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Modified Bohm's criterion in a collisional electronegative plasma having two-temperature non-extensive electrons

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Abstract
Plasma-material interaction has been a subject of interest for the past several decades due to its importance in various fields of research such as film deposition, surface nitriding, plasma etching, plasma reactors at low-pressure conditions etc. During this interaction, the presence of negative ions further plays a vital role to ease defect-free analysis of soft substrates. The response of plasma through the sheath formation, when a metallic plate or probe is inserted into it, depends on the plasma characteristics / parameters and the bias voltage used on the plate or probe. The Bohm's criterion decides such kind of interaction. The present work theoretically demonstrates a modified Bohm's criterion in an electronegative plasma which is collisional and has two-temperature non-extensive electrons (hot and cold electrons). The behaviour of positive ions is considered through their fluid equations, whereas the negative ions are taken to follow Boltzmann distribution. While writing the basic equations, ion source term and ionization rate are retained and Sagdeev's potential approach is employed to evaluate the Bohm's criterion which reveals a band for the positive ion velocity, means maximum and minimum values for the ion velocity. This modified Bohm's criterion is studied under the effects of various plasma parameters.

Keywords
Two-temperature electrons, Ionization rate, Ion source term, Non-extensive distribution, Modified Bohm's criterion

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1. Introduction

Significant modification in properties of the surface of a material can be done with the help of different processes such as chemical, mechanical, energetic or a combination of them. Each of these have their own pros and cons; like for mechanical processes, they require additional cleaning steps prior to further processing. Chemical treatments also make the manufacturing more complex because these processes are designed to a specific metal which results in a situation where each material requires its own chemical treatment. Energetic processes involve laser, flame and plasma treatments which can modify the surface being dry systems in nature. The most common plasma surface treatments include functions like surface cleaning, surface activation, surface etching, surface coating etc.

Plasma and nitriding parameters have been effectively optimized by growing Ti thin films using DC sputtering on a glass substrate and nitriding them in a hot cathode arc discharge plasma system [1, 2]. Sheet-moulded electron cyclotron resonance heating (ECRH) plasma enhanced with RF plasma source has been used for controlling the plasma boundaries and deposition rate of thin film [3]. Ni-like and Co-like X-rays transmitted from laser delivered Tin plasmas have been created. Most extreme X-ray conversion effectiveness was resulted as 3.54% at the power density of 5×10^{12} Wcm^{-2} into 2π steradian [4]. Under the laser operation at 1064 nm, EHYBRID simulation has been performed with pulse duration of 6 ns to simulate the soft X-rays emitted from Sn XII and Sn XIII ions [5, 6]. Oxygen and nitrogen plasmas have been used to optimize the Poly methyl methacrylate (PMMA) polymer surface where atomic force microscopic (AFM) and Fourier transform infrared spectrometer (ATR-FTIR) micrographs were used for the optimization [7, 8]. The basic utilizations of isotropic etching, RIE, and plasma aching/cleaning to shape definitively profiles of high-aspect-ratio contacts (HARC), gate stacks, and shallow trench isolation (STI) in the front end of line (FEOL) have been taken up by Abe et al. [9].

In such interactions, a sheath is formed on the surface of the material and it is significantly impacted in the presence of negative ions. For the production of negatively charged particles, Nitrogen has been used as an electronegative gas in a discharge process that was impacted by them discernibly [10]. In another cylindrical dc glow discharge plasma, it has been shown that the plasma potential, electron temperature, and floating potential are quite different in the situation of constant current mode and constant pressure modes and here electron density plays a vital role [11]. The analysis of thickness of the sheath formed due to collection of positive and negative ions on the surface of metallic conducting spherical probe has been carried out for different ions' temperature by treating both the ions at equal footing [12]. The sheath formation also takes place on the space vehicles moving at supersonic speeds [13, 14] and also in thruster devices [15] where acceleration of the charged particles is important. Hence, it has application in space propulsion as well. In this direction, a semi-analytical numerical method for rapid and assumptionless calculations of the magnetic field was developed for a thick rectangular coil and it was extended to a three-coil setup for enhancing thrust and nozzle's efficiency [16]. The efficiency is related to the plume that emerges out of the propulsion devices. For its control and enhanced thrust, Malik [17] has provided a concept of plasma detachment in a magnetic nozzle. However, the plasma in space propulsion devices is found to be unstable due to the inhomogeneity in plasma density and magnetic field [18–22]; similar situation arises in other cross-field plasmas [23].

Sheath has found applications in other plasma systems also; for example, microwave generated plasma where instability was driven by the energetic ions [24]. Strong field exists in the sheath and hence the sheath thickness is an important component in electronegative plasmas also, such as CF_4, O_2 and C_60 plasmas [12]. A distinctive nature of the plasma parameters has been observed due to the presence of the negative ions in different situations [25–29].

For only macroscopic equilibrium states, the Maxwellian distribution has been observed to be validated. In unmagnetized plasma, the long-range interactions cannot be described by Maxwellian distribution adequately. The existence of nonequilibrium stationary states has been recorded in such plasma systems. The electrons population in theoretical investigations and space plasma observations has been observed to deviate from their thermodynamic equilibrium [30, 31]. The gravitational and plasma systems, where long range interactions occur, are failed to be described by Boltzmann statistics theoretically and experimentally. Therefore, new statistics has been proposed by Tsallis [32], named as non-extensive statistics, to describe such systems adequately. Several measurements have validated this statistics for the systems having particle distribution deviated from the Maxwellian distribution. The q-nonextensive behaviour of the electrons [33, 34] present in the above plasmas can modify the whole phenomena of sheath formation, film deposition, coatings etc. Keeping in mind all the work conducted so far by the other investigators in such fields, in the present letter, we focus mainly on the Bohm's criterion which plays a vital role in these phenomena. For the completeness, we have considered ion source term, collisions and temperature of all the species in four-component plasma having hot and cold electrons in addition to negative ions. Under this realistic situation, Bohm's criterion is found to be modified and the sheath formation mechanism is expected to modify accordingly.

2. Basic fluid equations

An electronegative plasma with two-temperature electrons, namely cold and hot electrons, and positive ions is considered to evaluate the Bohm's sheath criterion in the presence of collision and ionization. The negative ions are assumed to follow their Boltzmann distribution. Considering their density as n_N along with background density n_{N0} (density in the plasma regime) and their temperature T_N, this is written as:

$$n_N = n_{N0} e^{\frac{e\phi}{k_B T_N}} \tag{1}$$

The cold electrons are specified in terms of n_c, n_{c0}, T_c and q as their density, background density, temperature and nonextensivity of the system, respectively. However, the hot electrons are specified in terms of n_h, n_{h0} and T_h. Their respective distributions are written below:

$$n_c = n_{c0}(1+(q-1)\frac{e\phi}{k_B T_c})^{\frac{q+1}{2(q-1)}} \tag{2}$$

$$n_h = n_{h0}(1+(q-1)\frac{e\phi}{k_B T_h})^{\frac{q+1}{2(q-1)}} \tag{3}$$

The positive ions are specified in terms of n_P, v_P and υ_{iz} as their density, velocity and ionization frequency, respectively. Their behaviour is governed by the usual fluid equations, means by the following continuity equation and equation of motion

$$\frac{\partial}{\partial x}(n_P v_P) - \upsilon_{iz} n_c = 0 \tag{4}$$

$$M_P n_P v_P \frac{\partial v_P}{\partial x} + Z_P e n_P \frac{\partial \phi}{\partial x} + k_B T_P \frac{\partial n_P}{\partial x} +$$

$$M_P n_P \upsilon_c v_P + M_P \upsilon_{iz} n_c v_P = 0 \tag{5}$$

Here M_P, Z_P and υ_c are mass of the positive ions, charge on the positive ions and collision frequency, respectively. In the above equation (5) the fourth term is the collisional force and fifth term is the ion source term, ϕ is the electric potential and T_P is the temperature of the positive ions.

Finally, the Poisson's equation in 1D is written as

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{e}{\varepsilon_0}(Z_P n_P - Z_N n_N - n_c - n_h) = 0 \tag{6}$$

Here we have taken the charge on the negative ions as Z_N. Considering n_{P0} as the positive ion background density, we can write the quasi-neutrality condition as

$$Z_P n_{P0} - Z_N n_{N0} - n_{c0} - n_{h0} = 0 \tag{7}$$

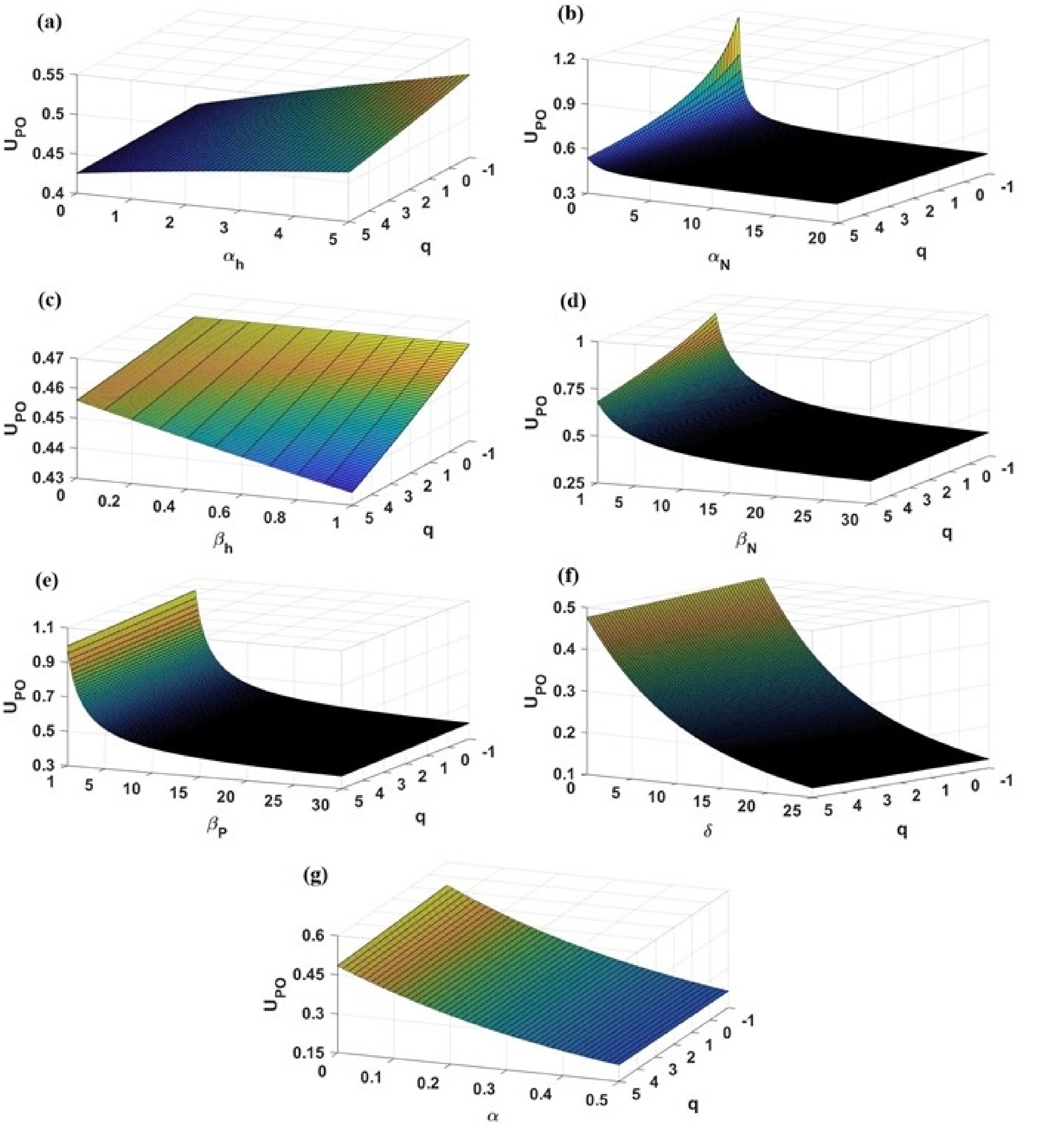

Figure 1. Normalized lower limit of the positive ion velocity at the sheath edge as a function of the non-extensive parameters with distinct α_h (a) when $Z_P = 1$, $Z_N = 1$, $\alpha = 0.05$, $\beta_N = 10$, $\beta_P = 10$, $\beta_h = 0.6$, $\alpha_N = 5$, $\delta = 1$ and $\psi'_0 = 0.1$; distinct α_N (b) when $Z_P = 1$, $Z_N = 1$, $\alpha = 0.05$, $\beta_N = 10$, $\beta_P = 10$, $\beta_h = 0.6$, $\alpha_h = 2$, $\delta = 1$ and $\psi'_0 = 0.1$; distinct β_h (c) when $Z_P = 1$, $Z_N = 1$, $\alpha = 0.05$, $\beta_N = 10$, $\beta_P = 10$, $\alpha_h = 2$, $\alpha_N = 5$, $\delta = 1$ and $\psi'_0 = 0.1$; distinct β_N (d) when $Z_P = 1$, $Z_N = 1$, $\alpha = 0.05$, $\beta_h = 0.6$, $\beta_P = 10$, $\alpha_h = 2$, $\alpha_N = 5$, $\delta = 1$ and $\psi'_0 = 0.1$; distinct β_P (e) when $Z_P = 1$, $Z_N = 1$, $\alpha = 0.05$, $\beta_N = 10$, $\beta_h = 0.6$, $\alpha_h = 2$, $\alpha_N = 5$, $\delta = 1$ and $\psi'_0 = 0.1$; distinct δ (f) when $Z_P = 1$, $Z_N = 1$, $\alpha = 0.05$, $\beta_N = 10$, $\beta_P = 10$, $\alpha_h = 2$, $\alpha_N = 5$, $\beta_h = 0.6$ and $\psi'_0 = 0.1$; and distinct α (g) when $Z_P = 1$, $Z_N = 1$, $\delta = 1$, $\beta_N = 10$, $\beta_P = 10$, $\alpha_h = 2$, $\alpha_N = 5$, $\beta_h = 0.6$ and $\psi'_0 = 0.1$. Here the prime denotes the first derivative.

3. Normalization parameters and normalized equations

The equations (1)-(7) are normalized with the appropriated normalized parameters, defined as follows $\psi = -\frac{e\phi}{k_BT_c}$, $\xi = \frac{x}{\lambda_{de}}$, $N_P = \frac{n_P}{n_{c0}}$, $N_N = \frac{n_N}{n_{c0}}$, $N_c = \frac{n_c}{n_{c0}}$, $N_h = \frac{n_h}{n_{c0}}$, $\alpha_N = \frac{n_{N0}}{n_{c0}}$, $N_{P0} = \frac{n_{P0}}{n_{c0}}$, $\alpha_h = \frac{n_{h0}}{n_{c0}}$, $U_P = \frac{v_P}{c_s}$, $\beta_P = \frac{T_c}{T_P}$, $\beta_N = \frac{T_c}{T_N}$, $\beta_h = \frac{T_c}{T_h}$, $\alpha = \frac{\lambda_{de}}{\Lambda}$, $\Lambda = \frac{c_s}{\upsilon_{iz}}$ and $\delta = \frac{\upsilon_c}{\upsilon_{iz}}$ together with $\lambda_{de} = \sqrt{\frac{\varepsilon_0 k_B T_c}{n_{c0}e^2}}$ and $c_s = \sqrt{\frac{k_BT_c}{M_P}}$.

Here, δ and α are respectively the collisional parameter and non-neutrality parameter. With the use of the above normalization parameters, the equations (1)-(7) in the dimensionless form appear as follows

$$N_N - \alpha_N e^{-\psi\beta_N} = 0 \tag{8}$$

$$N_c - [1-(q-1)\psi]^{\frac{q+1}{2(q-1)}} = 0 \tag{9}$$

$$N_h - \alpha_h[1-\beta_h(q-1)\psi]^{\frac{q+1}{2(q-1)}} = 0 \tag{10}$$

$$N_P\frac{\partial U_P}{\partial \xi} + U_P\frac{\partial N_P}{\partial \xi} - \alpha[1-(q-1)\psi]^{\frac{q+1}{2(q-1)}} = 0 \tag{11}$$

$$U_P\frac{\partial U_P}{\partial \xi} - Z_P\frac{\partial \psi}{\partial \xi} + \frac{\acute{N}_P}{\beta_P N_P} + U_P\alpha\delta + \frac{U_P\alpha[1-(q-1)\psi]^{\frac{q+1}{2(q-1)}}}{N_P} = 0 \tag{12}$$

$$\frac{\partial^2\psi}{\partial\xi^2} - Z_PN_P + Z_N\alpha_N e^{-\psi\beta_N} + [1-(q-1)\psi]^{\frac{q+1}{2(q-1)}} + \alpha_h[1-\beta_h(q-1)\psi]^{\frac{q+1}{2(q-1)}} = 0 \tag{13}$$

$$Z_PN_{P0} - Z_N\alpha_N - 1 - \alpha_h = 0 \tag{14}$$

4. Modified Bohm's criterion

The Bohm's criterion is derived here that reveals the appropriate values of U_{P0}. We multiply both sides of the Poisson's equation (13) by $\partial\psi/\partial\xi$ and then integrate it once to get

$$\frac{(\frac{\partial\psi}{\partial\xi})^2}{2} - \frac{(\frac{\partial\psi_0}{\partial\xi})^2}{2} = -V(\psi, U_{P0}) \tag{15}$$

Here $\partial\psi_0/\partial\xi$ is the normalized electric field at the sheath edge and $V(\psi, U_{P0})$ is the Sagdeev potential, given as

$$V(\psi,U_{p0}) = -Z_P\int_0^\psi N_P d\psi - \frac{Z_N\alpha_N e^{-\psi\beta_N-1}}{\beta_N} - \frac{2}{3q-1}\{[1-(q-1)\psi]^{\frac{3q-1}{2(q-1)}} - 1\} - \frac{2\alpha_h}{(3q-1)\beta_h}\{[1-\beta_h(q-1)\psi]^{\frac{3q-1}{2(q-1)}} - 1\} \tag{16}$$

Following boundary conditions must be satisfied by the Sagdeev potential at the sheath edge:

$$V(0,U_{P0}) = 0 and \frac{\partial}{\partial\psi}V(0,U_{P0}) = 0$$

Also, the maximizing condition for the Sagdeev potential V, i.e. $\partial^2 V(0,U_{P0})/\partial\psi^2 < 0$ yields

$$\frac{\partial^2 V(0,U_{P0})}{\partial\psi^2} = -\frac{Z_P\alpha}{\frac{\partial\psi_0}{\partial\xi}U_{P0}} + \frac{Z_PN_{P0}\frac{\partial U_{P0}}{\partial x}}{\frac{\partial\psi_0}{\partial\xi}U_{P0}} - Z_N\alpha_N\beta_N - \frac{q+1}{2} - (\frac{q+1}{2})\alpha_h\beta_h < 0$$

or

$$\frac{\partial U_{P0}}{\partial\xi} < [Z_N\alpha_N\beta_N + \frac{q+1}{2} + (\frac{q+1}{2})\alpha_h\beta_h]\frac{\frac{\partial\psi_0}{\partial\xi}U_{P0}}{Z_PN_{P0}} \tag{17}$$

Equation of motion (12) at the sheath edge reads

$$U_{P0}\frac{\partial U_{P0}}{\partial\xi} = Z_P\frac{\partial\psi_0}{\partial\xi} - \frac{\frac{\partial N_{P0}}{\partial\xi}}{\beta_PN_{P0}} - U_{P0}\alpha\delta - \frac{U_{P0}\alpha}{N_{P0}} \tag{18}$$

In the plasma, for the positive ions to enter in the sheath regime, the essential condition is $\partial\psi_0/\partial\xi > 0$ due to the existence of the neutral drag to the positive ions $\partial U_{P0}/\partial\xi \geq 0$. In other words, to overcome the impact of collisional drag, an accelerating field is necessary. Therefore, equation (18) becomes

$$U_{P0}^2(\frac{\alpha}{N_{P0}} + \alpha\delta) - Z_PU_{P0}\frac{\partial\psi_0}{\partial\xi} + \frac{\alpha}{N_{P0}\beta_P} \leq 0 \tag{19}$$

On solving equations (17) and (18), we have

$$\frac{\frac{\partial\psi_0}{\partial\xi}U_{P0}^2}{Z_PN_{P0}}[Z_N\alpha_N\beta_N + \frac{q+1}{2} + (\frac{q+1}{2})\alpha_h\beta_h] + (\frac{2\alpha}{N_{P0}} + \alpha\delta)U_{P0} - \{Z_P\frac{\partial\psi_0}{\partial\xi} + \frac{\frac{\partial\psi_0}{\partial\xi}}{\beta_PN_{P0}}[Z_N\alpha_N\beta_N +$$

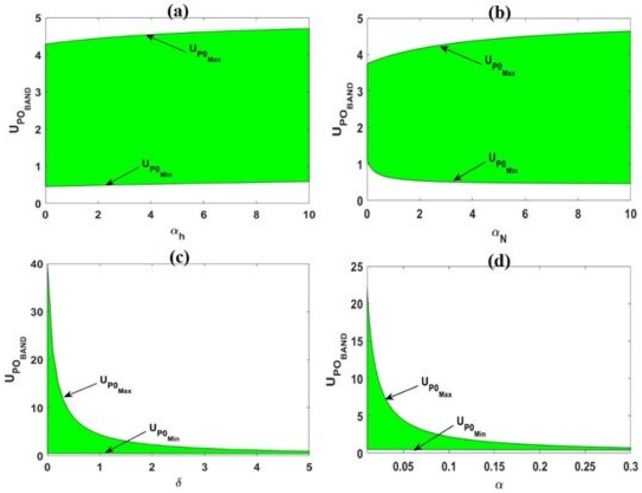

Figure 2. Normalized allowed band for the initial value of U_{P0}, i.e. $U_{P0_{max}} - U_{P0_{min}}$ as a function of α_h (a) when Z_P = 1, Z_N = 1, q = 0.5, α = 0.05, β_N = 10, β_P = 10, β_h = 0.6, α_N = 5, δ = 1 and $\acute{\psi}_0$ = 0.25; α_N (b) when Z_P = 1, Z_N = 1, q = 0.5, α = 0.05, β_N = 10, β_P = 10, β_h = 0.6, α_h = 2, δ = 1 and $\acute{\psi}_0$ = 0.25; δ (c) when Z_P = 1, Z_N = 1, q = 0.5, α = 0.05, β_N = 10, β_P = 10, β_h = 0.6, α_N = 5, α_h = 2 and $\acute{\psi}_0$ = 0.25; and α (d) when Z_P = 1, Z_N = 1, q = 0.5, β_N = 10, β_P = 10, β_h = 0.6, α_N = 5, α_h = 2, δ = 1 and $\acute{\psi}_0$ = 0.25. Here the prime denotes the first derivative.

$$\frac{q+1}{2} + (\frac{q+1}{2})\alpha_h\beta_h]\} \geq 0 \tag{20}$$

Equations (19a) and (19b) are of the form of $ax^2+bx+c=0$, whose solutions are

$$x = \frac{-b+\sqrt{b^2-4ac}}{2a} and \frac{-b-\sqrt{b^2-4ac}}{2a}$$

The first solution for both the equations is appropriate. This is because of the fact that the condition of $U_{P0} > 0$ is satisfied. Therefore, for four-component collisional electronegative warm plasma considered in the present model, the modified Bohm's criterion is obtained as

$$\frac{-\frac{(\frac{2\alpha}{N_{P0}}+\alpha\delta)}{\frac{\partial\psi_0}{\partial\xi}} + \sqrt{[\frac{(\frac{2\alpha}{N_{P0}}+\alpha\delta)}{\frac{\partial\psi_0}{\partial\xi}}]^2 + \frac{4}{Z_P N_{P0}}\{[Z_N\alpha_N\beta_N + \frac{q+1}{2} + (\frac{q+1}{2})\alpha_h\beta_h]\cdot\{Z_P + \frac{1}{\beta_P N_{P0}}[Z_N\alpha_N\beta_N + \frac{q+1}{2} + (\frac{q+1}{2})\alpha_h\beta_h]\}\}}}{\frac{2}{Z_P N_{P0}}[Z_N\alpha_N\beta_N + \frac{q+1}{2} + (\frac{q+1}{2})\alpha_h\beta_h]}$$

$$U_{P0} \leq \frac{Z_P\frac{\partial\psi_0}{\partial\xi} + \sqrt{(Z_P\frac{\partial\psi_0}{\partial\xi})^2 - 4(\frac{\alpha}{N_{P0}}+\alpha\delta)(\frac{\alpha}{N_{P0}\beta_P})}}{2(\frac{\alpha}{N_{P0}}+\alpha\delta)}$$

Evidently lower and upper limits for the positive ion velocity are seen at the sheath edge, leading to a velocity band for the Bohm's criterion.

We can discuss the limiting cases of the modified Bohm's criterion for its generality. In the absence of the hot electrons ($\alpha_h \to 0$) and negative ions ($\alpha_N \to 0$), the modified Bohm's criterion (20) for singly charged positive ions ($Z_P = 1$) reads

$$\frac{-\frac{2\alpha+\alpha\delta}{\frac{\partial\psi_0}{\partial\xi}} \sqrt{[\frac{2\alpha+\alpha\delta}{\frac{\partial\psi_0}{\partial\xi}}]^2 + 2(q+1)(1+\frac{q+1}{2\beta_P})}}{q+1} \leq U_{P0} \tag{21}$$

This result is the same as the one obtained by Bojaddaini and Chatei [35].

For single species of non-extensive electrons and cold electropositive collisionless plasma, i.e., in the limit $\alpha_h \to 0$, $T_P \to 0$ and $\alpha_N \to 0$, the lower limit of the equation (20) reads

$$\sqrt{\frac{2}{q+1}} \leq U_{P0} \tag{22}$$

This result is in agreement with the result of Tribeche et al. [33] and Gougam and Tribeche [34].

For the case of Maxwellian distribution of the electrons, i.e. in the limit q→ 1, one can find the following from equation (22)

$$1 \leq U_{P0} \tag{23}$$

This is the same result as obtained by Chen [36]. The above limiting cases substantiate the generality of the modified Bohm's criterion obtained here.

5. Results on modified Bohm's criterion

In order to get more insight into the modified Bohm's criterion, we plot here various graphs showing the variation of the minimum velocity and maximum velocity of the positive ions under the effect of various plasma parameters.

5.1 Lower limit on ion velocity

The normalized lower limit of the positive ion velocity U_{P0} at the sheath edge is given by Eq. (20) and its behaviour as a function of the non-extensive parameter q is portrayed in Fig. 1 with a different set of the parameters α_h (Fig. 1(a)), α_N ((Fig. 1(b))), β_h (Fig. 1(c)), β_N (Fig. 1(d)), β_P (Fig. 1(e)), δ (Fig. 1(f)) and α (Fig. 1(g)). In all the cases, value of U_{P0} is reduced slightly with an increased non-extensivity of the system. The velocity $P0$ is enhanced with an increment in the α_h, i.e. the hot-to-cold electron density. A significant reduction in the magnitude of U_{P0} is observed with the increasing β_P, β_N, δ, α and α_N, and an insignificant impact of β_h is obtained on the magnitude of U_{P0}. Most of these results are in agreement with the observation of other researchers, for example those of Refs. [37–41]. For our system of collisional electronegative warm plasma, the magnitude of U_{P0} is found to be different from that of the cold collisionless electropositive plasma obtained by Chen [36]. In the present case, the inequality obtained in Eq. (20) is termed as the modified Bohm's sheath criterion. A modification in Bohm's sheath criterion

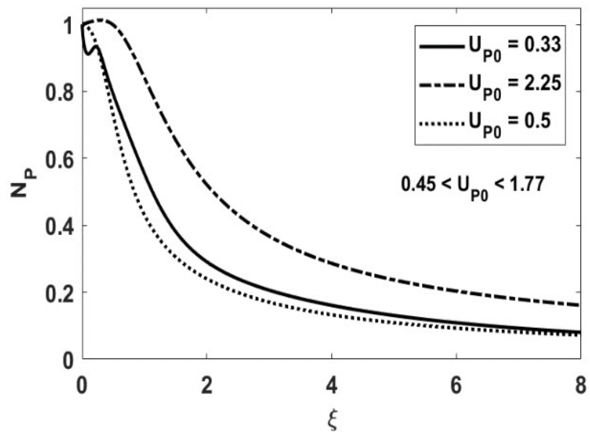

Figure 3. Normalized positive ion density as a function of the normalized distance from the sheath edge (ξ = 0) towards the wall/probe position for distinct initial values of the U_{P0} when Z_P = 1, Z_N = 1, q = 0.1, α = 0.05, β_N = 10, β_P = 10, β_h = 0.6, α_N = 5, α_h = 2, δ = 1 and $\acute{\psi}_0$ = 0.1. Here the prime denotes the first derivative.

results in modification of number of positive ions which are capable of entering into sheath region [12]. Consequently, a significant modification in the sheath formation mechanism is expected [25,37]. It is expected that the sheath thickness will increase with the increase of collisions and hot electrons temperature [25] whereas a reduced sheath thickness is expected to be observed for increasing non-extensivity [35], positive ion temperature [42], ionization [43] and cold electron density.

5.2 Allowed ion velocity band at the sheath edge, i.e. $U_{P0_{max}} - U_{P0_{min}}$

The allowed band for the initial values of U_{P0}, i.e. ($U_{P0_{max}} - U_{P0_{min}}$) as a function of α_h (Fig. 2(a)), α_N (Fig. 2(b)), δ (Fig. 2(c)) and α (Fig. 2(d)) is shown in Fig. 2. This band for the positive ion velocity is found to reduce significantly with increasing δ and α. Actually the band is quite large for the smaller values of δ and α, which becomes very narrow for their larger values. Thus, one must be careful while choosing the initial value of U_{P0} for higher collisional and non-neutrality parameters. However, the band does not show such a sharp variation with α_h and α_N.

The parameter δ describes the collisionality of the system and amount to the ratio of collisional frequency (v_c) to the ionization frequency (v_{iz}). The situation of $\delta \rightarrow 0$ corresponds to $v_c \rightarrow 0$, i.e. collision-less plasma. In collisionless or weakly collisional plasma, there are no restrictions on upper limits of positive ion velocity, i.e. their magnitude can be several times of ion sound velocity. Also, for such plasmas, it is only lower limit of positive ion velocity which is of greater interest. However, when number of collisions are considerable, i.e. δ takes a finite value then upper limit in addition to lower limits of positive ion velocity is strongly influenced. As the collisionality of the system increases, upper limit of the positive ion velocity decreases with an abrupt rate. The larger velocity band for the smaller values of δ corresponds to the transition of plasma from collisionless/weakly collisional to highly collisional plasma.

Negative ions help positive ions to get into the sheath region. Therefore, as negative ions are introduced in the system, i.e. α_N takes finite values, then an increment in the velocity band is seen. However, with increasing α_N, the system is dominantly governed by the negative ions only. Therefore, $U_{P0_{min}}$ does not change considerably and almost a constant velocity band is recorded for higher α_N.

5.3 Validation of the modified Bohm's criterion

The allowed range of U_{P0} for a given set of plasma parameters (α=0.05,q=0.1,β_N=10,β_P=10,β_h=0.6,α_N=5,δ=1,α_h=2 and ($\partial\psi_0/\partial\xi$=0.1) comes out to be 0.45 $\leq U_{P0} \leq$ 1.77. We have considered three different initial values of U_{P0}, i.e. U_{P0} = 0.33, 0.5 and 2.25, to validate the calculated modified Bohm's criterion, and also depicted the results in Fig. 3.

Case (a): When U_{P0} = 0.33, i.e., $U_{P0} < U_{P0_{min}}$, then the accelerating force, i.e., the electric force on the positive ions is dominated over the collisional force that decelerates the ions. Consequently, the positive ions are accelerated with a rapid rate near the sheath edge and accordingly a sharp reduction in the density of the positive ions is observed near the sheath edge to meet the law of conservation of flux.

Case (b): When U_{P0} = 2.25, i.e., $U_{P0} > U_{P0_{max}}$, then the deaccelerating collisional force on the positive ions is dominated over the electric force. Therefore, the positive ion velocity would be reduced near the sheath edge and increased N_P is observed there.

Case (c): When U_{P0} = 0.5, i.e. it is within the allowed range $U_{P0_{min}} < U_{P0} < U_{P0_{max}}$, then there is a balancing between the two forces, which in turn results in the smooth reduction of the positive ion density in the sheath regime.

The magnitude of $\partial\psi_0/\partial\xi$ turns out to be negative for the cases when U_{P0} = 0.33, i.e., $U_{P0} < U_{P0_{min}}$ and U_{P0} = 2.25, i.e., $U_{P0} > U_{P0_{max}}$. In other words, $\acute{U}_{P0} \leq 0$ or $\partial\psi_0/\partial\xi < 0$ is resulted for these cases. Consequently, a deceleration force will be experienced by the positive ions and they should move opposite to the probe or towards the plasma region. This, in turn, avoids the confinement of the positive ions within the sheath region. However, Fig. 3 portrayed a considerable concentration of the positive ions within the sheath region which seems to be physically invalid. Moreover, under the situation of $U_{P0} < U_{P0_{min}}$, an oscillatory behaviour has been seen, and under the situation of $U_{P0} > U_{P0_{min}}$, normalized positive ion density takes value greater than 1. Both of these behaviours are not physically acceptable because in the physically acceptable situations, a smooth variation of positive ions density must be recorded within the sheath region. For the cases (a) and (b), the Bohm's criterion is not satisfied. Also, for these cases, the shape of the profile of the positive ion density in the sheath regime is not physically acceptable, especially near the sheath edge. This incorrect value is because of the violation of the modified Bohm's criterion. Therefore, in reality no sheath would be formed for these two cases. On the other hand, for the case (c), the Bohm criterion is satisfied and a smooth reduction of the positive ion density in the sheath regime is observed. Therefore, a sheath will be formed only for the case (c).

The present investigation considered the q-nonextensive nature of the electrons in an electronegative plasma. This was done in an unmagnetized plasma, but the work can be extended in the presence of field having profile similar to the one taken in other works [44–46] and also can be verified by talking about the limiting cases to the investigations carried out elsewhere [47–50].

6. Conclusion

Our calculations show that the Bohm's criterion is modified when one considers a collisional electronegative plasma having two-temperature non-extensive electrons for the plasma-material interaction. This criterion puts an extra condition on the ion velocity that there is also a maximum value for the velocity in addition to its minimum allowed value at the sheath edge. More specifically, the ions whose velocity falls within a velocity-band decided by the Bohm's criterion would be able to enter into the sheath. This velocity-band for the positive ions is found to reduce significantly with increasing collision and ionization frequency in the plasma. For the decreased collision and ionization frequency, a larger band is seen, which, though does not show such a variation with hot-to-cold electron density ratio and the density ratio of negative ions and cold electrons.

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Conflict of interest statement:

The authors declare that they have no conflict of interest.

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