another work [25] studied the thermodynamic behavior of field equations for $f(\mathcal{R})$ gravity. Sadjadi and Jamil [29] applied logarithmic correction to the usual form of entropy and investigated the conditions that the presence of such modified terms in the entropy puts on other physical parameters the system such as the temperature of dark energy via requiring the validity of GSL. In another recent work, Sadjadi [30] considered spatially flat Friedmann-Robertson-Walker (FRW) universe in the framework of the modified Gauss-Bonnet gravity and obtained the conditions required for validity of generalized second law (GSL).

In the present work, we have taken different choices for the scale factor and examined whether the GSL holds for those choices. Details are presented in the subsequent sections. In a very recent work, Debnath et al. [31] investigated the validity of GSL for various choices of scale factor in fractional action cosmology by using, as well as without using, the first law of thermodynamics and considering different enveloping horizons. The present work is deviated from the study of [31] in the sense that instead of considering fractional action cosmology, we have chosen $f(\mathcal{R})$, a modified gravity, to investigate the validity of GSL.

Methodology

The generalized second law

In this section we are going to examine whether the generalized second law (GSL) will hold for various choices of scale factor and on various horizons under $f(\mathcal{R})$ gravity. The basic necessity for the validity of GSL is that the time derivative of the total entropy $\dot{S}_{\text{total}} = \dot{S}_{\text{H}} + \dot{S} \geq 0$, where $\dot{S}$ indicates the time derivative of normal entropy and $\dot{S}_{\text{H}}$ indicates the horizon entropy [22].

We consider the general class of FRW models with scalefactor $a(t)$ [32],

$$ds^2 = dt^2 + a^2(t)\left[\chi^2 + S_k^2(\chi)(d\theta^2 + \sin^2\theta d\psi^2)\right] \quad (1)$$

where $S_k(\chi) = \sin\chi,\ \chi,\ \sinh\chi$ are for closed, flat, and open models, respectively; and $a(t)$ is the scale factor.

For non-flat FRW universe, the field equations are

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \quad (2)$$

$$\dot{H} - \frac{k}{a^2} = -4\pi G(\rho + p) \quad (3)$$

The first law of thermodynamics (Clausius relation) [33] on the horizon is defined as $T_{\text{X}} dS_{\text{X}} = \delta Q = -d\text{E}_{\text{X}}$, where $d\text{E}$ is the amount of the energy flow through the cosmological horizon. From the unified first law, reference [34] derived that during the time internal dt, one can obtain the amount of energy crossing the cosmological horizon

$$-d\text{E}_{\text{X}} = T_{\text{X}} dS_{\text{X}} = A_c R_{\text{X}} H(\rho + p)dt, \quad (4)$$

where T_{X} and R_{X} are the temperature and radius of the horizons under consideration in the equilibrium thermodynamics. The area A_{c} of the cosmological horizon generalized to curved space is [32]

$$A_{\text{c}} = 4\pi a^2(t) S_k^2(\chi_c) \quad (5)$$

Subsequently, from Equations 4 and 5 the time derivative of the entropy on the horizon can be derived as

$$\dot{S}_{\text{X}} = \frac{4\pi R_{\text{X}} a^2(t) S_k^2(\chi_c) H}{T_{\text{X}}}(\rho + p) \quad (6)$$

Using Equation 3 in Equation 6 we get

$$\dot{S}_{\text{X}} = \frac{R_{\text{X}} a^2(t) S_k^2(\chi_c) H}{G T_{\text{X}}}\left(\frac{k}{a^2} - \dot{H}\right) \quad (7)$$

Time derivative of entropy inside the horizon is given by [35]

$$T\dot{S}_{\text{IX}} = (p + \rho)dV_c + V_c d\rho, \quad (8)$$

where volume V_{c} is within the cosmological horizon. Using the field equation we get

$$\dot{S}_{\text{IX}} = \left(\frac{p + \rho}{T}\right)(\dot{V}_{\text{c}} - 3HV_{\text{c}}) \quad (9)$$

where [32]

$$V_{\text{c}} = \begin{cases} 2\pi a^3(t)(\chi_{\text{c}} - \sin\chi_{\text{c}}\cos\chi_{\text{c}}) & \text{closed,} \\ \frac{4}{3}a^3(t)\chi_c & \text{flat,} \\ 2\pi a^3(t)(-\chi_c + \sinh\chi_c\cosh\chi_c) & \text{open.} \end{cases} \quad (10)$$

In the present work, we are going to consider four cosmological horizons, namely, (a) apparent, (b) Hubble, (c) event, and (d) particle horizons. For the above horizons, the radii are expressed as R_{A}, R_{H}, R_{E} and R_{P}, respectively. The above horizons are given by [35]

$$R_{\text{A}} = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}} \quad (11)$$

Here, $H = \frac{\dot{a}}{a}$ is the Hubble parameter. For $k = 0$ (i.e., flat universe) we get the radius of the Hubble horizon. Thus, the radius of the Hubble horizon is $R_{\mathrm{H}} = \frac{1}{H}$.

$$R_{\mathrm{E}} = a \int_t^{\infty} \frac{dt}{a} \tag{12}$$

$$R_{\mathrm{P}} = a \int_0^{t} \frac{dt}{a} \tag{13}$$

$$\begin{aligned} &\dot{R}_{\mathrm{A}} = -HR_{\mathrm{A}}^3 \left(\dot{H} - \frac{k}{a^2} \right) \ ; \ \dot{R}_{\mathrm{H}} = -\frac{\dot{H}}{H^2} \ ; \\ &\dot{R}_{\mathrm{E}} = HR_{\mathrm{E}} - 1 \ ; \ \dot{R}_{\mathrm{P}} = HR_{\mathrm{P}} + 1 \ ; \end{aligned} \tag{14}$$

Our target is to investigate whether $\dot{S}_{\mathrm{X}} + \dot{S}_{\mathrm{IX}} \geq 0$ holds for the cases to be considered.

Basic equations of $f(\mathcal{R})$ gravity

The action of $f(\mathcal{R})$ gravity is given by [28]

$$S = \int d^4x \sqrt{-g} \left[\frac{f(\mathcal{R})}{2\kappa^2} + \mathcal{L}_{\mathrm{matter}} \right] \tag{15}$$

where g is the determinant of the metric tensor $g_{\mu\nu}$, $\mathcal{L}_{\mathrm{matter}}$ is the matter Lagrangian and $\kappa^2 = 8\pi G$. The $f(\mathcal{R})$ is a nonlinear function of the Ricci curvature $\mathcal{R}$ that incorporates corrections to the Einstein-Hilbert action which is instead described by a linear function $f(\mathcal{R})$. The gravitational field equations in this theory are [28]

$$H^2 + \frac{k}{a^2} = \frac{\kappa^2}{3f'(\mathcal{R})} (\rho + \rho_{\mathrm{c}}) \tag{16}$$

$$\dot{H} - \frac{k}{a^2} = -\frac{\kappa^2}{2f'(\mathcal{R})} (\rho + p + \rho_{\mathrm{c}} + p_{\mathrm{c}}) \tag{17}$$

where ρ_{c} and p_{c} can be regarded as the energy density and pressure generated due to the difference of $f(\mathcal{R})$ gravity from general relativity given by [28]

$$\rho_{\mathrm{c}} = \frac{1}{8\pi f'} \left[-\frac{f - Rf'}{2} - 3Hf''\dot{R} \right] \tag{18}$$

$$p_{\mathrm{c}} = \frac{1}{8\pi f'} \left[\frac{f - \mathcal{R}f'}{2} + f''\ddot{\mathcal{R}} + f'''\ddot{\mathcal{R}}^2 + 6f''\dot{\mathcal{R}} \right] \tag{19}$$

where the scalar tensor $\mathcal{R} = -6 \left(\dot{H} + 2H^2 + \frac{k}{a^2} \right)$.

The choices of scale factor $a(t)$

In this paper we have considered three forms of the scale factor $a(t)$ in the $f(\mathcal{R})$ gravity to investigate the validity of the GSL of thermodynamics. The three choices, in literature, dubbed as *emergent*, *intermediate*, and *logamediate* respectively, are given by

1. *Emergent*, $a(t) = A \left(\eta + e^{Bt} \right)^n$ with $A > 0, \ B > 0, \ \eta > 0, \ n > 1$ [11].
2. *Intermediate*, $a(t) = \exp(Bt^{\beta})$ with $B > 0; \ 0 < \beta < 1$ [12].
3. *Logamediate*, $a(t) = \exp(A(\ln t)^{\alpha})$ with $A\alpha > 0, \ \alpha > 1$ [13].

For the above choices of scale factor, the forms of the Hubble parameter H are the following

$$H = \frac{Bne^{Bt}}{\eta + e^{Bt}}; \quad H = B\beta t^{-1+\beta}; \quad H = \frac{A\alpha (\ln t)^{-1+\alpha}}{t}. \tag{20}$$

It is clear from the above equations that we are first choosing various forms of scale factor and subsequently investigating the GSL in the corresponding scenarios. This 'reverse' way of investigations had earlier been used extensively by Ellis and Madsen [36], who chose various forms of scale factor and then found out the other variables from the field equations.

We choose the function $f(\mathcal{R})$ as [37]

$$f(\mathcal{R}) = \mathcal{R} + \xi \mathcal{R}^{\mu} + \zeta \mathcal{R}^{-\nu}. \tag{21}$$

We obtain the Ricci scalar $\mathcal{R}$ for the above three choices of scale factor leading to the forms of H obtained in (11). Subsequently we obtain $f(\mathcal{R})$ for all of the above choices as functions of time t. The radii of the various enveloping horizons of the universe are given below.

The radius of the apparent horizon is given by

$$R_{\mathrm{A}} = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}}$$

If we use $k = 0$, then we get the radius of the Hubble horizon $R_{\mathrm{H}} = \frac{1}{H}$. The radii of the particle R_{P} and the event R_{E} horizons are given by

$$R_{\mathrm{P}} = a \int_0^t \frac{dt}{a} \quad \text{and} \quad R_{\mathrm{E}} = a \int_0^{\infty} \frac{dt}{a}.$$

Discussions on the above radii of different horizons are available in [35]. Using the above forms of scale factors, the Ricci scalar $\mathcal{R}$ is reconstructed as follows:

For *emergent* scenario:

$$\begin{aligned} \mathcal{R} = 6 \Bigg[&-\frac{B^2 e^{2Bt} n}{(e^{Bt} + \eta)^2} + \frac{2B^2 e^{2Bt} n^2}{(e^{Bt} + \eta)^2} + \frac{B^2 e^{Bt} n}{e^{Bt} + \eta} \\ &+ \frac{k(e^{Bt} + \eta)^{-2n}}{A^2} \Bigg] \end{aligned} \tag{22}$$

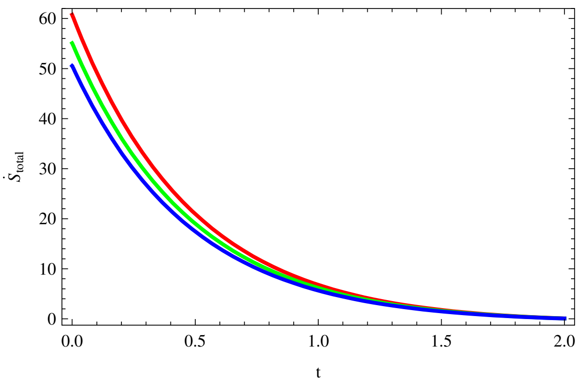

Figure 1 $\dot{S}_{\text{total}}$ **against cosmic time** t **for Hubble horizon in emergent scenario.**

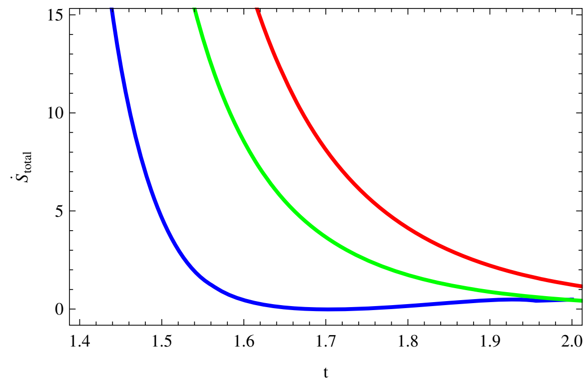

Figure 3 $\dot{S}_{\text{total}}$ **against cosmic time** t **for Hubble horizon in logamediate scenario.**

For *intermediate* scenario:

$$\mathcal{R} = 6\left[e^{-2Bt^{\beta}}k + Bt^{-2+\beta}(-1+\beta)\beta + 2B^{2}t^{-2+2\beta}\beta^{2}\right] \tag{23}$$

For *logamediate* scenario:

$$\mathcal{R} = 6\left[e^{-2A(\ln t)^{\alpha}}k + \frac{A(-1+\alpha)\alpha(\ln t)^{-2+\alpha}}{t^{2}} - \frac{A\alpha(\ln t)^{-1+\alpha}}{t^{2}} + \frac{2A^{2}\alpha^{2}(\ln t)^{-2+2\alpha}}{t^{2}}\right] \tag{24}$$

Now we have discussed the validity of the GSL of thermodynamics with the various choices of scale factor by obtaining the time derivatives of total entropy from for the universe enveloped by the different horizons and then plotted the time derivatives of the total entropy against cosmic time t to get the following twelve graphs shown in the Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 [Figures 1, 2, and 3 (for Hubble horizon), Figures 4, 5, and 6 (for apparent horizon), Figure 7, 8, and 9 (for particle horizon), and Figure 10, 11, and 12 (for event horizon)]. In all the plots we find that $\dot{S}_{\text{total}}$ is staying in the positive level. This indicates the validity of GSL of thermodynamics in all scenarios of the universe which are enveloped by Hubble, apparent, particle, and event horizons. In all the figures the red, green, and blue lines correspond to $k = -1,\ +1$, and $k = 0$, respectively.

Results and discussion

In the present work we have investigated the validity of generalized second law of thermodynamics in an universe enveloped by Hubble, apparent, particle and event horizons. Instead of considering FRW universe governed by Einstein gravity, we have considered a modified gravity in the form of $f(\mathcal{R})$ gravity. We have chosen the scale factors in three forms corresponding to emergent, intermediate, and logamediate scenarios. While investigating the validity of GSL of thermodynamics, we have not taken into account the first law of thermodynamics. For the purpose of the investigation of the validity of GSL, we have computed the entropy on the horizon, as well as inside the horizon, in the all twelve cases under consideration. We

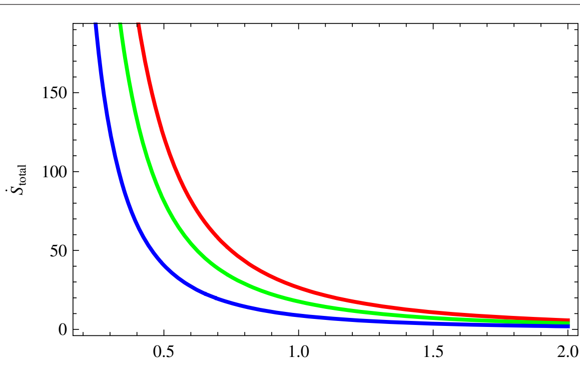

Figure 2 $\dot{S}_{\text{total}}$ **against cosmic time** t **for Hubble horizon in the intermediate scenario.**

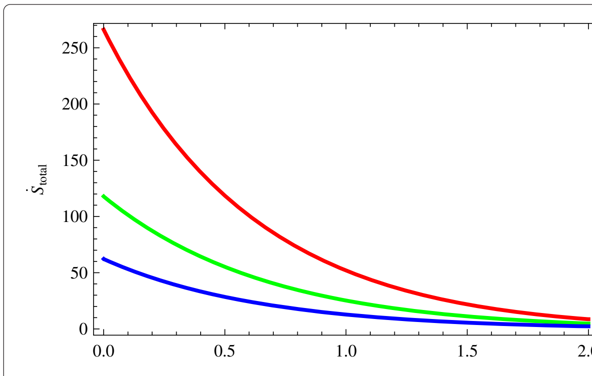

Figure 4 $\dot{S}_{\text{total}}$ **against cosmic time** t **for apparent horizon in the emergent scenario.**

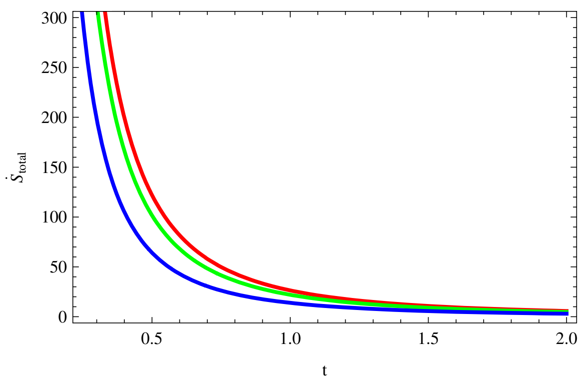

Figure 5 $\dot{S}_{\text{total}}$ **against cosmic time** t **for apparent horizon in the intermediate scenario.**

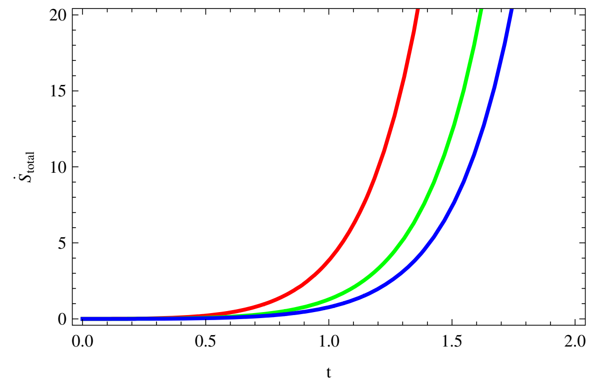

Figure 7 $\dot{S}_{\text{total}}$ **against cosmic time** t **for particle horizon in the emergent scenario.**

have kept the curvature of the universe under consideration. In all the possible three cases, we have examined the GSL for flat ($k = 0$), open ($k = -1$), and closed ($k = 1$) universes. We have plotted the time derivative of the total entropy $\dot{S}_{\text{total}}$ against the cosmic time t, in all of the cases under consideration.

In Figures 1, 2, and 3 we have considered three choices of scale factors in a universe enveloped by Hubble horizon and characterized by $f(\mathcal{R})$ gravity. In all of the three cases, $\dot{S}_{\text{total}}$ remains at the positive level and exhibit decaying behavior with the passage of cosmic time t. This indicates validity of GSL of thermodynamics in a universe characterized by $f(\mathcal{R})$ gravity and enveloped by Hubble horizon. Moreover, this holds irrespective of the curvature of the universe. It is further noted that for the intermediate and logamediate scenarios, the rate of decay of $\dot{S}_{\text{total}}$ is faster than in the case of emergent scenarios. It is further noted that this decaying behavior is significantly influenced by the curvature of the universe, in the case of logamediate scenario. From Figure 3 we find that, in logamediate scenario, $\dot{S}_{\text{total}}$ falls very sharply in case of flat universe ($k=0$). However, this rate is much lesser in open ($k = -1$) and closed ($k = 1$) universes.

In Figures 4, 5, and 6 we have considered apparent horizon. Although $\dot{S}_{\text{total}}$ stays at the positive level in all these three cases, the nature of its decay with cosmic time t has varied with the choice of scale factor. It has been observed that, in the case of logamediate scenario (Figure 6), $\dot{S}_{\text{total}}$ has fallen very sharply irrespective of the curvature. Whereas, in the the case of emergent scenario (Figure 4), the rate of change of $\dot{S}_{\text{total}}$ is much lesser. In this case the decaying of $\dot{S}_{\text{total}}$ is very slow for flat ($k = 0$). In the case intermediate scenario (Figure 5), the decaying of $\dot{S}_{\text{total}}$ is not significantly influenced by the curvature of the universe.

Figures 7, 8, and 9 confirm the validity of GSL of thermodynamics in a universe characterized by $f(\mathcal{R})$ gravity and enveloped by particle horizon. Here $\dot{S}_{\text{total}}$ has not shown any significant dependence on the curvature of the universe. However, $\dot{S}_{\text{total}}$ behavior exhibited significant changes in different scenarios of the universe. In the case of emergent scenario, it is increasing with cosmic time t;

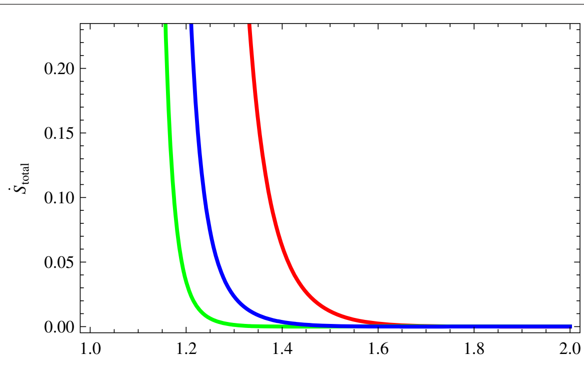

Figure 6 $\dot{S}_{\text{total}}$ **against cosmic time** t **for apparent horizon in the logamediate scenario.**

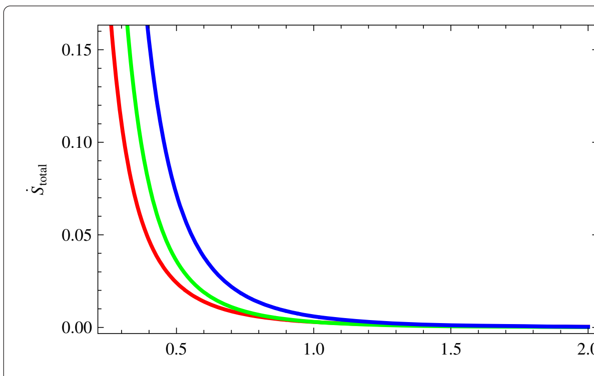

Figure 8 $\dot{S}_{\text{total}}$ **against cosmic time** t **for particle horizon in the intermediate scenario.**

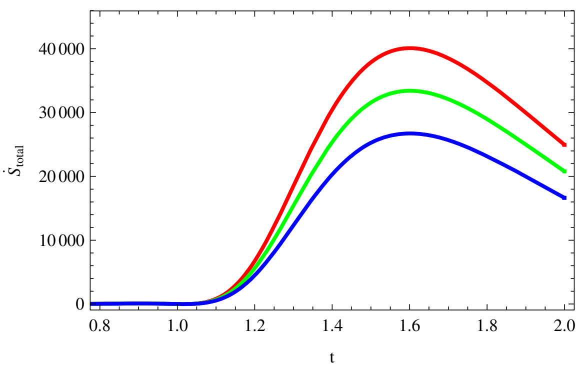

Figure 9 $\dot{S}_{\text{total}}$ **against cosmic time** t **for particle horizon in the logamediate scenario.**

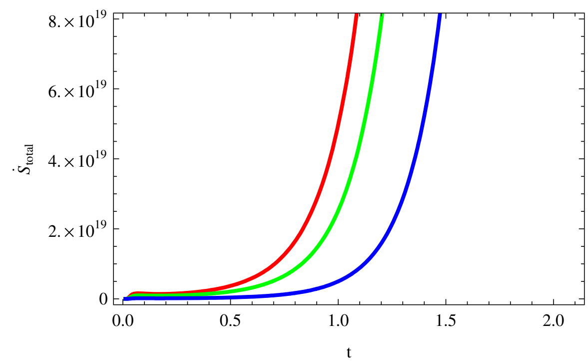

Figure 11 $\dot{S}_{\text{total}}$ **against cosmic time** t **for event horizon in the intermediate scenario.**

but in the case of intermediate scenario, it is decaying with cosmic time t. Although in the case of logamediate scenario (Figure 9), $\dot{S}_{\text{total}}$ behaves differently from the other scenarios. In Figure 9 we can see that $\dot{S}_{\text{total}}$ is decaying with cosmic time t after increasing up to a certain period of time.

In Figures 10, 11, and 12 we have plotted the time derivatives of total entropy for the universe in the emergent, intermediate, and logamediate scenarios, respectively, for the universe enveloped by the event horizon. These figures reveal that in $f(\mathcal{R})$ gravity the GSL of thermodynamics is valid for all the scenarios under consideration when we are assuming event horizon as the enveloping horizon of the universe.

Therefore, the rigorous study reported above reveals the validity of the GSL of thermodynamics in a universe governed by $f(\mathcal{R})$ gravity. Irrespective of the choice of scale factor, enveloping horizon and curvature of the universe, the time derivative of total entropy stays at the positive level.

Conclusions

While concluding the paper we now present a comparative analysis of the outcomes of the present paper with the existing works in this direction. In [28] the validity of GSL was investigated for $f(\mathcal{R})$ gravity on the apparent horizon, and it was shown that the GSL can be satisfied in both phantom and non-phantom phases of the universe. The present study deviates from the said study in the respect for choosing a model of $f(\mathcal{R})$ and considering various forms of the scale factor available in the literature. Moreover, here we have not confined ourselves to the apparent horizon only. We have also considered the other enveloping horizons like Hubble, particle, and event horizons. In all of our cases under consideration, the GSL of thermodynamics has been found to be satisfied. This study has adopted an approach similar to that of Debnath et al. [31]. However, the present study deviates from the said work in its choice of field equations. In the present work $f(\mathcal{R})$ has been considered, whereas in [31] the fractional action cosmology was considered. The conclusions

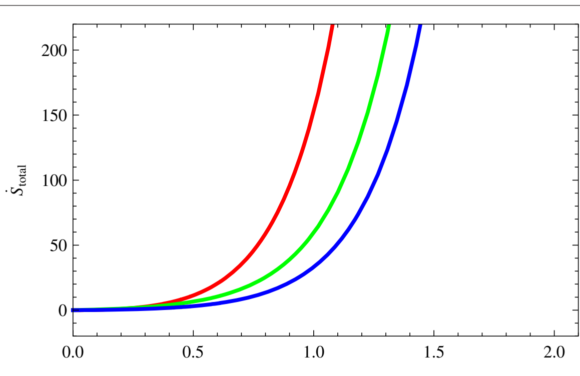

Figure 10 $\dot{S}_{\text{total}}$ **against cosmic time** t **for event horizon in the emergent scenario.**

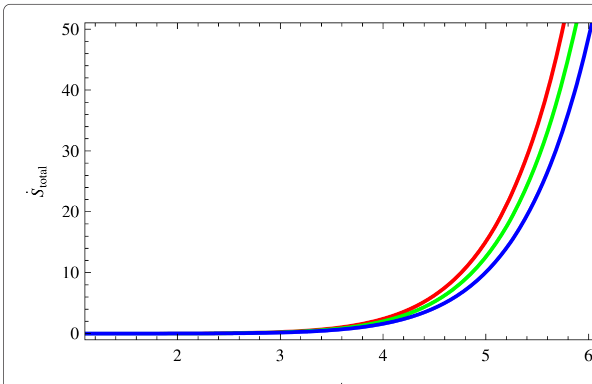

Figure 12 $\dot{S}_{\text{total}}$ **against cosmic time** t **for event horizon in the logamediate scenario.**

are deviated accordingly. In [35] logamediate and intermediate scenarios were considered in Einstein gravity and the validity of GSL was investigated. In the present work we observe that like in [35], the GSL is satisfied for both of the scenarios in spite of considering $f(\mathcal{R})$ instead of Einstein gravity.

In an old paper Barrow [38] looked at the validity of GSL in the context of viscous universes, where there is more diverse behavior and also the possibility for the creation of a dissipative structure. A study, highly relevant to be mentioned here, was done by Sadjadi [10], where the GSL was investigated in $f(\mathcal{R})$ with the scale factor $a(t) = a_0(t_s - t)^{-n}$ with $n > 0$ and derived some conditions for the validity of GSL, supposing the temperature to be proportional to the Gibbons-Hawking temperature and taking future event horizon as the horizon of the universe. In [10] $f(\mathcal{R}) = \alpha\mathcal{R}^m$ and $f(\mathcal{R}) = \beta\mathcal{R} + \alpha\mathcal{R}^m$ were considered with $\alpha,\ \beta,$ and $m \in \Re$ and finally, some conditions on $\alpha,\ \beta,$ and m were derived analytically for the validity of GSL. However, in the present paper we have not worked on deriving conditions on the model parameters. Rather, we have considered $f(\mathcal{R}) = R + \xi R^{\mu} + \zeta R^{-\nu}$ with $\xi,\ \mu,\ \zeta,$ and $\nu > 0$ following [37] and studied various forms of scale factor $a(t)$ taken from the existing literature. Instead of deriving conditions for model parameters, we have created plots for the time derivatives of the total entropy $\dot{S}_{\text{total}}$ and examined whether it is staying at positive levels or not. As future study we propose to work for generalization of the results and to derive restrictions on the model parameters required for GSL to be validated in various scenarios.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
RG carried out the literature survey, identified the problem and carried out some computations. Also, he took active role while drafting the manuscript. SC carried out computations, created figures and wrote various interpretations. Both authors read and approved the final manuscript.

Authors' information
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References

1. Copeland, EJ, Sami, M, Tsujikawa, S: Dynamics of dark energy. Int. J. Mod. Phys. D. **15**, 1753 (2006)
2. Riess, A, Filippenko, AV, Challis, P, Clocchiattia, A, Diercks, A, Garnavich, PM, Gilliland, RL, Hogan, CJ, Jha, S, Kirshner, RP, Leibundgut, B, Phillips, MM, Reiss, D, Schmidt, BP, Schommer, RA, Smith, RC, Spyromilio, J, Stubbs, C, Suntzeff, NB, Tonry, J: Observational evidence from supernovae for an accelerating universe and a cosmological constant. Astron. J. **116**, 1009 (1998)
3. Kluson, J: New models of $f(R)$ theories of gravity. Phys. Rev. D. **81**, 064028 (2010)
4. Nojiri, S, Odintsov, SD: Introduction to modified gravity and gravitational alternative for dark energy. Int. J. Geom. Meth. Mod. Phys. **4**, 115 (2007)
5. Mukhoyama, S: Horava-Lifshitz cosmology: a review. Class. Quantum Grav. **27**, 223101 (2010)
6. Paul, BC, Debnath, PS, Ghose, S: Accelerating universe in modified theories of gravity. Phys. Rev. D. **79**, 083534 (2009)
7. Nojiri, S, Odintsov, SD: Unifying inflation with ΛCDM epoch in modified $f(R)$ gravity consistent with Solar System tests. Phys. Lett. B. **657**, 238 (2007)
8. Nojiri, S, Odintsov, SD: Modified f(R) gravity unifying R^m inflation with the ΛCDM epoch. Phys. Rev. D. **77**, 026007 (2008)
9. Chattopadhyay, S, Debnath, U: Emergent universe in the chameleon, $f(R)$ and $f(T)$ gravity theories. Int. J. Mod. Phys. D. **20**, 1135 (2011)
10. Sadjadi, HM: Generalized second law in modified theory of gravity. Phys. Rev. D. **76**, 104024 (2007)
11. Mukherjee, S, Paul, BC, Dadhich, NK, Maharaj, SD, Beesham, A: Emergent universe with exotic matter. Class. Quantum Grav. **23**, 6927 (2006)
12. Barrow, JD, Nunes, NJ: Dynamics of "logamediate" inflation. Phys. Rev. D. **76**, 043501 (2007)
13. Barrow, JD, Liddle, AR: Perturbation spectra from intermediate inflation. Phys. Rev. D. **47**, 5219 (1993)
14. Lima, JAS, Alcaniz, JS: Thermodynamics, spectral distribution and the nature of dark energy. Phys. Lett. B. **600**, 191 (2004)
15. Silva, R, Alcaniz, JS, Lima, JAS: On the thermodynamics of dark energy. Int J. Mod. Phys. D. **16**, 469 (2007)
16. Sheykhi, A: Thermodynamics of interacting holographic dark energy with the apparent horizon as an IR cutoff. Class. Quantum Grav. **27**, 025007 (2010)
17. Chattopadhyay, S, Debnath, U: Generalized second law of thermodynamics in presence of interacting DBI essence and other dark energies. Int. J. Mod. Phys. A. **30**, 5557 (2010)
18. Setare, MR: Generalized second law of thermodynamics in quintom dominated universe. Phys. Lett. B. **641**, 130 (2006)
19. Setare, MR: Interacting holographic dark energy model and generalized second law of thermodynamics in a non-flat universe. J. Cosmol. Astropart. Phys. **01**, 023 (2007)
20. Setare, MR, Shafei, SM: A holographic model of dark energy and the thermodynamics of a non-flat accelerated expanding universe. J. Cosmol. Astropart. Phys. **09**, 011 (2006)

21. Elizalde, E, Nojiri, S, Odintsov, SD, Sáez-Gómez, D, Faraoni, V: Reconstructing the universe history, from inflation to acceleration, with phantom and canonical scalar fields. Phys. Rev. D. **77**, 106005 (2008)
22. Izquierdo, G, Pavon, D: Dark energy and the generalized second law. Phys. Lett. B. **633**, 420 (2006)
23. Jamil, M, Saridakis, EN, Setare, MR: The generalized second law of thermodynamics in. Horava Lifshitz cosmology. J. Cosmo. Astropart. Phys. P. **11**, 032 (2010)
24. Akbar, M, Cai, R-G: Friedmann equations of FRW universe in scalar-tensor gravity, $f(R)$ gravity and first law of thermodynamics. Phys. Lett. B. **635**, 7 (2006)
25. Akbar, M, Cai, R-G: Thermodynamic behavior of field equations for $f(R)$ gravity. Phys. Lett. B. **648**, 243 (2007)
26. Cai, R-G, Cao, L-M: Unified first law and the thermodynamics of the apparent horizon in the FRW universe. Phys. Rev. D. **75**, 064008 (2007)
27. Wu, S-F, Wang, B, Yang, G-H: Thermodynamics on the apparent horizon in generalized gravity theories. Nucl. Phys. B. **799**, 330 (2008)
28. Bamba, K, Geng, C-Q: Phys. Thermodynamics in $F(R)$ gravity with phantom crossing. Phys. Lett. B. **679**, 282 (2009)
29. Sadjadi, HM, Jamil, M: Generalized second law of thermodynamics for FRW cosmology with logarithmic correction. EPL. **92**, 69001 (2010)
30. Sadjadi, HM: On thermodynamics second law in the modified Gauss Bonnet gravity. Phys. Scripta. **05**, 055006 (2011)
31. Debnath, U, Jamil, M, Chattopadhyay, S: Fractional action cosmology: emergent, logamediate, intermediate, power law scenarios of the universe and generalized second law of thermodynamics. Int. J. Theor. Phys. **51**, 812 (2012)
32. Davis, TM, Davies, PCW, Lineweaver, CH: Black hole versus cosmological horizon entropy. Class. Quantum Grav. **20**, 2753 (2003)
33. Cao, Q-G, Chen, Y-X, Shao, K-N: Clausius relation and Friedmann equation in FRW universe model. JCAP. **05**, 030 (2010)
34. Cai, R-G, Kim, SP: First law of thermodynamics and Friedmann equations of Friedmann-Robertson-Walker universe. JHEP. **0502**, 050 (2005)
35. Das, A, Chattopadhyay, S, Debnath, U: Validity of generalized second law of thermodynamics in the logamediate and intermediate scenarios of the universe. Found. Phys. **42**, 266 (2011)
36. Ellis, GFR, Madsen, M: Exact scalar field cosmologies. Class. Quantum Grav. **8**, 667 (1991)
37. Nojiri, S, Odintsov, SD: Modified gravity with negative and positive powers of curvature: Unification of inflation and cosmic acceleration. Phys. Rev. D. **68**, 123512 (2003)
38. Barrow, JD: String-driven inflationary and deflationary cosmological models. Nucl. Phys. B. **310**, 743 (1988)

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