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Linear energy-momentum tensor for a scalar field in de Sitter space

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Abstract

In this work, under a small perturbation of the background metric with the squeezing of other fields (scalar field in our case), the linear form of the energy-momentum tensor is calculated in terms of the perturbation factor. The linear form of the Klein-Gordon equation for a scalar field with both minimally and conformally coupled cases in de Sitter and flat metrics are also calculated.

Keywords: Energy-momentum tensor in curved space, de Sitter spacetime, Klein-Gordon equation

Introduction

Studying scalar fields in cosmology is quite important because, for example, we can mention the inflation scenario at the early universe [1] and also the models of the slowly decaying cosmological constant [2]. They also play crucial roles in spontaneous symmetry breaking in particle physics and giving mass to the fields. In addition, scalar fields are present in most of the unifying theories of gravity with other fundamental forces. Mathematically, scalar fields are described by the Klein-Gordon equation wherein quanta are zero-spin particles.

The field equations can be straightforwardly generalized to be curved space-time in an entirely local and covariant manner. One of the most interesting models of curved space-times is described by the de Sitter (dS) model where the outcoming expanding universe carries scalar fields variously (mostly minimally and conformally) which couple with gravitation. Astrophysical data coming from type Ia supernova indicate that our universe is accelerating and can be well approximated by a world with a non-zero positive cosmological constant [3-6]. This means that our universe, in the first approximation, might be in a dS phase. However, there is no universal definition of the time-like Killing vector in dS space-time. Therefore, one cannot have a natural definition for a particle, e.g., various observers may disagree with the existence of particles. Definition of a particle strongly depends on its global nature, whereas the particle detectors are defined locally [7]. The absence of the general time-like Killing vector makes the particle's concept obscure. Thus, in such space-times, one needs a locally defined physical quantity that carries the significance of the particle. One such object of interest can be the energy-momentum tensor, $T_{\mu\nu}$, at point x. Therefore, obtaining the proper energy-momentum tensor becomes quite important. The history of finding the proper $T_{\mu\nu}$ for various fields in dS space-time is rich; for example, the case of scalar field can be found in [8,9]. On the other hand, some important physical quantities can be achieved by considering the linear form of the energy-momentum tensor.

In this paper, we first review the de Sitter space-time, and then the Klein-Gordon equation in this space-time is considered. After that, under the small perturbation of the background metric, we find the linear form of this equation in the dS background. In the global coordinates of dS (which cover this space-time entirely), the linear form of the energy-momentum tensor is calculated for a scalar field. Finally, a brief conclusion and an outlook for further investigation are presented.

de Sitter geometry

In $n+1$ dimensional flat space-time $\mathcal{M}^{n,1}$, by considering one constraint, one can illustrate the n dimensional de Sitter geometry. Let $x_\mu \in \mathbf{R}^{n+1}$, where $\mu = 0, 1, 2, \ldots n$; the following hypersurface is described by the following equation:

$$-x_0^2 + x_1^2 + \cdots + x_n^2 = constant(\equiv H^{-2}), \tag{1}$$

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where the equation describes the de Sitter geometry [10]; note that with $H^{-2} > 0$, it is the equation of a hyperboloid. H is the Hubble constant, and H^{-1} is the minimum radius of the hyperboloid and is also related to the cosmological constant by the following equation:

$$\Lambda = \frac{(D-2)(D-1)}{2} H^2. \tag{2}$$

de Sitter space-time is the vacuum solution of the Einstein equation with non-vanishing positive cosmological constant (anti-de Sitter and flat cases are followed by taking $\Lambda < 0$ and $\Lambda = 0$, respectively). It is the unique maximally symmetric curved space-time with ten Killing vectors (the same as the Minkowski space-time) and locally characterized by the following:

$$R_{\mu\nu\lambda\rho} = \frac{2\Lambda}{(n-1)(n-2)} (g_{\mu\lambda} g_{\nu\rho} - g_{\mu\rho} g_{\nu\lambda}). \tag{3}$$

$R_{\mu\nu\lambda\rho}$ is the Riemann curvature tensor. Using the relations $R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu}$, $R = g_{\mu\nu} R^{\mu\nu}$, we obtain the following:

$$\begin{aligned} R_{\mu\nu} &= \frac{2\Lambda}{n-2} g_{\mu\nu}, \\ R &= \frac{2n}{n-2} \Lambda, \end{aligned} \tag{4}$$

where R is the Ricci scalar. One can define many coordinate systems with different properties, say as, global, conformal, closed, etc. However, here, we consider the global coordinate system since, topologically, it covers the entire surface of the de Sitter hyperboloid. This is achieved by the following parametrization of $\mathcal{M}^{n,1}$ with defined coordinates $(\tau, \theta_1, \theta_2, \ldots \theta_n)$, as follows:

$$\begin{aligned} x^0 &= H^{-1} \sinh H\tau, \\ x^1 &= H^{-1} \cosh H\tau \cos\theta_1, \\ x^2 &= H^{-1} \cosh H\tau \sin\theta_1 \cos\theta_2, \\ x^i &= H^{-1} \cosh H\tau \left(\prod_{j=1}^{i-1} \sin\theta_j \right) \cos\theta_i, \\ x^n &= H^{-1} \cosh H\tau \left(\prod_{i=1}^{n-1} \sin\theta_i \right), \end{aligned} \tag{5}$$

where it follows by

$$\begin{aligned} & -\infty < \tau < \infty, \\ & \quad 0 \le \theta_j \le \pi, \quad 1 \le j \le n-2, \\ & \quad 0 \le \theta_{n-1} \le 2\pi. \end{aligned} \tag{6}$$

It is easy to check that the metric on the de Sitter space reads as follows:

$$\begin{aligned} ds^2 &= -d\tau^2 + \frac{\cosh^2 H\tau}{H^2} d\Omega_i^2, \qquad 1 \le i \le n-1, \\ d\Omega_i^2 &\equiv \sum_{i=1}^{n-1} \left(\prod_{j}^{i-1} \sin^2\theta_j d\theta_i^2 \right). \end{aligned} \tag{7}$$

In these coordinates, de Sitter geometry can be considered as an S^{n-1}-sphere at every fixed τ; it is infinitely large at $\tau = -\infty$, then shrinks to a minimal finite size at $\tau = 0$, and then grows again to infinity as long as τ goes to infinity [10].

By introducing the conformal time ρ as

$$\cosh H\tau \equiv \frac{1}{\cos\rho}, \tag{8}$$

the conformal metric in the de Sitter space is obtained:

$$ds^2 = \frac{1}{H^2 \cos^2\rho} \left(-d\rho^2 + d\Omega_i^2 \right). \tag{9}$$

Note that $-\frac{\pi}{2} < \rho < \frac{\pi}{2}$, and also, we choose this metric since it is easier to work with.

Klein-Gordon equation in de Sitter space-time

The scalar field action in curved space is given as follows:

$$S_\phi = \frac{1}{2} \int d^n x \sqrt{-g} \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + (m^2 + \zeta R)\phi^2 \right), \tag{10}$$

where g is the determinant of the metric tensor and ζ is the coupling constant between the scalar field and the gravitational field [7]:

$$\zeta = \frac{n-2}{4(n-1)}.$$

Note that we use the $(-, +, ...+)$ signature. Variation with respect to the scalar field gives us the Klein-Gordon equation:

$$(\Box - m^2 - \zeta R)\phi = 0, \tag{11}$$

in which $\Box = \frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} \partial^\mu$ is the d'Alembertian operator in curved space, which should be found for any case. For example, in a four-dimensional de Sitter space with the metric (7) (its conformal counterpart can be easily obtained by replacing Equation 8), it becomes the Laplace-Beltrami operator which is given as follows [11,12]:

$$\Box = -\frac{\partial^2}{\partial\tau^2} - 3H \tanh H\tau \frac{\partial}{\partial\tau} + \frac{H^2}{\cosh^2 H\tau} \triangle_3, \tag{12}$$

where $\triangle_3$ is the Laplace operator on hypersurface S^3 defined as follows:

$$\triangle_3 = \frac{\partial^2}{\partial\theta_1^2} + 2\cot\theta_1\frac{\partial}{\partial\theta_1} + \frac{1}{\sin^2\theta_1}\frac{\partial^2}{\partial\theta_2^2} + \frac{\cot\theta_2}{\sin^2\theta_1}\frac{\partial}{\partial\theta_2} + \frac{1}{(\sin\theta_1\ \sin\theta_2)^2}\frac{\partial^2}{\partial\theta_3^2}. \tag{13}$$

After separation of variables $\phi(x) = \chi(\tau)\mathcal{Y}_{Llm}(\Omega)$ and doing some calculation, the solution of the Klein-Gordon equation reads (more mathematical details can be found in [11,12]):

$$\mathcal{Y}_{Llm} = \left(\frac{(L+1)(2l+l)(L-l)!}{2\pi^2(L+l+1)!}\right)^{\frac{1}{2}} \times 2^l l!\,(\sin\theta_1)^l C_{L-l}^{l+1}(\cos\theta_1)Y_{lm}(\theta_2,\theta_3), \tag{14}$$

where $(L,l,m)\in \mathbf{N}\times\mathbf{N}\times\mathbf{Z}$ with $0\le l\le L$ and $-l\le m\le l$, and also C_n^λ are the Gegenbauer polynomials; Y_{lm} stand for the spherical harmonics:

$$Y_{lm}(\theta_1,\theta_2) = (-1)^m\left(\frac{(l-m)!}{(l+m)!}\right)^{\frac{1}{2}} P_l^m(\cos\theta_2)e^{im\theta_3}, \tag{15}$$

P_l^m are the associated Legender function.

For the $\chi(\rho)$ part, let us find it in its conformal form; after using (8), relation (12) turns to the following:

$$\Box = -H^2\cos^2\rho\left(\frac{\partial^2}{\partial\rho^2} + 2\tan\rho\frac{\partial}{\partial\rho} - \triangle_3\right). \tag{16}$$

The solution for θ_i is the same as the previous, and its ρ part becomes the following [11]:

$$\chi_{\lambda L}(\rho) = A_L(\cos\rho)^{\frac{3}{2}}\left[P^\lambda_{L+\frac{1}{2}}(\sin\rho) - \frac{2i}{\pi}Q^\lambda_{L+\frac{1}{2}}(\sin\rho)\right], \tag{17}$$

where P^λ and Q^λ are the associate Legender functions of first and second kind. A_L is given as follows:

$$A_L = H\frac{\sqrt{m}}{2}\left(\frac{\Gamma(L-\lambda+\frac{3}{2})}{\Gamma(L+\lambda+\frac{3}{2})}\right)^{\frac{1}{2}}. \tag{18}$$

Note that λ is a parameter defined by the following:

$$\lambda = \sqrt{\frac{9}{4}-\kappa} \qquad 0\le\kappa\le\frac{9}{4}, \qquad \lambda = i\sqrt{\kappa-\frac{9}{4}} \qquad \kappa\ge\frac{9}{4}. \tag{19}$$

Actually, κ is the expectation value of the Casimir operator for scalar field in de Sitter space which relates to ζ and m via $\kappa = (\frac{m}{H})^2 + 12\zeta$; here, we have introduced it as a parameter. Note that the case $\kappa = 0$ or $m^2 = -12H^2\zeta$ is a particular case which is not considered here. One can find more about this case in relevant papers.

Linearization of the Klein-Gordon equation

Let us suppose a small perturbation from the background:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \tag{20}$$

in which, $\bar{g}_{\mu\nu}$ and $h_{\mu\nu}$ are the background metric and a small perturbation, respectively, with the condition that in every point of the space-time, we have $h_{\mu\nu} \ll \bar{g}_{\mu\nu}$. Under this perturbation of metric, after doing some calculation, one can obtain the linearization of (10) as follows:

$$\begin{aligned} S_\phi &= \frac{1}{2}\int d^4x\sqrt{-\bar{g}}\left\{\mathcal{O}(h^0) + \mathcal{O}(h) + \cdots\right\}, \\ \mathcal{O}(h^0) &= \bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + (m^2+\zeta\bar{R})\phi^2, \\ \mathcal{O}(h^1) &= (\bar{g}^{\mu\nu}\frac{h}{2} - h^{\mu\nu})\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}m^2\phi^2 \\ &\quad + \zeta(R_L + \bar{R}\frac{h}{2})\phi^2. \end{aligned} \tag{21}$$

Now in order to find the linear form of the field equation, one can either take the variation of (21) with respect to ϕ or directly linearize the Klein-Gordon equation. Both of them give the same linear form of the Klein-Gordon equation. Here, we follow the second one. the linear form of the box operator becomes the following:

$$\begin{aligned} \Box\phi &= g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = g^{\mu\nu}\nabla_\mu\partial_\nu\phi \\ &= \left(\bar{g}^{\mu\nu} - h^{\mu\nu}\right)\left(\bar{\nabla}_\mu\partial_\nu\phi - (\Gamma^\alpha_{\mu\nu})_L\partial_\alpha\phi\right) \\ &= \bar{\Box}\phi - \left(h^{\mu\nu}\bar{\nabla}_\mu\partial_\nu\phi + \bar{g}^{\mu\nu}(\Gamma^\alpha_{\mu\nu})_L\partial_\alpha\phi\right) + \mathcal{O}(h^2). \end{aligned} \tag{22}$$

On the other hand, for the Ricci scalar, we have the following:

$$R = \bar{R} + R_L + \mathcal{O}(h^2), \tag{23}$$

where the linear part of the Christoffel connection and also the Ricci scalar is given as follows [13]:

$$\begin{aligned} (\Gamma^\alpha_{\mu\nu})_L &= \frac{1}{2}\bar{g}^{\alpha\lambda}(\overline{\nabla}_\mu h_{\nu\lambda} + \overline{\nabla}_\nu h_{\mu\lambda} - \overline{\nabla}_\lambda h_{\mu\nu}), \\ (R_{\mu\nu})_L &= \frac{1}{2}\left(\bar{\nabla}\cdot\bar{\nabla}_\mu h_\nu + \bar{\nabla}\cdot\bar{\nabla}_\nu h_\mu - \bar{\Box}h_{\mu\nu} - \bar{\nabla}_\mu\bar{\nabla}_\nu h\right), \\ R_L &= \bar{\nabla}\cdot\bar{\nabla}\cdot h - \bar{\Box}h - \Lambda h. \end{aligned} \tag{24}$$

Note that $h = \bar{g}_{\mu\nu}h^{\mu\nu}$, and $\bar{\nabla}\cdot A \equiv \bar{\nabla}_\mu A^\mu$.

Plugging (22) and (24) in (11) results in the following:

$$(\bar{\Box} - m^2 - \zeta\bar{R})\phi - \left[h^{\mu\nu}\bar{\nabla}_\mu\partial_\nu + \frac{1}{2}\left(2\bar{\nabla}\cdot h^\alpha - \bar{\nabla}^\alpha h\right)\partial_\alpha + \zeta\left(\bar{\nabla}\cdot\bar{\nabla}\cdot h - \bar{\Box}h - \Lambda h\right)\right]\phi = 0. \tag{25}$$

This is the linear form of the Klein-Gordon equation in generic background. It is worth to mention that the scalar field is supposed to be freezed during the metric perturbation.

With this linear form, one can evaluate the change in the observable quantities due to this perturbation by introducing a proper h.

Energy-momentum tensor for a scalar field

In generic background case

The energy-momentum tensor is given as follows:

$$T_{\mu\nu} = 2\sqrt{-g}\frac{\delta S_\phi}{\delta g^{\mu\nu}}. \tag{26}$$

The variation of both the background and linear part of (21) gives us the following:

$$\begin{aligned} T_{\mu\nu} =& (1+2\zeta)\partial_\mu\phi\partial_\nu\phi - \left(2\zeta+\frac{1}{2}\right)\bar{g}_{\mu\nu}\partial^\sigma\phi\partial_\sigma\phi \\ & - \frac{1}{2}\bar{g}_{\mu\nu}m^2\phi^2 + 2\zeta\phi\bar{\nabla}_\mu\partial_\nu\phi - 2\zeta\bar{g}_{\mu\nu}\phi\bar{\Box}\phi \\ & + \zeta(\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R})\phi^2 - \left(2\zeta+\frac{1}{2}\right) \\ & \times\left(h_{\mu\nu}\bar{g}^{\lambda\sigma} - \bar{g}_{\mu\nu}h^{\lambda\sigma}\right)\partial_\lambda\phi\partial_\sigma\phi - \frac{1}{2}m^2\phi^2 h_{\mu\nu} \\ & - 2\zeta\phi(\Gamma^\lambda_{\mu\nu})_L\partial_\lambda\phi + 2\zeta\phi\left(\bar{g}_{\mu\nu}\bar{g}^{\lambda\sigma}(\Gamma^\eta_{\lambda\sigma})_L\partial_\eta\phi\right. \\ & \left. - (h_{\mu\nu}\bar{g}^{\lambda\sigma} - \bar{g}_{\mu\nu}h^{\lambda\sigma})\bar{\nabla}_\lambda\partial_\sigma\phi\right) \\ & + \zeta\phi^2\left((R_{\mu\nu})_L - \frac{1}{2}(h_{\mu\nu}\bar{R} + \bar{g}_{\mu\nu}R_L)\right). \end{aligned} \tag{27}$$

Note that by putting h-parts equal to zero, one gets exactly the correct result for $T_{\mu\nu}$, (e.g., in de Sitter space-time with relevant $\bar{R}$ and $\bar{R}_{\mu\nu}$) for the a scalar field [8,9].

For the de Sitter space with given metric (9) and in the minimally coupled case, one obtains the following:

$$\begin{aligned} T_{\rho\rho} =& \frac{1}{2}(\partial_\rho\phi)^2 + \frac{1}{2}\mathcal{T} + \frac{1}{2}\left[h_{\rho\rho}(H\cos\rho)^2\left((\partial_\rho\phi)^2 - \mathcal{T}\right)\right. \\ & \left. - \frac{1}{(H\cos\rho)^2}h^{\lambda\sigma}\partial_\lambda\phi\partial_\sigma\phi\right], \end{aligned} \tag{28}$$

where

$$\begin{aligned} \mathcal{T} \equiv& (\partial_{\theta_1}\phi)^2 + \frac{1}{\sin^2\theta_1}(\partial_{\theta_2}\phi)^2 + \frac{1}{(\sin\theta_1\sin\theta_2)^2}(\partial_{\theta_3}\phi)^2 \\ & + \frac{1}{(H\cos\rho)^2}m^2\phi^2. \end{aligned} \tag{29}$$

As a particular case in which the scalar field is independent of θ_i, Equation 28 reduces to the following:

$$T_{\rho\rho} = \frac{1}{2}(\partial_\rho\phi)^2 + \frac{1}{2(H\cos\rho)^2}m^2\phi^2 - \frac{1}{2}m^2 h_{\rho\rho}\phi^2. \tag{30}$$

Flat background

In the flat background, let us consider the minimally coupled scalar field ($\zeta = 0$). For the scalar field, the zero-zero (or time-time) component of Equation 27, which has the interpretation of the energy density in flat space, reduces to the following:

$$\begin{aligned} T_{00} =& \frac{1}{2}\left\{(\partial_0\phi)^2 + (\partial_i\phi)^2 + m^2\phi^2\right. \\ & \left. - \left[h_{00}(\partial_\mu\phi)^2 + h^{\lambda\sigma}\partial_\lambda\phi\partial_\sigma\phi - h_{00}m^2\phi^2\right]\right\}. \end{aligned} \tag{31}$$

The first three terms are the usual T_{tt} in the Minkowski space, and the others belong to the effect of the metric perturbation. Therefore, at this stage, one can study the change in the energy levels due to this kind of perturbation as follows:

$$E = \int d^3x\ T_{tt} = E_0 + \Delta E, \tag{32}$$

where E_0 is the energy level of the spinless relativistic particle evaluated in the usual way in flat space; on the other hand, ΔE can be supposed as its correction due to the fluctuation of the metric and is given as follows:

$$\Delta E = -\frac{1}{2}\int d^3x\left[h_{00}(\partial_\mu\phi)^2 + h^{\lambda\sigma}\partial_\lambda\phi\partial_\sigma\phi - h_{00}m^2\phi^2\right]. \tag{33}$$

Conclusions

Having understood that in general, curved space-times-due to the lack of time-like Killing vector - has no common acceptable vacuum for all inertial observers; the next natural question is to ask what happens when one wants to consider the quantum effects of various kinds of fields. In considering quantum effects, the expectation value of the locally defined energy-momentum tensor becomes important. For example, in de Sitter space-time, one can consider the effect of the scalar field by finding its proper energy-momentum tensor. In view of some papers ([8,9,14]) on this subject, in the present work, we considered the effect of the metric perturbation on energy-momentum tensor. It was found that the effect of metric perturbation appears as small corrections in the original form of $T_{\mu\nu}$. As a suggestion with this additional part to the zero-zero component of the energy-momentum tensor, one can study the effect of the gravitational waves in the Casimir effect since in the Casimir effect, this additional part will appear as a force, which we leave to future works.

Competing interests
Both authors declare that they have no competing interests.

Authors' contributions

In the present work, MRT participated in the sequence alignment and drafted the manuscript. SM helped draft the manuscript. Both authors read and approved the final manuscript.

Authors' information

MRT has a PhD in theoretical physics and is appointed assistant professor of physics at IAUCTB in 2006. SM has received her Master of Science in theoretical physics.

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