

# A new method for the $\Omega_{ccb}$ baryon spectroscopy in the nonrelativistic quark model: ansatz approach

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## Abstract:

Triply heavy  $\Omega_{ccb}$  baryon is considerable theoretical interest in a baryonic analogue of heavy quarkonium because of the color-singlet bound state of three heavy quark ( $c, b$ ) combination inside. In this paper, we will discuss  $\Omega_{ccb}$  baryon in the nonrelativistic quark model based on the ansatz approach in hypercentral constituent quark model. The masses of the ground and excited states of the  $\Omega_{ccb}$  baryon are computed. The hypercentral potential is regarded as a combination of the color Coulomb plus linear confining term and the six-dimensional harmonic oscillator potential in this work. Also, we added the first order correction and the spin-dependent part to the hypercentral potential. The Regge trajectories has been plotted for this baryon and a detailed comparison with previous theoretical calculations is given. Further, using the computed spectroscopic data, the magnetic moments are determined for the ground state based on the nonrelativistic hypercentral constituent quark model.

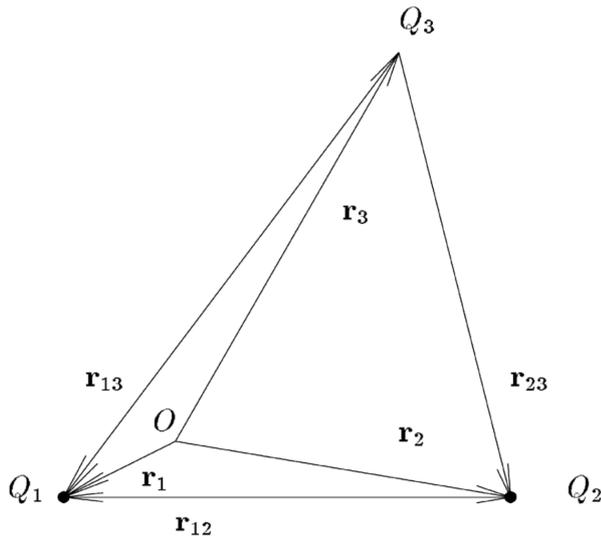
**Keywords:** Triply heavy baryon; Hypercentral constituent quark model; Regge trajectories; Magnetic moment

## 1. Introduction

In recent years, a large number of heavy baryon states, charmonium-like states and bottomonium-like states have been observed, which have attracted intensive attentions and have revitalized many works on the singly heavy, doubly heavy, triply heavy and quadruply heavy hadron spectroscopy. Many new states of heavy baryons as well as of heavy mesons were observed. For the first time, the LHCb collaboration observed the doubly charmed baryon state  $\Xi_{cc}^{++}$  in the  $\Lambda_c^+ k^- \pi^+ \pi^+$  mass spectrum [1]. The LHCb collaboration studied the doubly charmed tetraquark,  $T_{cc}^+$ , with a quark content  $cc\bar{u}\bar{d}$  [2] and the  $J/\psi J/\psi$  invariant mass distribution using  $pp$  collision data  $\sqrt{s} = 7, 8$  and 13 TeV was investigated by the LHCb collaboration in 2020. The narrow resonance structure  $X(6900)$  around 6.9 GeV and a broad structure just above the  $J/\psi J/\psi$  mass with global significances of more than  $5\sigma$  was observed [3]. The observation of the  $\Xi_{cc}^{++}$  and  $X(6900)$  provides some crucial experimental inputs on the strong correlation between the two charm quarks, which may shed light on the spectroscopy of the doubly heavy, triply heavy baryon states, doubly heavy, triply heavy, quadruply heavy tetraquark states, and pentaquark states [4]. As we know producing the triply charm/bottom

heavy baryons is very difficult and thus no experimental signal for any of them has yet been reported. Baranov et al. believed that ccc-baryons couldn't be observed in  $e^+e^-$  collisions and the expectations for triply-bottom baryons would be worse [5]. Bjorken in the 1980s [6] came to this conclusion as well. He offered hadron-induced fixed target experiments as the best strategy to observe the ground-state triply-charmed baryon,  $\Omega_{ccc}^{++}$ . On the other hand, estimates of the production cross section of triply heavy baryons in proton-proton [7] and heavy ion [8] collisions indicate that triply charmed  $\Omega_{ccc}$  baryons have good chances to be observed at LHC. The first theoretical study of heavy baryon spectroscopy was carried out by ref. [9] using the QCD motivated bag model. In this article, we discuss one of the triply heavy baryons which is  $\Omega_{ccb}$ . Many theoretical approaches have determined the masses of this baryon. They are, non-relativistic quark model [8], Fadeev approach [9, 10], Sum rules [11], Bag model [12], di-quark model [13], Lattice QCD [14, 15], relativistic quark model [16, 17] and variational cornell [18].

Since the solution of the hypercentral Schrödinger equation with Coulombic-like term plus a linear confining term potential cannot be obtained analytically, we have used the hypercentral constituent quark model (hCQM) with



**Figure 1.** The configuration of the three-quark system ( $Q_1Q_2Q_3$ ).  $O$  is the string-junction point.  $Q_1, Q_2$  are the heavy quarks  $b$  or  $c$ . The quark  $Q_3$  is treated as a heavy quark  $b, c$  in the triply heavy baryon or a light quark  $q$  in the doubly heavy baryon.

Coulombic-like term plus a linear confining term and the harmonic oscillator potential [19, 20] (the six-dimensional hyper-Coulomb potential is attractive for small separations, while at large separations a hyper-linear term gives rise to quark confinement. The six-dimensions harmonic oscillator potential, which has a two-body character, and turns out to be exactly hypercentral). We also added the first order correction and the spin-dependent part (the spin-spin, spin-orbital and tensor interaction are considered for hyperfine splitting) to the potential and spectra have been generated for the ground states and radial excited states ( $2S$ - $5S$ ) of  $\Omega_{ccb}$  for  $J^P = (1/2)^+$  and  $J^P = (3/2)^+$ . For the orbital states,  $1P$ - $5P$ ,  $1D$ - $4D$  and  $1F$ - $2F$  orbital excited states are calculated.

The present manuscript is arranged as follows. We describe briefly in Sec. 2 the theoretical framework and the hypercentral interaction potentials. In Sec. 3, we present the quasi-exact analytical solution of the radial Schrödinger equation for our proposed potential. In Sec. 4, we calculate the masses of the ground, orbitally, and radially excited states of triply heavy  $\Omega_{ccb}$  baryon and compare our results with previous calculations. Also, we draw Regge trajectories for  $\Omega_{ccb}$  baryon. The magnetic moments of  $\Omega_{ccb}$  baryon has been presented in Sec. 5. Finally, Sec. 6 contains our conclusions.

## 2. Theoretical framework and interaction potentials

The ground states and excited states of doubly heavy baryons in charm and bottom sector have been distinguished using hypercentral constituent quark model in our past study [19, 21, 22]. We would predict that the suggested model will present a same degree of accuracy for triply heavy  $\Omega_{ccb}$  baryon too. The relevant degrees of freedom for the motion of three heavy quarks are related by the Ja-

cobi coordinates  $\rho$  and  $\lambda$  and the respective reduced masses are given by  $m_\rho$  and  $m_\lambda$  [19–24]. The configuration of the three-quark system ( $Q_1Q_2Q_3$ ) is shown in Fig. 1.

The constituent quark masses used in our calculations are  $m_1 = m_2 = 1.345$  GeV (mass of  $c$  quark) and  $m_3 = 4.902$  GeV (mass of  $b$  quark). To describe three-quark dynamics, we define hyper radius  $x = \sqrt{\rho^2 + \lambda^2}$  and hyper angle  $\xi = \arctan(\rho/\lambda)$ . In this paper, we consider the confining hypercentral potential as a combination of the color Coulomb [25] plus linear confining term [26] and the six-dimensional harmonic oscillator potential [27, 28] with a first order correction [27] and the spin-dependent interaction [29],

$$V(x) = V^{(0)}(x) + \left(\frac{1}{m_\rho} + \frac{1}{m_\lambda}\right)V^{(1)}(x) + V_{SD}(x),$$

$$V^{(0)}(x) = \frac{\tau}{x} + \beta x + kx^2,$$

$$V^1(x) = C_F C_A \frac{\alpha_s^2}{4x^2},$$

$$V_{SD}(x) = V_{SS}(x)(\mathbf{S}_\rho \cdot \mathbf{S}_\lambda) + V_{\gamma S}(x)(\boldsymbol{\gamma} \cdot \mathbf{S}) + V_T(x) \left[ S^2 - \frac{3(\mathbf{S} \cdot \mathbf{x})(\mathbf{S} \cdot \mathbf{x})}{x^2} \right] \tag{1}$$

$$V_{SS}(x) = \frac{1}{3m_\rho m_\lambda} \nabla^2 V_V, \text{ and}$$

$$V_{\gamma S} = \frac{1}{2m_\rho m_\lambda x} \left( 3 \frac{dV_V}{dx} - \frac{dV_S}{dx} \right), \text{ and}$$

$$V_T(x) = \frac{1}{6m_\rho m_\lambda} \left( \frac{3d^2V_V}{d^2x} - \frac{1}{x} \frac{dV_V}{dx} \right).$$

where  $\tau$  is the hyper-Coulomb strength,  $\beta (= 0.048)$  corresponds to the string tension for baryons and  $\alpha_s$  is the strong running coupling constant. The parameters  $C_F$  and  $C_A$  are the casimir charges of the fundamental and adjoint representation [30]. The spin dependent part  $V_{SD}(x)$  contains three types of the interaction terms described as [31]. In above equation  $V_V(x) = \tau/x$  and  $V_S(x) = \beta x + kx^2$ . We can obtain the baryon masses by  $M_B = \sum m_{Q_i} + \langle H \rangle$  [32]. Now we want to solve the hyperradial Schrödinger equation for the three-body potential interaction Eq. (1).

## 3. Quasi-exact analytical solution of the hyperradial Schrödinger equation

The dynamics of the baryonic system are considered in the wave-function  $\psi_{V\gamma}(x)$  which is the solution of hyperradial

**Table 1.** The Quark mass (in GeV) and the fitted values of the parameters used in our calculations.

$m_\mu = m_d$	$\tau$	$\beta$	$\omega$
0.302	0.418	0.871	0.152

**Table 2.** Ground state masses of  $\Omega_{ccb}$  for both  $J^P = 1/2^+$  and  $J^P = 3/2^+$  (in GeV).

State	Our Calc	[36]	[17]	[8]	[10]	[14]	[15]	[37]	[38]	[13]	[16]	[18]
$J^P = 1/2^+$	8.006	8.005	7.984	8.245	7.867	8.007(29)	8.005(17)	8.004	8.301	8.005	8.018	8.018
$J^P = 3/2^+$	8.038	8.049	7.999	8.265	7.963	8.037(29)	8.026(18)	8.023	8.301	8.027	8.025	8.046

Schrödinger equation

$$\left(\frac{d^2}{dx^2} + \frac{5}{x} \frac{d}{dx} - \frac{\gamma(\gamma+4)}{x^2}\right) \psi_{v\gamma}(x) = -2m[E - V(x)] \psi_{v\gamma}(x) \tag{2}$$

where  $\gamma$  is the grand angular quantum number and given by  $\gamma = 2n + l_\rho + l_\lambda$ ,  $n = 0, 1, \dots$ ;  $l_\rho$  and  $l_\lambda$  are the angular momenta associated with the  $\rho$  and  $\lambda$  variables and  $v$  denotes the number of nodes of the space three-quark wave functions. The transformation  $\psi_{v\gamma}(x) = x^{-5/2} \phi_{v\gamma}$  [33] reduces Eq. (2) to the following form

$$\begin{aligned} \dot{\phi}_{v\gamma}(x) + [\varepsilon - a_1x^2 - a_2x - \frac{a_3}{x} - \frac{a_4}{x^2} - \frac{a_5}{x^3} + \frac{a_6}{x^5} + \\ a_7 - \frac{(2\gamma+3)(2\gamma+5)}{4x^2}] \phi_{v\gamma}(x) = 0 \end{aligned} \tag{3}$$

The hyperradial wave function  $\phi_{v\gamma}(x)$  is a solution of the reduced Schrödinger equation for each of the three identical particles with the mass  $m$  and interacting potential (1), where

$$\begin{aligned} \varepsilon = 2mE, \quad a_1 = 2mk, \quad a_2 = 2m\beta, \quad a_3 = 2m\tau, \\ a_4 = 2m \left( \frac{1}{m_\rho} + \frac{1}{m_\lambda} \right) \left( -C_f C_A \frac{\alpha_s^2}{4} \right), \end{aligned} \tag{4}$$

$$a_5 = 2m \left[ \frac{2\tau}{3m_\rho m_\lambda} (S_\rho \cdot S_\lambda) - \frac{3\tau}{2m_\rho m_\lambda} (\boldsymbol{\gamma} \cdot \mathbf{s}) + \frac{7\tau}{6m_\rho m_\lambda} s^2 \right],$$

$$a_6 = 2m \frac{21\tau}{6m_\rho m_\lambda} (\mathbf{s} \cdot \mathbf{x})(\mathbf{s} \cdot \mathbf{x}),$$

$$a_7 = 2m \left( \frac{\beta + 2k}{2m_\rho m_\lambda} (\boldsymbol{\gamma} \cdot \mathbf{s}) \right)$$

We suppose  $\phi_{v\gamma} = f(x) \exp[r(x)]$  for the wave function and we make use of the ansatz for the  $f(x)$  and  $r(x)$  [38, 39],

$$\begin{aligned} f(x) = \prod (x - u_l^v), \quad v = 1, 2, \dots, \\ f(x) = 1, \quad v = 0 \end{aligned} \tag{5}$$

$$r(x) = g \ln x + qx^2 + cx + \frac{d}{x}$$

By calculating  $\dot{\phi}_{v\gamma}(x)$  from ansatz approach and comparing with Eq. (3), we obtain

$$\begin{aligned} \left[ a_1x^2 + a_2x + \frac{a_3}{x} + \frac{a_4}{x^2} + \frac{a_5}{x^3} - \frac{a_6}{x^5} - a_7 + \frac{(2\gamma+3)(2\gamma+5)}{4x^2} - \varepsilon \right] \\ = \left[ \left\{ 4q^2x^2 + 4cqx + \frac{(2ac-4dq)}{x} + \frac{(a^2-a-2cd)}{x^2} + \right\} \right. \\ \left. \left\{ \frac{2d(1-a)}{x^3} + \frac{d^2}{x^4} + (c^2 + 2q + 4ac) \right\} \right] \end{aligned} \tag{6}$$

We used the ansatz approach, which is a simplified version of the quasi-exact Lie algebra. The Ansatz approach, despite its wide applications in case of many interactions, has its own challenges. To be more precise, the arising set of equations, which is based on finding the solution of an associated Riccati equation, is subjected to many restricting conditions and the parameters cannot be determined independent of each other. Also, the higher states are very hard to calculate. By equating the corresponding powers of  $x$  on both sides of Eq. (6) and regarding  $k = (m\omega^2)/2$ , we can obtain the energy eigenvalues for the mode  $v = 0$  and grand angular momentum  $\gamma$  as follows

$$E_{v\gamma} = -\frac{\omega}{2} - \left( \frac{\beta + m\omega^2}{2m_\rho m_\lambda} (\boldsymbol{\gamma} \cdot \mathbf{s}) \right) - \frac{\beta^2}{2m\omega} - \frac{m\omega^2\tau}{\beta} \tag{7}$$

In a similar manner we can continue for other modes ( $v = 1, 2, 3, \dots$ ).

**Table 3.** The masses of radial excited states for  $\Omega_{ccb}$  (in GeV).

Baryon	State	$J^P$	Our Calc	[36]	[17]	[37]	[38]	[8]	[10]
$\Omega_{ccb}$	2S	$1/2^+$	8.591	8.621	8.361	8.455	8.647	8.537	8.337
$\Omega_{ccb}$	3S	$1/2^+$	9.174	9.224					
$\Omega_{ccb}$	4S	$1/2^+$	9.791	9.823					
$\Omega_{ccb}$	5S	$1/2^+$	10.370	10.424					
$\Omega_{ccb}$	2S	$3/2^+$	8.609	8.637	8.366	8.468	8.600	8.553	8.427
$\Omega_{ccb}$	3S	$3/2^+$	9.188	9.232					
$\Omega_{ccb}$	4S	$3/2^+$	9.799	9.828					
$\Omega_{ccb}$	5S	$3/2^+$	10.372	10.424					

**Table 4.** The masses of orbital excited states of  $\Omega_{ccb}$  (in GeV).

State	Our Calc	[34]	[17]	[8]	[35]	[36]	[37]	[10]
$(1^4P_{1/2})$	8.411	8.400	8.250	8.418	8.36	8.306	8.491	8.164
$(1^4P_{3/2})$	8.370	8.383	8.262	8.420	8.36	8.306	8.491	8.275
$(1^4P_{5/2})$	8.350	8.365	8.267	8.432		8.311	8.491	
$(2^4P_{1/2})$	8.969	8.992						
$(2^4P_{3/2})$	8.943	8.976						
$(2^4P_{5/2})$	8.915	8.955						
$(3^4P_{1/2})$	9.555	9.585						
$(3^4P_{3/2})$	9.528	9.569						
$(3^4P_{5/2})$	9.502	9.547						
$(4^4P_{1/2})$	10.179	10.181						
$(4^4P_{3/2})$	10.151	10.164						
$(4^4P_{5/2})$	10.138	10.140						
$(5^4P_{1/2})$	10.756	10.775						
$(5^4P_{3/2})$	10.730	10.758						
$(5^4P_{5/2})$	10.711	10.735						
$(1^4D_{1/2})$	8.842	8.848	8.405					
$(1^4D_{3/2})$	8.821	8.831	8.412		8.536	8.647		
$(1^4D_{5/2})$	8.798	8.808	8.473	8.568	8.536	8.647		
$(1^4D_{7/2})$	8.771	8.780	8.473	8.568	8.538	8.647		
$(2^4D_{1/2})$	9.429	9.437						
$(2^4D_{3/2})$	9.410	9.420						
$(2^4D_{5/2})$	9.387	9.396						
$(2^4D_{7/2})$	9.356	9.368						
$(3^4D_{1/2})$	10.01	10.031						
$(3^4D_{3/2})$	9.982	10.012						
$(3^4D_{5/2})$	9.961	9.988						
$(3^4D_{7/2})$	9.946	9.957						
$(4^4D_{1/2})$	10.615	10.622						
$(4^4D_{3/2})$	10.601	10.605						
$(4^4D_{5/2})$	10.580	10.582						
$(4^4D_{7/2})$	10.563	10.553						
$(1^4F_{3/2})$	9.273	9.280						
$(1^4F_{5/2})$	9.247	9.254						
$(1^4F_{7/2})$	9.211	9.222						
$(1^4F_{9/2})$	9.189	9.184						
$(2^4F_{3/2})$	9.841	9.864						
$(2^4F_{5/2})$	9.820	9.840						
$(2^4F_{7/2})$	9.789	9.809						
$(2^4F_{9/2})$	9.740	9.773						

For calculating the best triply heavy  $\Omega_{ccb}$  baryon mass predictions, the values of  $m_\mu = m_d$ ,  $\omega$ ,  $\tau$  and  $\beta$  (which are listed in Table 1) are selected using genetic algorithm.

#### 4. Triply heavy $\Omega_{ccb}$ baryon spectra and regge trajectories

As the first step, we have calculated the masses of the ground state 1S, for  $J^P = 1/2^+$  and  $J^P = 3/2^+$ . Our calculated masses are obtained by using the hypercentral potential Eq. (1) in the hypercentral constituent quark model. Our calculated masses for the ground states of  $\Omega_{ccb}$  baryon are presented and compared with previous theoretical predictions in Table 2. The predicted mass of the ground state  $\Omega_{ccb}$  baryon (for  $J^P = 1/2^+$ ) range from 7.867 to 8.301 GeV and for  $J^P = 3/2^+$ , range from 7.963 to 8.301 GeV. Our predictions for these masses are well inside both ranges. We compare our predictions with previous calculations for the radial excited states (2S-5S) for both  $J^P = 1/2^+$  and  $J^P = 3/2^+$  in Table 3.

We can easily observe that our 2S-5S states show a smaller difference in the range of  $\approx 60$  MeV with Ref. [34]. The

**Table 5.** Fitted slopes and intercepts (in  $\text{GeV}^2$ ) of the Regge trajectories ( $n, M^2$ ) plane.

Baryons	$J^P$	State	$\beta$	$\beta_0$
$\Omega_{ccb}$	$1/2^+$	S	$0.0920 \pm 0.0018$	$-5.8326 \pm 0.155$
	$1/2^-$	P	$0.0889 \pm 0.0017$	$-6.5796 \pm 0.170$
	$5/2^+$	D	$0.0868 \pm 0.0018$	$-7.2338 \pm 0.171$

hypercentral constituent quark model was employed in Ref. [34]. In Ref. [17], the triply heavy baryons was investigated in the framework of the relativistic quark model based on the quark-diquark picture in the quasipotential approach in QCD. To calculate the mass spectra of  $\Omega_{ccb}$  baryon, the constituent quark model, which employs the Gaussian expansion method and the variational principle to solve the nonrelativistic three-body problem, was used in Ref. [36]. In Ref [37], for investigating baryons with heavy quarks, the renormalization group procedure for effective particles was applied. The nonrelativistic quark model with the harmonic oscillator wave functions and perturbative account of the relativistic corrections were used for the calculation of the triply heavy baryon masses by Ref. [8]. In Ref. [10] the relativistic Faddeev equation with the rainbow-ladder truncated kernel was employed.

The orbital excited states of  $\Omega_{ccb}$  baryon are determined from 1P-5P, 1D-4D and 1F-2F states. Our results are given in Table 4. Our 1P state mass for  $J^P = 1/2^-$  is 7 MeV lower than Ref. [8] and the 1P ( $3/2^+$ ) state value is 10 MeV higher than that of Ref. [35]. Our 1P state  $J^P = 5/2^-$  shows 15 MeV difference with Ref. [34]. Our outcomes are slightly different from [34], i.e. for 1D state  $J^P = 1/2^+$  shows 6 MeV, for  $J^P = 3/2^+$  and  $J^P = 5/2^+$  show 10 MeV, for  $J^P = 7/2^+$  shows 9 MeV. For both radial and orbital excited states of triply heavy  $\Omega_{ccb}$  baryon, our calculations are in accordance with Shah et al. [34] and the minimum and maximum percentage of relative error values are 0.018% and 0.47% between our calculations and their reported masses. Our model is somehow similar to the Shah's et al. model. However, we added the six-dimensional harmonic oscillator potential term to the confining hypercentral potential and analytically solved Schrödinger equation, while they solved the six-dimensional Schrödinger equation using the Mathematica notebook numerically.

Recently, some singly and doubly heavy baryons have been established experimentally, but none of the triply heavy

**Table 6.** Fitted slopes and intercepts (in  $\text{GeV}^2$ ) of the Regge trajectories ( $J, M^2$ ) plane.

Baryons	$J^P$	State	$\alpha$	$\alpha_0$
$\Omega_{ccb}$	$3/2^+$	S	$0.0579 \pm 0.0018$	$-3.0123 \pm 0.187$
	$5/2^+$	P	$0.05857 \pm 0.0019$	$-3.425 \pm 0.182$
	$7/2^+$	D	$0.05924 \pm 0.0017$	$-4.003 \pm 0.178$

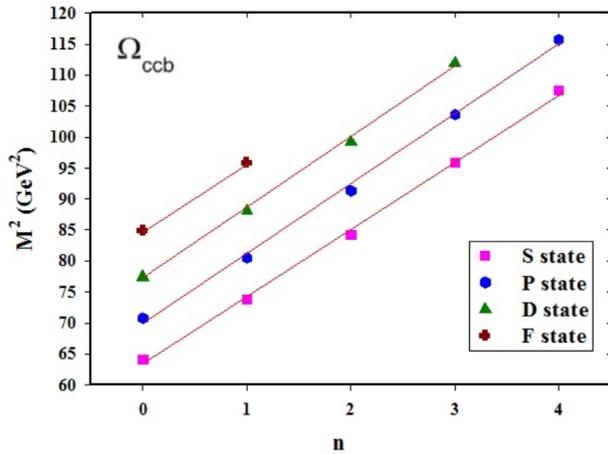


Figure 2. Regge trajectory ( $n, M^2$ ) plane for  $\Omega_{ccb}$  baryon.

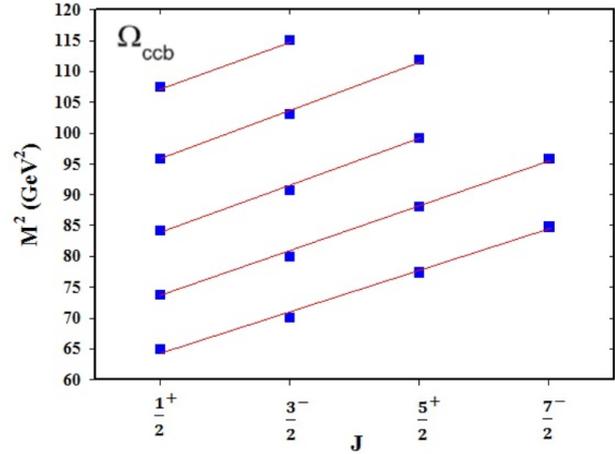


Figure 3. Regge trajectory ( $J, M^2$ ) plane of  $\Omega_{ccb}$  baryon for natural parity.

baryons have been observed. The Regge trajectories are the one of the most important properties of baryons and play a vital role in identifying any new (experimentally) excited state as well as in providing information about the quantum numbers of particular states. Regge trajectories represent the different states with identical  $J^P$  value on the same plane. In this study we obtained both radial and orbital excited states masses of  $\Omega_{ccb}$  baryon up to  $L = 3$ . This will allow us to construct the heavy baryon Regge trajectories in ( $n, M^2$ ) [see Fig. (2)] and ( $J, M^2$ ) for natural ( $J^P = 1/2^+, J^P = 3/2^-, J^P = 5/2^+, J^P = 7/2^-$ ) and unnatural ( $J^P = 3/2^+, J^P = 5/2^-, J^P = 7/2^+, J^P = 9/2^-$ ) parities [see Figs. (3-4)] plane. Then we can test their linearity, parallelism, and equidistance and determine their parameters: Regge slopes and intercepts. Their determination is of great importance, since they provide a better understanding of the hadron dynamics. Moreover, their knowledge is also important for nonspectroscopic problems such as, e.g., hadron production and high-energy scattering. The ground and radial excited states S ( $J^P = 1/2^+$ ) and the orbital excited state P ( $J^P = 1/2^-$ ), D ( $J^P = 5/2^+$ ) and F ( $J^P = 7/2^-$ ) are plotted from bottom to top in ( $n, M^2$ ). We use where  $\beta, \alpha$ , and  $\beta_0, \alpha_0$  are the slope and intercept, respectively.  $J$  is the baryon spin,  $M$  is the baryon mass, and  $n = \mathbf{n} - 1$ , where  $n$  is the principal quantum number. Straight lines were obtained by the linear fitting in all figures. The values of  $\beta$  and  $\beta_0$  are shown in Table 5 and the values of  $\alpha$  and  $\alpha_0$  are shown in Table 6.

As the results revealed the squares of the computed masses match very well with the linear trajectory and provide almost parallel states, equidistant in S, P, D, and F. The trajectories are useful in that and the slope and intercepts of the

plots will also identify the principal quantum number of the state. Our trajectories are effective in determining spin and parity of discovered baryons.

$$n = \beta M^2 + \beta_0, \quad J = \alpha M^2 + \alpha_0 \quad (8)$$

### 5. Magnetic moment

Baryon magnetic moment places a crucial role in providing information regarding the structure and shape of baryon [43]. In this study the magnetic moments of triply heavy  $\Omega_{ccb}$  baryons are calculated based on the nonrelativistic hypercentral constituent quark model using the spin-flavor wave functions of the constituent quark and their effective masses. The expression of magnetic moment of baryon can be obtained by operating the expectation value equation [40],

$$\mu_{Baryon} = \sum_q \langle \varphi_{sf} | \hat{\mu}_{qz} | \varphi_{sf} \rangle, \quad q = c, c, b \quad (9)$$

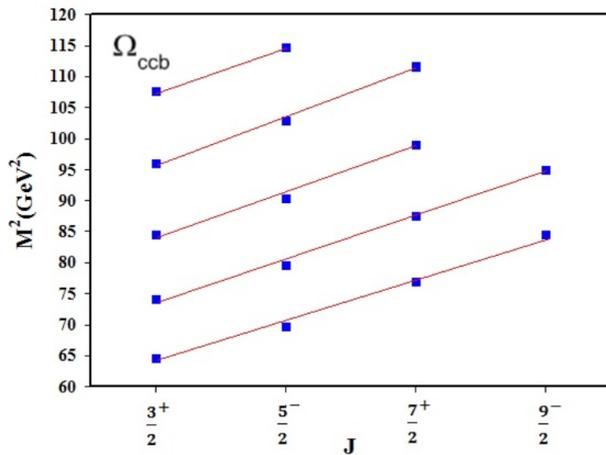
where,  $\varphi_{sf}$  is the spin-flavor wave function of  $\Omega_{ccb}$ ,  $\hat{\mu}_{qz}$  is the magnetic moment operator and  $\mu_q = e_q / (2m_q^{eff}) \sigma_q$ . Where,  $e_q$  and  $\sigma_q$  are charge and spin of the individual constituent quark respectively and  $m_q^{eff}$  [34] is the effective mass of constituent quark. The spin-flavor wave functions [44] and magnetic moments for the ground states mass (for both  $J^P = 1/2^+$  and  $J^P = 3/2^+$  state) are calculated and shown in Table 7.

### 6. Conclusion

In this work, we used the nonrelativistic quark model based on the ansatz approach in the Hypercentral Constituent

Table 7. Magnetic moments (in nuclear magnetons).

Baryons	Function [40]	Our	[34]	[41]	[42]	[40]	[13]
$\Omega_{ccb}^+$	$4/3\mu_c - 1/3\mu_b$	0.587	0.606	0.505	0.522	0.502	
$\Omega_{ccb}^{*+}$	$\mu_b + 2\mu_c$	0.798	0.819	0.659	0.703		0.807



**Figure 4.** Regge trajectory ( $J, M^2$ ) plane of  $\Omega_{ccb}$  baryon for unnatural parity.

Quark Model for the computation of the mass spectra of triply heavy  $\Omega_{ccb}$  baryon. This investigation, help us to understand the interaction between the heavy quarks in baryons. For this aim we have analytically solved the radial Schrödinger equation for three particles under the effective hypercentral potential. Our proposed potential is regarded as a combination of the Coulombic-like term plus a linear confining term and the harmonic oscillator potential. We also added the first order correction and the spin-dependent part to the potential. Our model has succeeded to assign the  $J^P$  values to the ground and excited states of doubly heavy baryons so far [21, 22]. In a similar way, the possible predictions of triply heavy  $\Omega_{ccb}$  baryon are achieved in this study. The calculated masses were compared with past lattice QCD and other quark model calculations. Reasonable agreement with lattice results was found. We construct the Regge trajectories for the  $\Omega_{ccb}$  baryon in ( $n, M^2$ ) and ( $J, M^2$ ) planes and find their slopes and intercepts to identify the unknown  $J^P$  values for all baryons. We can see that the trajectories are linear, parallel and equidistant. In addition, we have calculated the magnetic moments for the ground state of  $\Omega_{ccb}$  baryon and they are fairly in agreement with other calculations.

#### Conflict of interest statement

The authors declare that they have no conflict of interest.

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