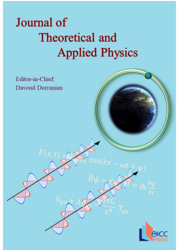

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Ion acoustic cnoidal waves in electron-positron-ion plasmas with q-nonextensive electrons and positrons and high relativistic ions

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Abstract:
In this paper propagation of the nonlinear cnoidal ion-acoustic waves in unmagnetized electron-positron-ion plasma have been studied. The nonextensivity distribution function was used to describe the plasma electrons and positrons, while plasma ions are taken high relativistic. We have used the reductive perturbation method (RPM) to study the characteristic of ionacoustic cnoidal waves in this three-component plasma. The Korteweg-de Vries equation, which describes the nonlinear waves in such plasma, has been derived. In this work, we have investigated the effects of relativistic ions and q-nonextensive distribution of electrons and positrons on the characteristics of the ionacoustic periodic (cnoidal) wave, such as the amplitude, wavelength, and frequency.

Keywords: Plasma nonextensivity; Electron-positron-ion plasma; Ion acoustic nonlinear wave; Cnoidal wave; Sagdeev potential; High relativistic plasma; Weakly relativistic plasma

1. Introduction

In plasma physics, an ion acoustic wave is one type of longitudinal oscillation of the ions and electrons in a plasma, much like acoustic waves traveling in neutral gas. However, because the waves propagate through positively charged ions, ion acoustic waves can interact with their electromagnetic fields, as well as simple collisions. In fluid dynamics, a ion acoustic cnoidal wave (IACW) is a nonlinear and exact periodic wave solution of the equation in terms of Jacobin elliptical-functions *cn*, which is why they are coined cnoidal waves. The cnoidal wave solutions were derived by Korteweg and de Vries, in their 1895 paper in which they also propose their dispersive long-wave equation, now known as the Korteweg–de Vries equation [1]. In the limit of infinite wavelength, the cnoidal wave becomes a solitary wave. These periodic functions are believed to be generated in defocusing regime of plasmas and have many important applications in diverse areas of physics such as nonlinear transport phenomena [2–4]. The characteristics of nonlinear IACW in a field free or magnetized plasma has been investigated frequently in recent years [5–8].
In this study, The IACW has been considered in electron-positron-ion (e–p–i) plasma. Recently, the (e-p-i) plasmas have attracted the attention of several authors [9, 10]. They have studied linear and nonlinear wave propagation in (e–p–i) plasmas using different models. The presence of positrons can lead to the reduction in frequency and amplitude wave in such system [11]. The response of plasma does not exist in electron-ion (e–i) plasmas. Since the positron annihilation time could be much larger than the characteristic time scale for the ion acoustic wave, the (e–p–i) plasma can also appear in laboratory plasma such as in tokamaks to probe particle transport and other magnetic confinement systems. It is well known that the e–p–i plasma may exist in several astrophysical situations such as active galactic nuclei, bipolar outflows, pulsars magnetosphere, early universe, etc [12–15].
Present long-range interactions, long-time memory or fractality of the corresponding space-time/phase-space within the conventional Boltzmann-Gibbs (BG) statistics are disobedient [16, 17]. The main reason for this inability is that BG statistics is an additive or extensive formalism. In dealing with the statistical properties of systems with long-range correlations, Tsallis extended BG thermodynamics by generalizing the concept of entropy to nonextensive

regimes [18]. Nonextensivity means that the entropy of the composition $(A+B)$ of two independent systems A and B is equal to $S_Q^{(A+B)} = S_Q^{(A)} + S_Q^{(b)} + (1-Q)S_Q^{(A)}S_Q^{(B)}$, where the parameter Q that underpins the generalized entropy of Tsallis is linked to the underlying dynamics of the system and provides a measure of the degree of its correlation among the various physical systems where connections with the Tsallis entropy have been found are plasma physics, long-range Hamiltonian systems, gravitational systems and many other applications. As investigated by several researchers, the Q-nonextensive formalism may be very important for systems with long-range interactions such as astrophysics and plasma physics. In this regard, they showed that the experimental results point to non-Maxwellian velocity distribution. Since that, the Tsallis q-entropy and the generalized statistics have been used with success in plasma physics [19–38]. To study all possible astrophysical scenarios,it is wise to follow the nonextensive distribution. As electrons and positrons have the same mass but opposite charge, it is expected that they will be described by a similar distribution.

Most of investigations on linear and nonlinear phenomena are limited to nonrelativistic plasmas. But when the electron or ion velocity (Ve,i) approaches the velocity of light, relativistic effects may significantly modify the IACW behavior [39–41]. In relativistic plasmas relativistic corrections to a particle's mass and velocity are important.Such corrections typically become important when a significant number of electrons or ion reach speeds greater than 0.86 c (Lorentz factor Γ=2).Such plasmas may be created either by heating a gas to very high temperatures or by the impact of a high-energy particle beam. Relativistic plasma with a thermal distribution function has temperatures greater than around 260 keV. The primary changes in plasma's behavior as it approaches the relativistic regime are slight modifications to the equations which describe a nonrelativistic plasma and to collision and interaction cross sections. The equations may also need modifications to account for pair production of electron-positron pairs (or other particles at the highest temperatures). Relativistic plasmas occur in a variety of situations, e.g. in laser–plasma interaction [42], space-plasma phenomena [43], plasma sheet boundary layer of earth's magnetosphere [44], in the Van Allen radiation belts [45].

In 1984 Das and Paul studied ion-acoustic solitary waves in relativistic plasmas and derived a Kortewag-de Vries (KdV) equation for collisionless plasma with cold ions and without electron inertia [46]. Nejoh has investigated the same with warm plasma [47]. Kalita et al. have investigated the existenceof solitons considering the complete fluid equation of electrons [48].

In 1987 Roychoudhury and Sikha found solitary wave solutions in a relativistic plasma [49] Gill et al. investigated ion-acoustic solitary waves in weakly relativistic plasmas containing electron-positron-ion plasma with Boltzmann positrons and electrons and cold ions [50]. Pakzad researched the same situation in weakly relativistic plasma with thermal positrons, nonextensive electrons, and cold ions [51] and in another paper Pakzad et al. studied ion-acoustic solitary waves in electron-positron-ion plasmas with Q-nonextensive electrons and high relativistic ions [52]. In 2006 Biswajit Sahu et al. studied electron acoustic solitons in a relativistic plasma with nonthermal electrons [53]. Hafez et al. studied ion acoustic shock and solitary waves in highly relativistic plasmas with nonextensive electrons and positrons [54].

The research in this field includes two categories, one is those that assume only ions to be relativistic and the role of inertia of electrons in a relativistic plasma is usually ignored, such as Nejoh and Sanuki have considered large amplitude Langmuir and ion-acoustic waves in a relativistic two fluid plasmas deriving the pseudopotential [55]. Of course, large amplitude relativistic ion acoustic waves are discussed for the simple case m_e= 0 and T_i= 0 without the pressure variation equation. The second are those that consider both electrons and ions to be relativistic at simultaneously. The equation of pressure changes is used. Such as Kuehl and Zhang have considered first the effect of electron inertia in a relativistic plasma [56]. No investigation has been reported on the study of IACW in relativistic plasmas. As electron–positron, plasmas with ion possessing relativistic velocities are frequently observed in astrophysical and space environments, so there is a need to study IACW in such plasma system.

In this work, we study IACW in electron–positron plasma with high relativistic ion it is assumed only ions to be relativistic and the role of inertia of electrons ignored. Our aim of this study therefore to recognize effects of nonextensive Q-parameter of electrons with relativistic ions on the IACW in plasmas unmagnetized. Our research may be very helpful for astronomers and can help us to find better knowledge about the effects of relativistic particles in plasmas.

The manuscript is organized as follows: following the introduction in Sec. 1 the basic equations, governing our plasma system, are presented in Sec. 2. Using the reductive perturbation method, the KdV equation is derived in Sec. 3. Cnoidal solution of the KdV equation is discussed in Sec. 4. Sec. 5 is devoted to results and discussion, and finally, we present our main conclusion in Sec. 6.

2. Basic equations

We assume unmagnetized and collision less plasma consisting of a mixed fluid with nonextensive distributed positrons and electrons and cold ions. Moreover, it is assumed that ion velocity has high relativistic effect, and the ion acoustic wave propagate in the z direction. The nonlinear dynamics of the ion acoustic waves is governed by the continuity and motion equations for ion fluid, and the Poisson's equation:

$$\partial_t N + \partial_z(NV) = 0 \tag{1}$$

$$\partial_t(\Gamma V) + V\partial_z(\Gamma V) = -\partial_z \Phi \tag{2}$$

$$\partial_z^2 \Phi = N_e - \Pi N_p - (1-\Pi)N \tag{3}$$

$$N_e = [1+(Q-1)\Phi]^{\frac{Q+1}{2(Q-1)}} \tag{4}$$

$$N_p = [1-(Q-1)T\Phi]^{\frac{Q+1}{2(Q-1)}} \tag{5}$$

They approximated for small Φ

$$N_e = 1+\left(\frac{Q+1}{2}\right)\Phi+\frac{(Q+1)(3-q)}{8}\Phi^2+... \quad (6)$$

$$N_p = 1-\left(\frac{Q+1}{2}\right)T\Phi+\frac{(Q+1)(3-q)}{8}T^2\Phi^2+... \quad (7)$$

The normalization has been made by the following non-dimensional variables:

$$V \to \frac{V}{\sqrt{\frac{kT_e}{m}}}, \qquad \Phi \to \frac{e\Phi}{kT_e}$$

$$t \to \frac{t}{\sqrt{\frac{m\varepsilon_0}{N_0 e^2}}}, \qquad z \to \frac{z}{\sqrt{\frac{kT_e\varepsilon_0}{N_0 e^2}}}$$

Ion density N, electron density N_e, and positron density N_p are normalized with their corresponding equilibrium densities respectively.
In the above equations:

$$\Pi = \frac{N_{p0}}{N_{e0}} \quad (8)$$

Π: The fractional concentration of positron with respect to electron in the equilibrium

$$T = \frac{T_e}{T_p} \quad (9)$$

T: The temperature ratio of electron to positron.
T_p: The temperature of positron
T_e: The temperature of electron
N_e: The density of electron
N_{e0}: The equilibrium density of electron
N_p: The density of positron
N_{p0}: The equilibrium density of positron
N: The density of ion
Φ: The electrostatic potential
k: Boltzmann's constant
Γ is Lorentz relativistic factor and for weakly relativistic plasma it is approximated by its expansion up to second term ($V \ll c$) .

$$\Gamma = \left(1-\frac{V^2}{c^2}\right)^{-\frac{1}{2}} \cong 1+\frac{V^2}{2c^2} \quad (10)$$

and for a high relativistic plasma it is approximated by its expansion up to second term

$$\Gamma = \left(1-\frac{V^2}{c^2}\right)^{-\frac{1}{2}} \cong 1+\frac{V^2}{2c^2}+\frac{3V^4}{8c^4} \quad (11)$$

To model the effect of electron nonextensivity, we refer to the following Q-distribution function given by several authors [57–60].

$$f_e(v_e) = P_Q\left[1+(1-Q)\left(\frac{m_e v_e^2}{2T_e}-\frac{e\Phi}{T_e}\right)\right]^{\frac{1}{Q-1}} \quad (12)$$

The constant of normalization P_Q is given by

$$P_Q = N_{e0}\frac{\Gamma(\frac{1}{1-Q})}{\Gamma(\frac{1}{1-Q}-\frac{1}{2})}\sqrt{\frac{m_e(1-Q)}{2\pi T_e}}, \quad \text{for} \quad -1<Q<1 \quad (13)$$

$$P_Q = N_{e0}(\frac{1+Q}{2})\frac{\Gamma(\frac{1}{Q-1}+\frac{1}{2})}{\Gamma(\frac{1}{Q-1})}\sqrt{\frac{m_e(Q-1)}{2\pi T_e}}, \quad \text{for} \quad Q>1 \quad (14)$$

Here, the parameter Q defines for the strength of nonextensivity. It may be useful to note that for $Q<-1$, the Q-distribution is unnormalizable. In the extensive limiting case ($Q=1$), the Q-distribution reduces to the well-known Maxwell-Boltzmann distribution.

3. Derivation of KdV equation

To derive the KdV equation from the basic set of equations (1)-(5), We have used reductive perturbation method (RPM) in the present investigation. Besides RPM, homotopy perturbation method [61–66] and modified Lindstedt–Poincaré method [67–70] can also be powerfully applied. Homotopy perturbation method make full advantages of traditional perturbation method and homotopy techniques. Similarly, modified Lindstedt–Poincaré methods introduces a new transformation of the independent variables so as to avoid the occurrence of secular term in the perturbation series solution. RPM used here enable us to look for long waves. Mathematically we build scalelength in the original equations of motion by rescaling both space and time co-ordinates which are appropriate for the dispersion of the long wave length phenomena. For large class of dispersion system, KdV equation is the equation which governs such weak nonlinear long behavior. Suppose z is scaled in a certain way, then it is dispersion relation which gives information of how the time part of the system reacts. The dispersion relation for harmonic waves is found from the linearized version of the original set of equations. For example, for solution of the form $\exp(i\theta)$, we have

$$\theta = kz-\omega(k)t \quad (15)$$

where $\omega(k)$ is dispersion relation and k are wave number. For long wave (small k), we use limiting form of dispersion relation and write $k=\zeta^p K$ where K is new wave number and p is unknown number to be determined later. Then Eq. (8) becomes

$$\theta = K\zeta^p z-\omega(\zeta^p k)t \quad (16)$$

For dispersive system, Taylor expansion of $\omega(k)$ in Eq. (8) yields

$$\theta = K\zeta^p(z-\acute{\omega}(0)t)-k^3\zeta^{3p}\acute{\acute{\acute{\omega}}}(0)t \quad (17)$$

First and third derivative terms are constant, then above equation yield natural scaling for z and t as

$$\varsigma = \zeta^p(z-u_0 t), \quad \tau = \zeta^{3p}t \quad (18)$$

It often turns out that when KdV equation occurs, the p usually takes the value of 1/2. It may further be noted that this prescription is closely related to validity of hyperbolic approximation and similarity transformation [71]. It may be noticed that there are many complicated examples, where different types of asymptotic expansions are used in different regions of space. The scaling of space and time variables may also change from region to region. After a above description, according to RPM, the independent variables are scaled as [50]:

$$\varsigma = \zeta^{\frac{1}{2}}(z-u_0 t) \quad (19)$$

and

$$\tau = \zeta^{\frac{3}{2}} t \tag{20}$$

where ζ is a small parameter that characterizes the strength of the nonlinearity, and u_0 is the phase velocity of the wave to be determined later. Dependent variables are expanded as follows:

$$N = 1 + \zeta N_1 + \zeta^2 N_2 + \zeta^3 N_3 + \dots \tag{21}$$

$$V = V_0 + \zeta V_1 + \zeta^2 V_2 + \zeta^3 V_3 + \dots \tag{22}$$

$$\Phi = \zeta \Phi_1 + \zeta^2 \Phi_2 + \zeta^3 \Phi_3 + \dots \tag{23}$$

On substituting the expansion (22) into Eqs. (1)-(5), using Eqs. (19) and (20), and equating terms with the same powers of ζ, we obtain a set of equations. The set of equations at the lowest order is;

$$-u_0 \partial_\varsigma N_1 + \partial_\varsigma (V_1 + N_1 V_0) = 0 \tag{24}$$

$$-(u_0 - V_0)\Gamma_1 \partial_\varsigma V_1 + \partial_\varsigma \Phi_1 = 0 \tag{25}$$

$$\Phi_1 \left(\frac{Q+1}{2}(1+\Pi T) \right) - (1-\Pi) N_1 = 0 \tag{26}$$

Integrating Eqs. (24) and (25) with respect to ζ for continuous wave which can have finite perturbation ever at $\zeta \to \pm\infty$, we get a relationship among the first order perturbed quantities as

$$N_1 = \left[\frac{\frac{Q+1}{2}(1+\Pi T)}{1-\Pi} \right] \Phi_1 \tag{27}$$

$$V_1 = (u_0 - V_0) \left[\frac{\frac{Q+1}{2}(1+\Pi T)}{1-\Pi} \right] \Phi_1 \tag{28}$$

For the existence of a nontrivial solutions of the first order quantities, from Eqs. (27) and (28), we require

$$u_0 = \left[\frac{\frac{Q+1}{2}(1+\Pi T)\Gamma_1}{1-\Pi} \right] \Phi_1 + V_0 \tag{29}$$

where u_0 is the phase velocity of the cnoidal wave in the ion-acoustic wave frame. Taking $Q = 1$, Eq. (29) will be exactly the same with the equation which was reported by Chawlaet. al for nonextensive plasma [72].
The next higher-order equations are

$$-(u_0 - V_0)\partial_\varsigma N_2 + \partial_\tau N_1 + \partial_\varsigma V_2 + \partial_\varsigma (N_1 V_1) = 0 \tag{30}$$

$$-(u_0-V_0)\Gamma_1 \partial_\varsigma V_2 + \Gamma_1 \partial_\tau V_1 + [\Gamma_1 - 2\Gamma_2(u_0-V_0)] V_1 \partial_\varsigma V_1 + \partial_\varsigma \Phi \tag{31}$$

$$\partial_\varsigma^2 \Phi_1 = \left(\frac{Q+1}{2}(1+\Pi T) \right) \Phi_2$$
$$+ \left(\frac{(3-Q)(Q+1)}{4}(1-\Pi T^2) \right) \frac{\Phi_1^2}{2} - (1-\Pi) N_2 \tag{32}$$

where for high relativistic plasma we have

$$\Gamma_1 = 1 + \frac{3V_0^2}{2c^2} + \frac{15V_0^4}{8c^4} \tag{33}$$

and

$$\Gamma_2 = \frac{3V_0}{2c^2} + \frac{30V_0^3}{8c^4} \tag{34}$$

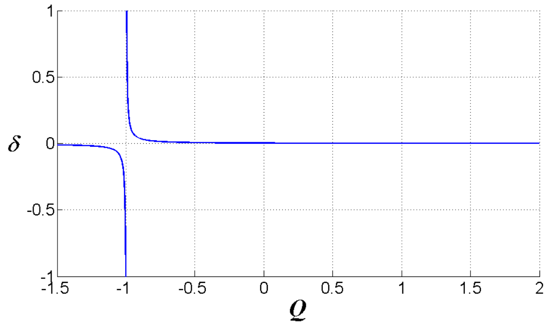

Figure 1. Plot of δ versus Q having $T = 0.1$, $\Pi = 0.1$, $V = 0.0075$, ζ= 0.8 and $\varepsilon = -0.002$

After some algebraic manipulations, second order quantities are eliminated and Φ_1 is found to satisfy the following KdV equation

$$\partial_\tau \Phi_1 + a\Phi_1 \partial_\varsigma \Phi_1 + b\partial_\varsigma^3 \Phi_1 = 0 \tag{35}$$

which is the required KdV equation and describes the evolution of the first order perturbed potential (Φ_1). The coefficients a and b are given by

$$a = \frac{3}{2\Gamma_1(u_0 - V_0)} + \frac{(u_0 - V_0)}{2} \left[\frac{(3-Q)(1-\Pi T^2)}{2(1+\Pi T)} \right] - \frac{\Gamma_2}{\Gamma_1} \tag{36}$$

$$b = \frac{1}{2} \left[\frac{(u_0 - V_0)}{\frac{Q+1}{2}(1+\Pi T)} \right] \tag{37}$$

a and b are the coefficient of nonlinear term and dispersion term of KdV equation respectively. Taking $V_0 = 0$, Eq. (36) and (37) will be exactly the same with the equation which was reported for nonrelativistic plasma and taking $Q = 1$, Eq. (36) and (37) will be exactly the same with the equation which was reported by Chawla et al. for nonextensive plasma [72]. Generally, the first term of a in Eq. (36) and the relation of b in Eq. (37) have the same form in all unmagnetized plasmas and the effect of electrons distribution function appears in the second term. Of course, u_0 is always under the influence of electrons distribution functions.

4. Cnoidal wave solution of KdV equation

In order to find the steady state cnoidal and solitary wave solutions of the KdV Eq. (35), we follow the same procedure as already done in Refs [73, 74], we consider the new

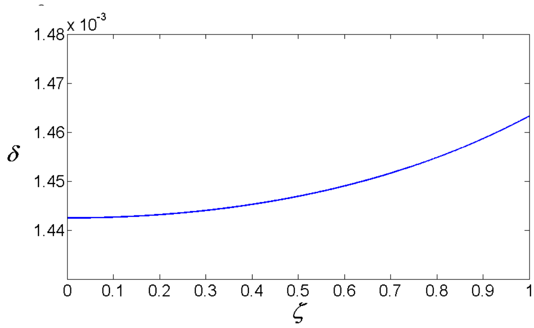

Figure 2. Plot of δ versus ζ having $T = 0.1$, $\Pi = 0.1$, $V = 0.0075$, Q= 0.1 and $\varepsilon = -0.002$

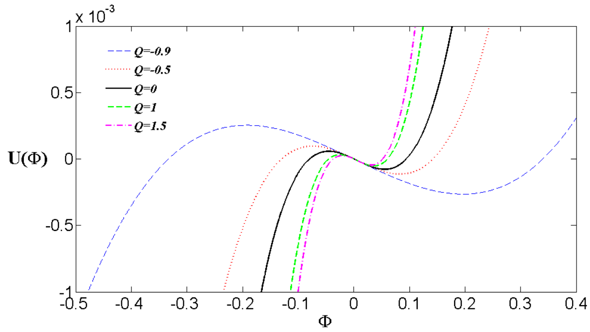

Figure 3. Plot of $U(\Phi)$ versus Φ for different Qs, having $T = 0.1$, $\Pi = 0.1$, $V = 0.0075$, $\zeta = 0.8$ and $\varepsilon = -0.002$

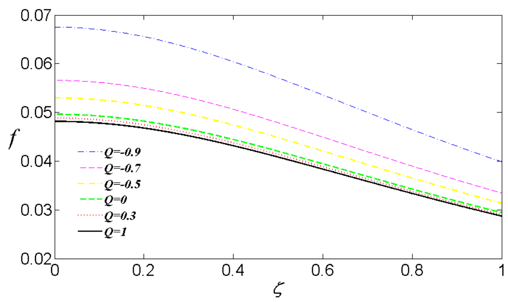

Figure 5. Plot of frequency of periodic wave (cnoidal) versus ζ for different Qs, having $T = 0.1$, $\Pi = 0.1$, $V = 0.0075$ and $\varepsilon = -0.002$

variable as $k = \varsigma - V\tau$ where V is the velocity of the nonlinear structure moving with the frame. Then for $\Phi_1 = \Phi_1(k)$ the function we obtain

$$b\partial_k^3\Phi_1 + \partial_k\left(a\frac{\Phi_1^2}{2} - V\Phi_1\right) = 0 \tag{38}$$

After integration of equation (38), we obtain the following equation of the conservative nonlinear oscillator:

$$\partial_k^2\Phi_1 = -\partial_{\Phi_1}U \tag{39}$$

where its potential energy is defined as

$$U(\Phi_1) = \frac{a}{6b}\Phi_1^3 - \frac{V}{2b}\Phi_1^2 + \varepsilon\Phi_1 \tag{40}$$

Here ε is the integration constant and the potential are defined to within the constant. To have the potential well, the following condition should be ful filled:

$$\left(\frac{V^2}{a^2} - 2\varepsilon\frac{b}{a}\right) > 0 \tag{41}$$

After the integration of equation (40), we obtain the following energy conservation law:

$$\frac{1}{2}\left(\frac{\partial\Phi_1}{\partial k}\right)^2 = \frac{1}{2}e_0^2 - U(\Phi_1) \tag{42}$$

where $e_0^2/2$ is the integration constant having the meaning of total energy of oscillations. In other words, ε and e_0 are

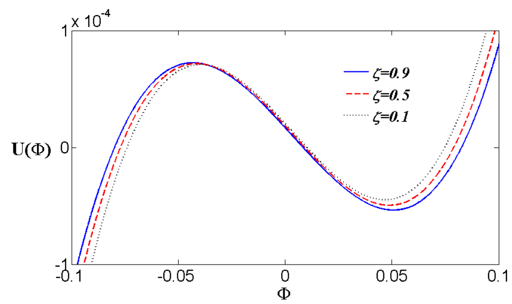

Figure 4. Plot of $U(\Phi)$ versus Φ for different ζs having $T = 0.1$, $\Pi = 0.1$, $V = 0.0075$, $Q = 0.1$ and $\varepsilon = -0.002$

the charge density and the electric field when the potential Φ_1 vanishes, respectively Using Eq. (40) in Eq. (42), we have

$$\left(\frac{\partial\Phi_1}{\partial k}\right)^2 = e_0^2 - \frac{a}{3b}\Phi_1^3 + \frac{V}{b}\Phi_1^2 - 2\varepsilon\Phi_1 \tag{43}$$

Let us consider the following initial conditions $\Phi_1(0) = \gamma_0$ and $(d\Phi_1(0)/dk = 0)$. Then we can define

$$e_0^2 = \frac{a}{3b}\gamma_0^3 + \frac{V}{b}\gamma_0^2 - 2\varepsilon\gamma_0 \tag{44}$$

Substituting Eq. (44) into Eq. (43), and after factorization, we have

$$\left(\frac{\partial\Phi_1}{\partial k}\right)^2 = \frac{a}{3b}(\gamma_0 - \Phi_1)(\Phi_1 - \gamma_1)(\Phi_1 - \gamma_2) \tag{45}$$

where

$$\gamma_{1,2} = \frac{3}{2}\left[\frac{V}{a} - \frac{\gamma_0}{3} \pm \sqrt{\frac{1}{3}(\theta_1 - \gamma_0)(\gamma_0 - \theta_2)}\right] \tag{46}$$

and

$$\theta_{1,2} = \frac{V}{a} \pm 2\sqrt{\delta} \tag{47}$$

and

$$\delta = \left(\frac{V}{a}\right)^2 - 2\varepsilon\frac{b}{a} \tag{48}$$

The γ_0, γ_1 and γ_2 are the real roots of Sagdeev potential the following inequalities should be kept [58].

$$\theta_1 \le \gamma_0 \le \theta_2 \quad \text{or} \quad \theta_2 \le \gamma_0 \le \theta_1$$

From Eqs. (38)-(42), we have the following relation,

$$V = \frac{a}{3}(\gamma_0 + \gamma_1 + \gamma_2) \tag{49}$$

The periodic (cnoidal) wave solution of Eq. (38) is given by [59, 60].

$$\Phi(k) = \gamma_1 + (\gamma_0 - \gamma_1)cn^2(Dk, m) \tag{50}$$

where cn is Jacobian elliptic function, whereas the parameters J $(0 < J < 1)$ and B are defined as

$$J^2 = \frac{\gamma_0 - \gamma_1}{\gamma_0 - \gamma_2} \qquad 0 < J < 1 \tag{51}$$

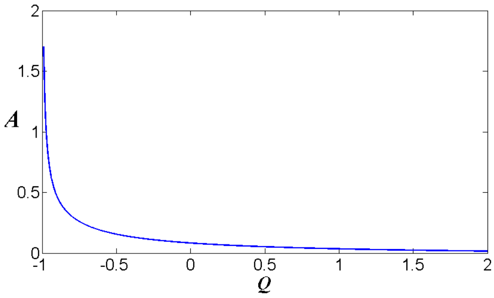

Figure 6. Plot of amplitude of periodic wave (cnoidal) versus Q, having $T = 0.1$, $\Pi = 0.1$, $V = 0.0075$, $\zeta = 0.8$ and $\varepsilon = -0.002$

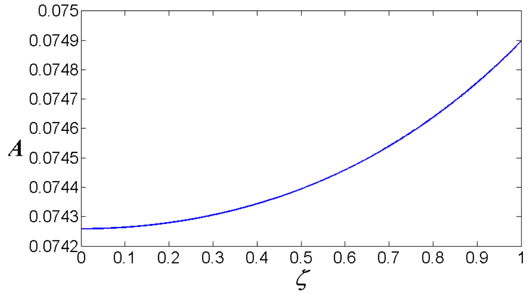

Figure 7. Plot of amplitude of periodic wave (cnoidal) versus ζ having $T = 0.1$, $\Pi = 0.1$, $V = 0.0075$, $Q= 0.1$ and $\varepsilon = -0.002$

$$B = \sqrt{\frac{a}{12b}(\gamma_0 - \gamma_2)} \tag{52}$$

Physically, the elliptic parameter J (the modulus) may be viewed as a fair indicator of the nonlinearity with the linear limit being $J \to 0$ and the extreme nonlinear limit being $J \to 0$. The conditions for the existence of a cnoidal solution of Eq. (48) require that

$$\gamma_0 > \gamma_1 > \gamma_2 \qquad \text{and} \qquad \gamma_1 \leq \Phi_1 \leq \gamma_0$$

Furthermore, the amplitude A, the wavelength λ and the frequency f of the cnoidal wave are defined as

$$A = \gamma_0 - \gamma_1 \tag{53}$$

$$\lambda = 4\sqrt{\frac{3b}{a(\gamma_0 - \gamma_2)}} K(J) \tag{54}$$

$$f = \frac{\Lambda}{\lambda} \tag{55}$$

where $K(J)$ is the first kind of complete elliptic integral and $\Lambda = u_0 + u$ is the velocity of the cnoidal wave in the laboratory frame.

5. Results and discussion

We have considered the nonlinear propagation of IACW in three component high relativistic plasma. Using the fluid model for ions and reductive perturbation technique, we have derived the KdV equation for investigating small amplitude cnoidals. The nonlinear coefficient of KdV equation depends upon the strength of nonextensivity (Q), positron density(Π), relative temperature (T) and relativistic factor ($\zeta = V_0/c$). In this paper, among of different parameters, effects of relativistic factor (ζ) and the strength of nonextensivity (Q) have been investigated.

Effects of ζ and Q may be extracted from Eqs. (36) and (37). A numerical computation has been performed to extract more information and display the results from equations to show that how these parameters influence the formation of cnoidals. Cnoidal wave may generate and propagate in plasma medium only if Sagdeev potential has three real roots. In this case the domain of real roots should be found from the inequalities $\delta > 0$ and $0 < J < 1$ [59]. The acceptable values for Q in which Sagdeev potential [i.e. Eq. (40)] has three real roots are shown in Fig. 1. This figure shows that for all values of $Q > -1$ periodic wave (Cnoidal) may be formed. This investigation shows that the set of allowable Q values in relativity mode is more than non-relativity state, which is determined by comparing the data of this paper with the reference [11, 73]. δ versus ζ is shown in Fig. 2. Ion relativity has no limiting effect on formation of IACW and for all magnitudes of ζ periodic wave may generate and propagate in plasma medium.

In Fig. 3 Sagdeev potential versus Φ for different values of Q has been plotted. This figure shows that by increasing plasma nonextensivity the width and depth of the potential well is decreased. The width of Sagdeev potential well is proportional with IACW amplitude. According to this result with increasing the plasma nonextensivity, energy of IACW will decrease. Furthermore, with increasing Q and the width of Sagdeev potential well, three roots of potential equation tend to each other, leads to formation of ion acoustic solitary wave in the plasma medium.

In Fig. 4 Sagdeev potential versus Φ for different values of Q has been plotted. This figure shows that by increasing relativistic factor the width and depth of the potential well is negligible increased.

Both the nonextensivity and the relativistic plasma have a decreasing effect on the IACW frequency.In Fig. 5, the frequency of ion acoustic periodic wave (cnoidal) versus ζ for different values of Qs has been plotted. By increasing

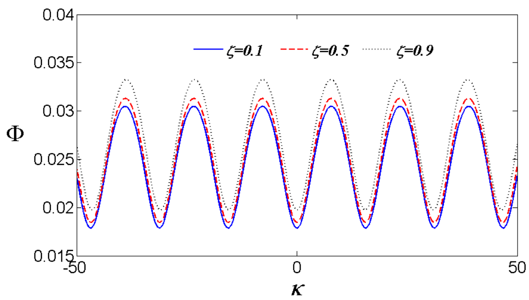

Figure 8. Plot of the potential of the cnoidal wave Φ versus k for different ζs, having $T = 0.1$, $\Pi = 0.1$, $V = 0.0075$, $Q= 0.1$ and $\varepsilon = -0.002$

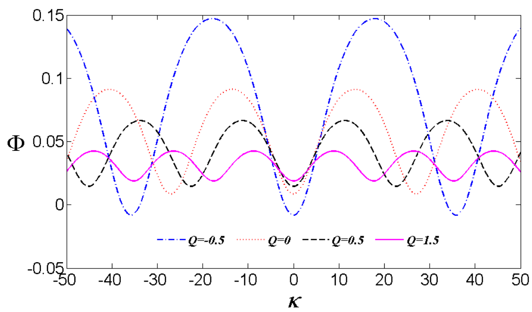

Figure 9. Plot of the potential of the cnoidal wave Φ versus k for different Qs having $T = 0.1$, $\Pi = 0.1$, $V = 0.0075$, ζ= 0.8 and $\varepsilon = -0.002$

Q and ζ the frequency of periodic wave (cnoidal) will decrease. With increasing ζ the mass of ions increases leads to decrease the frequency of plasma ions oscillation, and with increasing Q the stability of system deduces leads to decrease the frequency of IACW.

Figures 6 and 7 present the amplitude of periodic wave (cnoidal) versus Q and ζ respectively. By increasing the nonextensiivity of plasma, the amplitude of periodic wave (cnoidal) will decrease. Variation of the amplitude of IACW mainly occurs in the super extensive regime when $Q < 1$. In the subextensive regime, the amplitude of IACW is almost independent of plasma nonextensivity. For all magnitudes of Q, the amplitude is positive, in other words only the compressive IACW may generates and propagated in the plasma. By increasing ζ, the amplitude of periodic wave (cnoidal) slightly increased. It can be explained by the fact that with increasing ζ the energy of ions increases. For all magnitudes of ζ also the amplitude if positive, this leads to generation of compressive IACW in plasma medium. Comparing Figs. 6 and 7, it is clear that the amplitude of IACW is under the influence of plasma nonextensivity rather than the relativistic effect.

Effects of ζ and Q on the wave pattern of IACW are presented in Figs. 8 and 9. In any case, for all magnitudes of Q and ζ, IACW is compressive. This is a general case and in all nonextensive plasmas, ion acoustice waves (cnoidal or solitary) are compressive [11,51,52,73]. As was mentioned before, the amplitude and frequency of IACW varies with Q rather that ζ. This was observed by Pakzad et al. for the case of ion acoustic solitary waves in relativistic plasma [51,52,74].

6. Conclusion

Propagation of IACW in collisionless, unmagnetized high relativistic plasmas with nonextensive electrons and positrons has been studied. Investigating the factors affecting the cnoidal waves, it is found that the relativistic effects of ions not only do not limit the formation of periodic waves, but the range of permissible Q values for this wave are formed is expanded compared to nonrelativistic states. Amplitude of the cnoidal wave and its width has been derived as functions of plasma parameters. An increase in the nonextensive factor has a decreasing effect on the amplitude and energy of cnoidal waves, which occurs in the subextensive regime, but with the increase in the energy of ions and relativistic effects, the amplitude and energy of periodic wave increases, and finally, the nonextensive effects are appeared more dominant than the relativistic effects. In the investigation of the potential well, it was found that the width and depth of the well decreases with the increase of the nonextensive factor, which indicates the decrease of the wave amplitude, With the decrease of the width of the well, the three roots of the Sagdeev potential equation become closer together and cause the periodic waves to tend to solitons. While the increase of the relativistic factor, the width and depth of the well negligible increased. With the increase of relativistic effects and the velocity of ions, which has a direct effect on mass of ions, as well as the increase of the Q parameter, which reduces the stability of the system, it is natural to see a decrease in the frequency of the cnoidal waves. From the positive amplitude and the pattern of the cnoidal waves it is shown that cnoidal waves are also compressive in this the relativistic nonextensive plasma. Results show that plasmas containing nonextensive positrons and nonextensive electrons, high relativistic ions are a capable medium for different modes, which has not been studied yet. Investigating the propagation of cnoidal waves in this medium can help us to find better knowledge about the effects of relativistic particles in plasmas.

Conflict of interest statement

The authors declare that they have no conflict of interest.

References

[1] D. J. Korteweg and G. De Vries. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, **39**:422, 1895.

[2] A. V. Gurevich and L. Stenflo. *Physica Scripta*, **38**:855, 1988.

[3] Y. H. Ichikawa. *Physica Scripta*, **20**:296, 1979.

[4] K. Konno, T. Mitsuhashi, and Y. H. Ichikawa. *Journal of the Physical Society of Japan*, **46**:1907, 1979.

[5] L. L. Yadav, R. S. Tiwari, K. P. Maheshwari, and S. R. Sharma. *Physical Review E*, **52**:3045, 1995.

[6] R. S. Tiwari, S. L. Jain, and J. K. Chawla. *Physics of Plasmas*, **14**:022106, 2007.

[7] T. Kaladze and S. Mahmood. *Physics of Plasmas*, **21**:032306, 2014.

[8] J. Y. Wang, X. P. Cheng, X. Y. Tang, J. R. Yang, and B. Ren. *Physics of Plasmas*, **21**:032111, 2014.

[9] Y. N. Nejoh. *Australian Journal of Physics*, **50**:309, 1997.

[10] M. Salahuddin, H. Saleem, and M. Saddiq. *Physical Review E*, **66**:36407, 2002.

[11] F. Farhadkiyaei and D. Dorranian. *Contributions to Plasma Physics*, **58**:42, 2018.

[12] A. M. Mirza, A. Hasan, M. Azeem, and H. Saleem. *Physics of Plasmas*, **10**:4675, 2003.

[13] H. R. Miller and P. J. Witta. *Active Galactic Nuclei, Proceedings of a Conference Held at the Georgia State University, Atlanta, Georgia*, :202, 1987.

[14] F. Curtis Michel. *Theory of Neutron Star Magnetospheres*. University of Chicago Press, 1th edition, 1991.

[15] G. W. Gibbons, S. W. Hawking, and S. T. C. Siklos. *Theory of Neutron Star Magnetospheres*. Cambridge University Press, 1th edition, 1982.

[16] J. L. Morán-López and J. M. Sánchez. *New trends in magnetism, magnetic materials, and their applications*. Springer, 1th edition, 1994.

[17] C. Tsallis. *Chaos, Solitons Fractals*, **6**:539, 1995.

[18] C. Tsallis. *Journal of Statistical Physics*, **52**:479, 1988.

[19] J. A. S. Lima, R. Silva Jr., and J. Santos. *Physical Review E*, **61**:3260, 2000.

[20] V. Mun oz. *Nonlinear Processes in Geophysics*, **13**:237, 2006.

[21] V. Latora, A. Rapisarda, and C. Tsallis. *Physical Review E*, **64**:056134, 2001.

[22] A. Taruya and M. Sakagami. *Physical Review Letters*, **90**:181101, 2003.

[23] S. Abe and Y. Okamoto. *Nonextensive Statistical Mechanics and Its Applications*. Springer Berlin, Heidelberg, 1th edition, 2001.

[24] J. M. Liu, J. S. De Groot, J. P. Matt, T. W. Johnston, and R. P. Drake. *Physical Review Letters*, **72**:2717, 1994.

[25] J. Du. *Physics Letters A*, **329**:262, 2004.

[26] L. Liyan and D. Jiulin. *Physica A: Statistical Mechanics and its Applications*, **387**:4821, 2008.

[27] B. Liu and J. Goree. *Physical Review Letters*, **100**:055003, 2008.

[28] Z. Liu, L. Liu, and J. Du. *Physics of Plasmas*, **16**:072111, 2009.

[29] Z. Liu and J. Du. *Physics of Plasmas*, **16**:123707, 2009.

[30] M. Tribeche, L. Djebarni, and R. Amour. *Physics of Plasmas*, **17**:042114, 2010.

[31] R. Amour and M. Tribeche. *Physics of Plasmas*, **17**:063702, 2010.

[32] M. Tribeche and L. Djebarni. *Physics of Plasmas*, **17**:124502, 2010.

[33] L. AitGougam and M. Tribeche. *Astrophysics and Space Science*, **331**:181, 2011.

[34] A. S. Bains, M. Tribeche, and T. S. Gill. *Physics of Plasmas*, **18**:022108, 2011.

[35] R. Amour and M. Tribeche. *Physics of Plasmas*, **18**:033706, 2011.

[36] M. Tribeche and A. Merriche. *Physics of Plasmas*, **18**:034502, 2011.

[37] L. AitGougam and M. Tribeche. *Physics of Plasmas*, **18**:062102, 2011.

[38] A. S. Bains, M. Tribeche, and T. S. Gill. *Physics Letters A*, **375**:2059, 2011.

[39] S. Singh T. Honzawa. *Physics of Fluids B: Plasma Physics*, **5**:2093, 1993.

[40] H. K. Malik, S. Singh, and R P. Dahiya. *Physics of Plasmas*, **1**:1137, 1994.

[41] T. S. Gill, H. Kaur, and N. S. Saini. *Journal of Plasma Physics*, **71**:23, 2004.

[42] J. Arons. *Space Science Reviews*, **24**:417, 1979.

[43] C. Grabbe. *Journal of Geophysical Research*, **94**:17299, 1989.

[44] J. I. Vette. *Particles and Fields in the Magnetosphere*, **17**:350, 1970.

[45] H. Ikezi. *Physics of Fluids*, **16**:1668, 1973.

[46] G. C. Das and S. N. Paul. *Physics of Fluids*, **28**:823, 1985.

[47] Y. Nejoh. *Journal of Plasma Physics*, **37**:487, 1987.

[48] B. C. Kalita, S. N. Barman, and G. Goswami. *Physics of Plasmas*, **3**:145, 1996.

[49] R. K. Roychoudhury and S. Bhattacharyya. *Physics of Fluids*, **30**:2582, 1987.

[50] T. S. Gill, A. Singh, H. Kaur, N. S. Saini, and P. Bala. *Physics Letters A*, **361**:364, 2007.

[51] H. R. PakzadAstrophys. *Space Science*, **32**:677, 2009.

[52] H. R. Pakzad and K. Javidan. *Indian Journal of Physics*, **87**:7, 2013.

[53] B. Sahu and R. Roychoudhurya. *Physics of Plasmas*, **13**:072302, 2006.

[54] M. G. Hafez, M. R. Talukder, and M. Hossain Ali. *Physics of Plasmas*, **23**:012902, 2016.

[55] Y. Nejoh and H. Sanuki. *Physics of Plasmas*, **1**:2154, 1994.

[56] H. H. Kuehl and C. Y. Zhang. *Physics of Plasmas*, **3**:26, 1991.

[57] T. Kaladze, S. Mahmood, and H. Ur-Rehman. *Physica Scripta*, **86**:035506, 2012.

[58] T. Kaladze and S. Mahmood. *Physics of Plasmas*, **21**:032306, 2014.

[59] E. F. El-Shamy. *Physical Review E*, **91**:033105, 2015.

[60] V. I. Karpman. *Non-linear Waves in Dispersive Media*. Pergamon, Oxford, 1th edition, 1975.

[61] J. H. He. *Applied Mathematics and Computation*, **151**:28, 2004.

[62] J. H. He. *Applied Mathematics and Computation*, **135**:73, 2002.

[63] J. H. He. *Computer Methods in Applied Mechanics and Engineering*, **178**:257, 1999.

[64] J. H. He. *International Journal of Non-Linear Mechanics*, **35**:37, 2000.

[65] J. H. He. *International Journal of Nonlinear Sciences and Numerical Simulation*, **6**:207, 2000.

[66] M. El-Shahed. *International Journal of Nonlinear Sciences and Numerical Simulation*, **6**:163, 2005.

[67] J. H. He. *International Journal of Nonlinear Sciences and Numerical Simulation*, **2**:317, 2001.

[68] J. H. He. *International Journal of Non-Linear Mechanics*, **37**:309, 2002.

[69] J. H. He. *International Journal of Non-Linear Mechanics*, **37**:315, 2002.

[70] H. M. Liu. *Chaos Solitons Fractals*, **23**:573, 2005.

[71] T. Taniuti. *Progress of Theoretical Physics Supplements*, **55**:1, 1974.

[72] J. K. Chawla and M. K. Mishra. *Physics of Plasmas*, **17**:102315, 2010.

[73] F. Farhad Kiyaei and D. Dorranian. *Physics of Plasmas*, **24**:012107, 2017.

[74] H. R. Pakzad. *Indian Journal of Physics*, **83**:1605, 2009.