Volume 8, Issue 4, 082424 (1-7)

Signal Processing and Renewable Energy (SPRE)

https://dx.doi.org/10.57647/j.spre.2024.0804.24

Joint non-iterative beamforming schemes for SER minimization in Ad-Hoc network

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Original Research

Received:
30 October 2024
Revised:
19 November 2024
Accepted:
23 November 2024

Abstract:

This research paper proposes three joint transmit and receive (TX/RX) beamforming schemes for the minimization of symbol error rates of Ad-Hoc networks. The first beamforming scheme is based on minimizing the optimization problem of the total mean squared error between the transmitted and received data symbols. The second beamforming scheme is based on the projection of the effective channel on the space spanned by the channel matrix. The third one is the introduction of a practical beamforming scheme that is based on the strongest eigenvalues of the channel matrix. This beamforming scheme proves its optimality in its operation under the condition that the number of transmit antennas equal to or larger than the total number of receive antennas. The proposed joint TX/RX beamforming schemes are formulated in closed forms. Numerous numerical results are provided using the NS2 platform. The proposed beamforming schemes are compared with the widely adopted block diagonalization transmission scheme. The simulation results demonstrated that the proposed beamforming schemes achieved better symbol error rate performance than the block diagonalization scheme. Accordingly, more robust communications can be achieved for Ad-Hoc Networks. This research paper provides an efficient manner to meet the demanded reliability of Ad-Hoc networks which is a symbol error rate of 10^{-9}.

Keywords: Block Diagonalization (BD); Singular Value Decomposition(SVD); Minimum Mean Squared Error Detector (MMSE); Ad-Hoc Network (AHN)

1. Introduction

Increasing demand for various industries today is improving the effectiveness of operational activities. Industrial communication is a research track within Ad-Hoc networks (AHNs). There are critical requirements for Industrial communication systems such as increasing the operational efficiency of the industrial control process. In addition, more flexible operations should be supported with the reduction of capital expenditure [1–3]. Lately, the recent advances in wireless communication technology have gained much more attention to be applied into industrial applications, more specifically, in wireless sensor networks (WSN) [1]. It is expected that Industrial wireless communication can provide a high degree of real-time dynamic control for industrial processes [3]. In addition, the advantages of wireless technologies such as the retrofitting ability and inherent flexibility offer great potential for the future of industrial communication [2–4]. In order to coordinate transmissions to multiple users simultaneously, multi-user MIMO is a key component of future wireless communication systems [5]. Linear precoding techniques such as zero-forcing (ZF) algorithm can achieve high sum-rate capacity with low complexity [6]. However, for single antenna users, zero forcing is proposed. For transmission to multiple users simultaneously, block diagonalization is suggested to achieve such a goal [7]. It can be considered as a generalization of zero-forcing transmitters [8]. Multi-antenna users receive multiple streams and are involved in interference cancellations [9]. Because of the relation between transmit and receive antennas, the total number of accommodated slave nodes is limited in block diagonalization [10]. This can be considered as a disadvantage of sucha transmission method. Coordinated beamforming plays a major role in relaxing such dimensionality constraint of block diagonalization [11–13]. This can be achieved by jointly optimizing the receive combining vectors and transmit beamforming. However, most of these coordinated beamforming schemes are iterative in their operation which results in a high complexity and consumes a lot of time [14]. Furthermore, the coordinated beamforming schemes are not guaranteed to converge and they are sensitive to the stopping criteria [15]. Therefore, noniterative coordinated beamforming schemes are proposed [12]. Coordination of beamforming information from the master node antennas to slave nodes in the downlink process is a limitation of iterative and non-iterative beamforming schemes. One method to coordinate beamforming information is a user-specific pre-coded pilot symbols [16]. However, the associated overhead of pre-coded pilot symbols linearly increases with the number of users [14]. Also, inter-user interference (IUI) and inter-antenna interference (IAI) degrade multi-user multiple-input multiple-output (MU-MIMO) throughput [17]. Block diagonalization has been widely studied since it is computationally efficient but its performance is limited because of the presence of residual inter-antenna interference [18]. In this paper, three joint non-iterative beamforming schemes are proposed for Ad-Hoc networks. The first proposed beamforming scheme is based on the hybrid between the block diagonalization scheme and singular value decomposition, which is denoted as (BD/SVD). In BD, the interference from other user signals is canceled in the precoding process. The basic idea is to minimize the mean squared error between the transmitted and received symbols in 16 quadrature amplitude modulation (16QAM) systems. The second proposed beamforming scheme is based on the space spanned by the channel matrix formed by the strongest singular values of the channel matrix and its left singular vector. Furthermore, based on the strongest singular values of the channel matrix and the projection of the effective channel on the right singular vector, the third beamforming scheme is proposed. The proposed schemes are applicable for any number of master node antennas communicating with slave nodes. The proposed beamforming schemes are non-iterative and expressed in closed forms, so they are marked by low computational complexity. The channel state information is assumed to be perfectly known at both the transmitter and the receiver ends, respectively. The proposed joint TX/RX beamforming schemes achieved a better symbol error rate performance than the conventional block diagonalization scheme. The paper is organized as follows: In Section 2, the proposed system model is presented. The signal representation is formulated in Section 3. In Section 4, the proposed new beamforming schemes are introduced. The simulation results are demonstrated in Section 5. The conclusion is provided in Section 6.

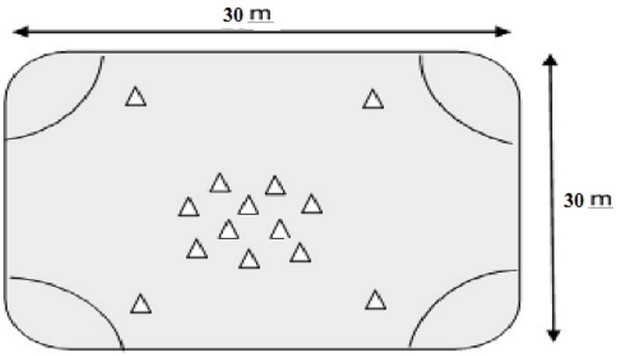

Figure 1. The proposed system model of an ad-hoc network for a factory hall.

2. System model

Fig. 1 clarifies the proposed system model for an Ad-Hoc network of spatial dimension (30m × 30m) that represents a factory hall. The slave nodes (SNs) are distributed properly in a uniform manner over the network region. The master node (MN) is connected to a high-speed and high-capacity link through distributed antennas that are perfectly synchronized to each other. In this model, a total number of K slave nodes is utilized each of which is equipped with $\mathrm{n_k}$ antenna elements. The SNs are collocated in the center of the network. While the MN is equipped with $\mathrm{N_t}$ antenna elements. It is assumed that channel state information (CSI) is known at both the MN and SNs (Fig. 2).

3. Signal representation

In this section, the signal representation of the proposed beamforming schemes is expressed. The MN, which acts as a baseband unit (BBU) with transmit antennas communicates with K slave nodes. Each slave node has $\mathrm{n_k}$ receive antennas. Thereby, the total number of receiving antennas is $\mathrm{N_r}$ such that $\mathrm{N_r = K.n_k}$. There are K slave nodes scheduled on the same resource block. The block diagram of the proposed system model is shown in Fig. 3. The downlink channel $\mathrm{H_k} \in \mathrm{C}^{\mathrm{n_k} \times \mathrm{N_t}}$ from the master node to each slave node k is assumed to be a narrow-band flat fading channel. The data symbol is transmitted from the master node such

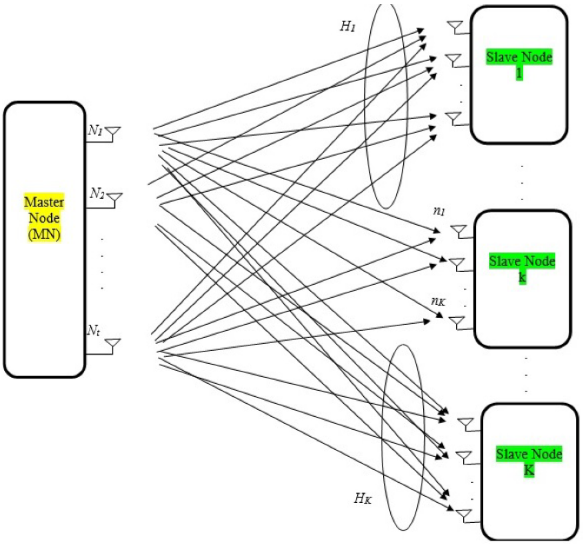

Figure 2. The proposed signal model for a master node with $\mathrm{N_t}$ transmit antennas and K slave nodes, each with $\mathrm{n_k}$ receive antennas.

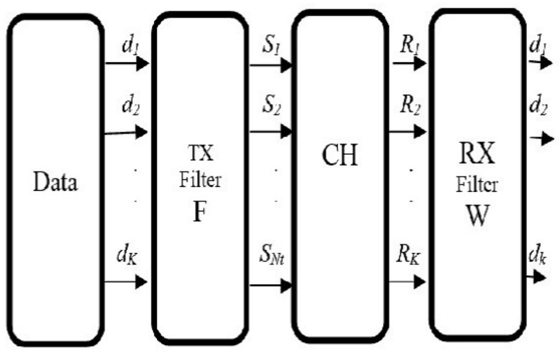

Figure 3. Block diagram of the proposed system model for a master node with N_t transmit antennas and K slave nodes.

that $S \in C^{N_t \times 1}$. In the frequency domain:

$$S = \sqrt{\frac{2E_s}{T_S}}.F.D \tag{1}$$

where $S = [s_1\ s_2\ ...s_{N_t}]^T \in C^{N_t \times 1}$.$E_s = \hat{P}_t.T_s$ T_s is the normalized transmitted symbol energy. T_s is the symbol duration. $F \in C^{N_t \times N_r}$ is the transmit filter matrix. D is the data vector transmitted for the slave nodes such that $D = [d_1\ d_2\ . . .\ d_{N_r}]^T \in C^{N_r \times 1}$. Specifically, for a slave node k, let the symbol transmitted to slave node k is d_k, the transmit beam former is f_k and the receive combining vector is w_k. At the slave node, the signal at slave node k is given by:

$$y_k = w_k^H H_k f_k d_k + w_k^H H_k \sum_{i=1, i \neq k}^{K} f_i d_i + w_k^H n_k \tag{2}$$

The first term is the useful signal to slave node k. The second term is the interference from other nodes. The last term is the noise contribution. Thus, in general, at the slave nodes, the frequency domain received signal vector $R \in C^{N_r \times 1}$.

$$R = H.S + n \tag{3}$$

where $S \in C^{N_t \times 1}$ is the transmitted signal from the master node. $H \in C^{N_r \times N_t}$ is the channel matrix between the MN antennas and K slave nodes. $n \in C^{N_r \times 1}$ represents Gaussian random variables with zero mean and $\frac{2N_0}{T_s}$ variance. N_0 is additive white Gaussian noise (AWGN) power spectrum density. The frequency domain soft output symbol vector $\tilde{D} \in C^{N_r \times 1}$ is obtained by applying the receive combining filter on R:

$$\tilde{D} = W.R \tag{4}$$

where $\tilde{D}$= $[d_1\ d_2\ . . .\ d_{N_r}]^T \in C^{N_r \times 1}$ and $W \in C^{N_r \times N_r}$ is the receive combining matrix. $R \in C^{N_r \times 1}$ is the received signal at the slave nodes. Let the singular value decomposition (SVD) of H_k be given by:

$$H_k = U_k \sigma_k V_K^H \tag{5}$$

where σ_k is a diagonal matrix with $\sigma_{ij} = 0\ \forall\ i \neq j$ and $\sigma_{ij} = \sigma_i \geq 0\ \forall$ i=j i.e. $\sigma_k = \text{diag}(\sigma_1, \sigma_2, . . . , \sigma_k)$, where $(\sigma_1, \sigma_2, . . . , \sigma_k)$ are the eigenvalues of the matrix H_k in a descending order. U_k and V_k are orthogonal matrices such that $U_k U_k^H = U_k^H U_k = I$ and $V_k V_k^H = V_k^H V_k = I$. The columns of V_k and U_k arethe right-singular vectors and the left-singular vectors of H_k, respectively.

4. The proposed beamforming schemes

4.1 Block diagonalization/singular value decomposition

In this section, the transmit/receive filter matrices are presented. These filters minimize the total mean squared error (MSE) between the transmit data vector D and the output receive data vector $\tilde{D}$. The problem of minimization of the total MSE under the total transmitpower constraint is formulated as follows:

$$\min_{F,W} \underbrace{E\left[\sum_{k=1}^{K} \text{tr}\left[\left(D - \frac{\tilde{D}}{\sqrt{\frac{2E_s}{T_s}}}\right)\left(\left(D - \frac{\tilde{D}}{\sqrt{\frac{2E_s}{T_s}}}\right)^H\right)\right]\right]}_{\varepsilon} \tag{6}$$

$$\text{s.t.} \sum_{k=1}^{K} \text{tr}(F.F^H) = I_k \tag{7}$$

where ε is the total MSE between D and $\tilde{D}$ In the block diagonalization/singular value decomposition (BD/SVD) scheme, the inter-user interference (IUI) is removed by block diagonalization at the master node. The BD precoding matrix corresponding to slave node k is

$$\tilde{F} = \bar{V}_n(:, i), i = 1, 2, ..., n_k \tag{8}$$

where $\bar{V}_n \in C^{N_t \times n_k}$ represents the right null space of $\bar{H}_k$ such that $\bar{H}_k = [H_1^T, H_2^T, ..., H_{n_k}^T]$
The equivalent channel matrix after BD $H_k = H_k.\bar{V}_n$ which can be considered as a single-user MIMO channel matrix because IUI does not occur. By applying singular value decomposition to the equivalent channel matrix H_k, the transmit filter matrix can be considered as:

$$F_k = V.\bar{V}_n \tag{9}$$

where $V \in C^{N_t \times N_t}$ is the unitary matrix whose each column has the right singular vector of H_k.

4.1.1 Derivation of an optimal receive filter

To minimize the MSE between the transmitted data vector and received data vector, we apply SVD to the downlink channel H similar to equation (5).

$$H = [U_{\text{signal}k} U_{\text{null}}] \begin{bmatrix} \sigma_k \\ 0 \end{bmatrix} V^H \tag{10}$$

where $U_{\text{signal}k} \in C^{N_r \times n_k}$ and $U_{\text{null}} \in C^{N_r \times (N_r - n_k)}$. Under the assumption that the transmit filter matrix is F = V which performs eigenmode transmission. By substitution, equation (3) can be rewritten as:

$$R = \sum_{k=1}^{K} H_k S_k + n = \sqrt{\frac{2E_s}{T_s}} [H_1 F_1 ... H_k F_k] \begin{bmatrix} D_1 \\ \vdots \\ D_k \end{bmatrix} + n \tag{11}$$

$$R=\sqrt{\frac{2E_s}{T_s}}[U_{signal1}\ldots U_{signalk}].\begin{bmatrix}\sigma_1 & & \\ & \ddots & \\ & & \sigma_k\end{bmatrix}D+n \quad (12)$$

$$R=\sqrt{\frac{2E_s}{T_s}}U_{signal}.\sigma_k D+n \quad (13)$$

MIMO for WSN, U_{signal} is not a unitary matrix. We multiply pseudo-inverse matrix of U_{signal} to remove IAI and IUI at the slave node such that $(U^H_{signal}U_{signal})^{-1}U^H_{signal}$. By substituting in equation (**??**):

$$\varepsilon=\sum_{k=1}^{K}\mathrm{tr}[(I_{nk}-WU_{signal}\sigma_k)\times(I_{nk}-WU_{signal}\sigma_k)^H]\gamma^{-1}$$
$$\sum_{k=1}^{K}\mathrm{tr}(WW^H) \quad (14)$$

Under the assumption that transmit filter matrices F_k are given, the above equation can be minimized when $\partial\varepsilon/\partial W$=0. Thus, by applying the minimum mean squared error (MMSE) criterion, the receive filter matrix can be expressed as:

$$W=\{(H_kF_k)^HH_kF_k+\gamma I_{nk}\}^{-1}\times(H_kF_k)^H \quad (15)$$

where γ is a parameter related to the power ratio between the signal and noise n.

4.2 Proposed beamforming scheme 1

In this section, we propose a beamforming scheme. In single-user MIMO, the transmit beam former and the combining vector are F_k and W, respectively. Based on the decoding of single-user MIMO, where the decoding is a function of the projection of the channel H_k on the left singular vector U. In a similar manner, let's set the beam former in a multiuser setting to be a function of the right singular vector V and its projection on the effective channel H_k. Thus, in general, the pre-precoder is set to be:

$$\tilde{F}_k=\frac{VH_k^H}{\|VH_k^H\|} \quad (16)$$

where V can be obtained from singular value decomposition SVD of H_k. Thus, the effective channel to slave node k reduces to

$$\tilde{F}_k.H_k=\frac{VH_k^H}{\|VH_k^H\|}.H_k \quad (17)$$

The multi-user system sum rate can be maximized by eliminating inter-user interference IUI. Thus, the optimal beam former must be chosen based on the space spanned by the matrix H_k such that:

$$H_j^H.F_k=0 \quad \text{for}\forall j\neq k, k=1,2,\ldots,n_k \quad (18)$$

The beam former that maximizes the signal-to-noise ratio and eliminates inter user interference (IUI) must be the projection of the effective channel on the space basis formed by the strongest singular values and the left singular vector so the beam former is:

$$F_k=\alpha.V.H_k^H.\sigma(:,i).U_k \quad (19)$$

where α is a scalar quantity. $V\in C^{N_t\times N_t}$ is a unitary matrix whose columns are the right singular vector of Hk and σ(:, i) $\in C^{n_k\times n_k}$ is the singular values of H_k where i = 1, 2, . . . , n_k. $U_k\in C^{n_k\times n_k}$ is the left singular vector of H_k for slave node k. The combining vector uses the minimum mean squared error (MMSE) criterion to eliminate inter-antenna interference (IAI). The receive combining vector is set to be a function of the pre-coder such that:

$$W=\{(H_kF_k)^HH_kF_k+\gamma I_{nk}\}^{-1}\times(H_kF_k)^H \quad (20)$$

4.3 Proposed beamforming scheme 2

In this section, an optimal beamforming scheme is proposed. In multi-user MIMO, the optimal beam former is F_k and the optimal receive combining vector is W. Inspired by the decoding of a single-user MIMO in Block diagonalization/ Singular value decomposition (BD/SVD), the combining vector in multi-user settings is set to be:

$$Z=\frac{H_kF_k}{\|H_kF_k\|} \quad (21)$$

where F_k is the optimal beamformer for a multiuser MIMO system. Applying the combining vector, the effective channel to slave node k is reduced to:

$$Z^HH_k=\alpha_k\sigma_k(:,i)F_k^H, i=1,2,\ldots,n_k \quad (22)$$

where α_k is a scalar quantity. σ_k(:, i) is the singular value of the channel matrix H_k. The optimal beamformer that eliminates inter-user interference (IUI) and inter-antenna interference (IAI) and at the same time maximizes the signal-to-noise ratio and minimize the system symbol error rate (SER) must be the projection of the effective channel H_k based on the right singular vector V_k such that:

$$F_k=V_k\sigma_k(:,i), i=1,2,\ldots,n_k \quad (23)$$

where $V_k\in C^{N_t\times N_t}$ is a unitary matrix whose columns are the right singular vectors. $\sigma_k\in C^{N_t\times n_k}$ is a matrix containing the strongest singular values of H_k. Using the minimum mean squared error criterion, the optimal combining vector that eliminates IAI is:

$$W=\{Z^H.Z+\gamma.I_{n_k}\}^{-1}\times Z^H \quad (24)$$

where γ is a parameter related to the power ratio between the signal and noise n. The equivalent channel after applying the proposed beamforming and combining vectors reduces to a parallel single user SISO channel as follows:

$$\begin{bmatrix}Z_1^HH_1\\ Z_2^HH_2\\ \vdots\\ Z_k^HH_k\end{bmatrix}.\begin{bmatrix}F_1^H\\ F_2^H\\ \vdots\\ F_k^H\end{bmatrix}=\begin{bmatrix}\alpha_1\sigma_1 & 0 & \cdots & 0\\ 0 & \alpha_2\sigma_2 & \cdots & 0\\ \vdots & \vdots & \ddots & 0\\ 0 & 0 & \cdots & \alpha_k\sigma_k\end{bmatrix} \quad (25)$$

where σ_1, σ_2, σ_k are the singular values of H_k. In this case, the beam former is optimal when the number of transmitted antennas is larger than the total number of slave nodes receive antennas.

In this section, as reliability is of an important concern for Ad-Hoc networks, the symbol error rate (SER) performance of the proposed schemes is provided. The simulation platform is NS2. A master node with the number of transmit antennas equal to 4 is used and the number of slave nodes is 2. Each slave node has two antennas. The used modulation is a 16 quadrature amplitude modulation (16QAM). Fig. 4 shows the simulation results for the proposed beamforming schemes. The proposed beamforming schemes are compared with the Block Diagonalization (BD) transmission method for multi-user MIMO settings. As shown in Fig. 4, the BD/SVD scheme achieves better performance than block diagonalization at a low signal-to-noise ratio (SNR). However, the performance is deteriorated at a high signal-to-noise ratio. This is attributed to the increased inter-antenna interference at a high SNR. Furthermore, the figure demonstrates that the new proposed schemes 1 and 2 provide superior performance in comparison to the block diagonalization scheme. The reason for that outperformance is the optimum design of the beamforming schemes and the criterion used in the detection methods. It can be seen that proposed scheme 1 has a coding gain of about 5dB at SER = 10^{-3} while proposed scheme 2 achieves a gain of about 9dB at SER = 10^{-3} in comparison to the BD transmission method.

The proposed beamforming schemes are scalable. They can be applied to massive MIMO systems. The proposed schemes are applied to the following system such that the total number of master node antennas is 32 while the number of slave nodes is 16. Fig. 5 shows that the proposed beamforming scheme 2 outperforms the other schemes. This is attributed to that the beam former is a function of the strongest singular values. It can be seen also that proposed scheme 1 has a good performance in comparison to the BD/SVD scheme. This results from the dependability of the beam former on the strongest singularvalues besides the optimality of the receive combining vector. For the proposed scheme 2, a system of 64 master node antennas transmitting information to 16 slave nodes is proposed for proposed scheme 2.

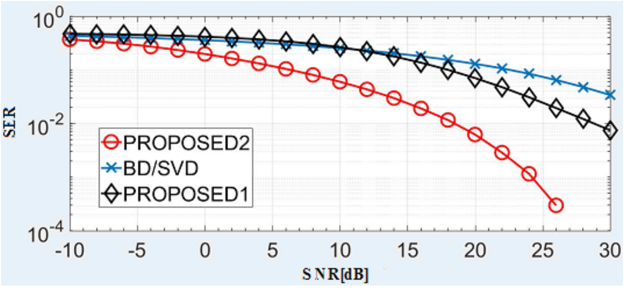

Figure 5. SER performance of the proposed schemes for massive MIMO systems.

Fig. 6 shows the performance for 64 master node antennas where SER = 10^{-6} can be achieved at about SNR = 20dB which plays a great role in achieving the demanded reliability of SER = 10^{-9} for Ad-Hoc networks. Another comparison parameter is the average processing of a packet using the proposed beamforming scheme. BD transmission schemes take about 0.0014 sec using an NS2 simulation program running on an intel core-i7 2.5GHz CPU. BD/SVD beamforming scheme requires 0.0016 sec to process a single packet. The consumed time for a proposed beamforming scheme 1 is 0.0014 sec, while it takes only 0.0013 sec for proposed scheme 2. As we can see the proposed scheme 2 has the optimal performance in terms of low computational complexity and lower SER performance.

Table 1 compares the proposed methods with other existing methods in terms of symbol error rate performance, computational complexity, and scalability for a signal-to-noise ratio of 20 dB with 64 master node antennas along with 16 slave nodes.

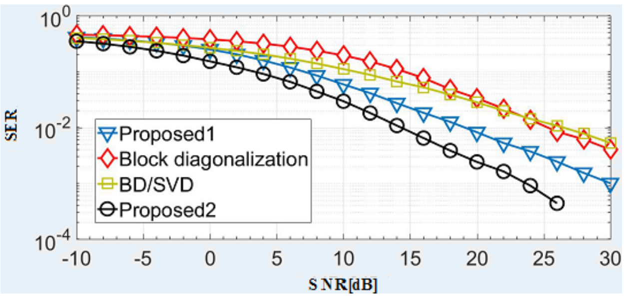

Figure 4. SER performance of the proposed schemes.

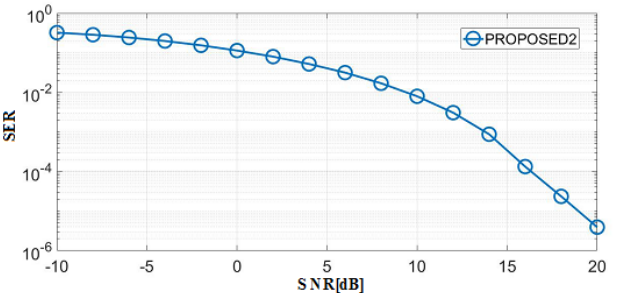

Figure 6. SER performance of the new proposed scheme 2.

Table 1. Comparison between proposed methods with other existing methods

System model	SER	computational complexity (10^{-4}sec)	scalability
Proposed Scheme 1	10^{-5}	13	scalable
Block Diagonalization	10^{-4}	14	Not scalable
BD/SVD	10^{-4}	16	Not scalable
Proposed Scheme 2	10^{-6}	11	scalable

5. Conclusion

In this paper, joint transmit/receive (TX/RX) filtering schemes for Ad-Hoc networks are proposed. Simulation results are provided and showed that the proposed beamforming schemes outperform the widely adopted block diagonalization scheme. The simulation results are presented in terms of symbol error rate. The symbol error rate performance demonstrated that more robust communication can be achieved with the proposed joint TX/RX filtering schemes. The proposed schemes are applied to a massive MIMO system. Simulation results exposed that the demanded reliability of Ad-Hoc networks can be met. The proposed schemes play a pivotal role in the standardization of Ad-Hoc networks.

Authors contributions
Not applicable

Availability of data and materials
The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflict of interests
The author declares that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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