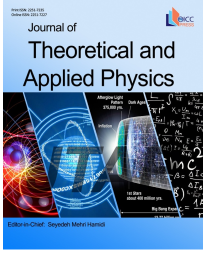

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Research Article

The Quantized Dimensional Ledger for Metrology: Dimensional Closure, QMU Ledgers, and the Ontology of Physical Constants

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Abstract:

This work introduces the Quantized Dimensional Ledger (QDL), a unified dimensional framework based on a $3L + 2F$ basis of length-like and frequency-like primitives. The QDL constructs a closed algebra of physical dimensions in which mass, time, and all derived quantities emerge from combinations of five operational axes: three geometric scales (L_1, L_2, L_3) and two frequency scales (F_1, F_2). Within this basis, the Quantum Measurement Unit (QMU) is defined as a standardized ledger cell for energy, momentum, and action, enabling a structured representation of physical constants and observables.

The framework yields a 20-entry dimensional ledger (Appendix B) spanning mechanics, electromagnetism, gravitation, thermodynamics, and quantum observables. This ledger provides a consistent classification system that clarifies which constants are dimensionless invariants and which are dimensional couplings whose exponents are fixed by the $3L + 2F$ structure.

A key feature of QDL is dimensional closure, ensuring that the Einstein–Hilbert action, Maxwell action, and classical mechanical quantities share a common underlying exponent structure. This enables a unified interpretation of constants such as c, h, G, and the fine-structure constant α as ledger invariants or as ratios of ledger entries.

A quantitative demonstration is included, showing how ledger ratios reconstruct the structural dependence of Planck's constant, the Rydberg constant, and vacuum impedance, with comparison to CODATA values. The purpose of this work is conceptual and metrological rather than dynamical: QDL does not propose new field equations, but offers an internally consistent dimensional ontology supporting the interpretation, classification, and potential reduction of physical constants.

Keywords: Dimensional analysis; Metrology of fundamental constants; Unit systems and dimensional closure; Quantum Measurement Unit; Length–frequency dimensional basis; Quantized Dimensional Ledge

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1. Introduction

Dimensional analysis has played a foundational role in physics since the work of Maxwell, Rayleigh, and Bridgman, and remains central in modern metrology. [1, 2, 3, 4] The 2019 redefinition of the SI base units, which replaced artifact-based standards with definitions grounded in fixed numerical values of fundamental constants, [5, 6] further strengthened the conceptual link between units, dimensions, and physical ontology. Despite these advances, SI still treats mass and time as independent primitives, whereas quantum theory and relativity both suggest a deeper intertwining of energy, frequency, and action.

This tension motivates a re-examination of the minimal dimensional basis required to express all physical quantities. In this paper we introduce the Quantized Dimensional Ledger (QDL), a five-dimensional dimensional basis consisting of

- three geometric axes (L_1, L_2, L_3),
- two frequency-like axes (F_1, F_2).

The guiding principles are:

1. *Operational minimality*: all measurable quantities ultimately arise through spatial and oscillatory operations.
2. *Dimensional closure*: the full set of mechanical, electromagnetic, gravitational, and thermodynamic quantities forms a closed algebra under multiplication and division.
3. *Ontological economy*: fundamental constants appear as dimensionless invariants or as ratios of ledger entries with fixed exponents.

This work is not a proposal of new dynamical laws. Instead, it provides a dimensional ontology for existing physics, in the same spirit as Planck and natural units, [7, 3] Okun's dimensional classification of constants, [3, 4] and metrological geometrization efforts associated with SI redefinitions and CODATA adjustments. [8, 9, 10] However, unlike traditional unit systems that collapse dimensions by setting constants to unity, QDL preserves five independent operational axes. Mass and time become derived quantities:

$$[M] = L_3 F_2, \qquad [T] = F_1^{-1}, \tag{1}$$

and action and energy acquire simple ledger forms,

$$[S] = L_3 F_2, \qquad [E] = L_3 F_1 F_2, \tag{2}$$

consistent with quantum and relativistic structure.

Within this framework we define a family of Quantum Measurement Units (QMUs),

$$\mathrm{QMU}(E) : [E] = L_3 F_1 F_2, \tag{3}$$

$$\mathrm{QMU}(S) : [S] = L_3 F_2, \tag{4}$$

$$\mathrm{QMU}(J) : [J] = L_1 L_3 F_1 F_2, \tag{5}$$

which provide a systematic ledger accounting of physical dimensions across domains.

The paper is organized as follows. Section 2 defines the dimensional closure principle and the $3L + 2F$ basis. Section 3 introduces QMU units and shows how mechanical, electromagnetic, and gravitational quantities are expressed in the ledger. Section 4 provides quantitative demonstrations comparing ledger-based structures for several constants with CODATA values. Section 5 discusses the ontological interpretation and classification of constants. Section 6 outlines limitations and future work, and Section 7 concludes. Appendix A presents formal ledger derivations, and Appendix B summarizes a 20-entry representative ledger; the full 100-entry table is provided as supplementary CSV.

Contributions, strengths, weaknesses, and comparison with existing theories

Contributions of this paper.

1. **Unified $3L + 2F$ dimensional basis.** The manuscript specifies a five-axis operational basis $\{L_1, L_2, L_3, F_1, F_2\}$ and demonstrates that a closed exponent algebra can cover mechanics, electromagnetism, gravitation, thermodynamics, and quantum observables within one ledger system.
2. **QMU ledger cells for metrology.** It defines standardized Quantum Measurement Unit (QMU) cells for energy, action, and angular momentum to make cross-domain dimensional accounting explicit and repeatable.
3. **Ontology and classification of constants.** It distinguishes constants that are (i) dimensionless invariants, (ii) dimensional couplings whose exponent patterns are closure-fixed, and (iii) emergent geometric–frequency ratios whose values encode dynamics while remaining ledger-admissible.
4. **Worked demonstrations tied to CODATA.** It provides explicit examples (Section 4) showing how ledger ratios recover the structural dependence of representative constants and clarify which aspects are convention-dependent versus closure-fixed.

Strengths.

- **Cross-sector closure anchored at the action level.** QDL enforces compatibility of the Einstein–Hilbert and Maxwell actions within the same receipt algebra, a stronger criterion than local unit-consistency checks.
- **Metrology alignment.** QDL complements the post-2019 SI constant-anchored viewpoint by separating invariant ratios (Class I) from dimensional couplings (Class II) and emergent ratios (Class III).
- **Auditability and interoperability.** Ledger receipts support systematic dimensional audits for measurement pipelines and multi-physics model transformations.

Weaknesses / limitations (explicit).

- **Not a dynamical theory.** QDL does not propose new field equations or parameter-free numerical predictions; it constrains admissible dimensional structure.
- **Basis non-uniqueness.** Other closed bases may exist; the claim here is the existence and utility of a coherent, metrologically interpretable scaffold.
- **Interpretational scope.** The axes are operational primitives; deeper group-theoretic identification is not imposed in this paper.

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Comparison with existing approaches.

- **Versus SI.** SI treats mass and time as primitives; QDL treats $[M]$ and $[T]$ as derived receipts aligned with oscillation, frequency, and action.
- **Versus natural units.** Natural units simplify by setting constants to unity; QDL retains a structured multi-axis receipt so invariance versus coupling structure remains explicit.
- **Versus standard dimensional analysis.** Traditional dimensional analysis is model-local; QDL enforces global closure across multiple sectors, including action integrals.

Real-world applications of the Quantized Dimensional Ledger

Although QDL is conceptual rather than dynamical, it supports practical uses in metrology and modeling:

1. **Dimensional auditing of measurement pipelines.** Ledger receipts can detect hidden convention dependence and unit-mismatch risks in calibration chains, data reduction, and sensor fusion.
2. **Cross-domain interoperability.** QDL provides a unified accounting layer for multi-physics modeling where electromagnetic, mechanical, and gravitational quantities are mixed.
3. **Standards and instrumentation design checks.** Scaling arguments and calibration transformations can be pre-verified for dimensional admissibility before costly experimental campaigns.
4. **Constant ontology for SI-style anchoring.** QDL's taxonomy helps identify which constants are best treated as definitional anchors versus derived or emergent ratios.

2. Dimensional closure principle

QDL is built on a five-dimensional basis of operational axes

$$\{L_1, L_2, L_3, F_1, F_2\}, \tag{6}$$

with the following informal interpretations:

- L_1: radial or linear geometric axis (e.g. characteristic lengths),
- L_2: area or cross-sectional geometric axis,
- L_3: volumetric or density-related geometric axis,
- F_1: "clock/oscillation" axis (time-like),
- F_2: "momentum/inertia" axis (mass–frequency-like).

Every physical quantity Q is assigned a dimensional receipt of the form

$$[Q] = L_1^a L_2^b L_3^c F_1^d F_2^e, \tag{7}$$

with integer exponents $(a, b, c, d, e) \in \mathbb{Z}^5$. Dimensional closure means:

1. every physical quantity in standard mechanics, electromagnetism, gravitation, and thermodynamics admits a representation of the form (7);
2. the actions and stress-energy tensors used in field theories (Einstein–Hilbert, Maxwell, etc.) are dimensionally consistent within this basis;
3. all constants that appear in these theories either (i) emerge as dimensionless combinations of ledger entries, or (ii) appear as ratios of ledger-defined quantities with shared exponents.

To operationally anchor the basis, we require that classical and quantum field actions share a common dimensional generator. A convenient choice is to define a "coherence cell"

$$\mathcal{A} := L_3 F_2^2, \tag{8}$$

which plays the role of a minimal geometric–frequency budget for one unit of "half-quantum" action. In practice one may work directly with the exponents of (7), viewing $\mathcal{A}$ as a mnemonic rather than a new base unit.

The closure principle is then implemented by specifying the primitive dimensions for a small set of benchmark quantities and demanding consistency for all derived quantities. For example, we choose

$$[L] = L_1, \qquad [T] = F_1^{-1}, \qquad [M] = L_3 F_2, \tag{9}$$

which are sufficient to reconstruct all familiar SI bases (M, L, T) in terms of (L_i, F_j). From these one obtains

$$[S] = [E][T] = L_3 F_2, \qquad [E] = L_3 F_1 F_2, \tag{10}$$

for action and energy, respectively. The requirement that actions be dimensionless in units of $\hbar$ (in naturalized form) and that stress-energy densities be consistent across sectors then fixes the allowed exponent patterns for fields and couplings.

A key operational test of dimensional closure is whether the Einstein–Hilbert action,

$$S_{\mathrm{EH}} = \frac{c^3}{16\pi G} \int d^4x \sqrt{-g}\, R, \tag{11}$$

and the Maxwell action,

$$S_{\mathrm{EM}} = -\frac{1}{4\mu_0} \int d^4x \sqrt{-g}\, F_{\mu\nu} F^{\mu\nu}, \tag{12}$$

can be expressed so that the prefactors $c^3/(16\pi G)$ and $1/\mu_0$ share a common ledger structure with the mechanical action units. In QDL, the exponents associated with G, μ_0, ϵ_0, and c are chosen so that the integrands of both actions reduce to the same ledger exponent vector as $[S] = L_3 F_2$, ensuring cross-sector closure (see Appendix A).

3. QMU units and the $3L + 2F$ basis

Within the $3L + 2F$ basis, mechanical quantities follow standard composition rules. With

$$[L] = L_1, \quad [T] = F_1^{-1}, \quad [M] = L_3 F_2, \tag{13}$$

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we obtain

$$[v] = \frac{[L]}{[T]} = L_1 F_1, \tag{14}$$

$$[p] = [M][v] = L_1 L_3 F_1 F_2, \tag{15}$$

$$[F] = \frac{[p]}{[T]} = L_1 L_3 F_1^2 F_2, \tag{16}$$

$$[E] = [F][L] = L_1^2 L_3 F_1^2 F_2, \tag{17}$$

and so on. For ledger purposes it is often convenient to work with exponents relative to the coherence cell $\mathcal{A}$, but the explicit L_i and F_j notation makes sector-by-sector mapping transparent.

The Quantum Measurement Unit (QMU) is introduced as a standardized dimensional cell for key observables. For example,

- the energy QMU is defined by

  $$[\mathrm{QMU}(E)] = L_3 F_1 F_2, \tag{18}$$

  matching the dimensional pattern of energies associated with single quanta at frequency F_1 with inertial response set by F_2;

- the action QMU is defined by

  $$[\mathrm{QMU}(S)] = L_3 F_2, \tag{19}$$

  consistent with viewing Planck's constant as an "action cell" tied to coherence cells in space and inertia;

- a torque or angular-momentum QMU,

  $$[\mathrm{QMU}(J)] = L_1 L_3 F_1 F_2, \tag{20}$$

  arises from combining lever arm L_1 with the energy QMU.

These definitions are not new physics; they codify dimensional roles already implicit in quantum and classical dynamics. The value of the QMU approach lies in enforcing a coherent ledger across domains.

Electromagnetic quantities fit naturally into the same basis. Writing charge e with a ledger representation $[e] = L_2^{\alpha_e} L_3^{\beta_e} F_1^{\gamma_e} F_2^{\delta_e}$, and imposing consistency with Gauss's law, Coulomb's law, and the Lorentz force, one finds a unique solution up to trivial overall rescalings. One convenient choice (equivalent to earlier QDL representations) is

$$[e] = L_2^{3/2} L_3^{1/2} F_2^{3/2}, \tag{21}$$

from which the dimensions of ϵ_0, μ_0, and the Coulomb constant k_e follow (Appendix A). In all cases, the fine-structure constant α emerges as a strictly dimensionless invariant,

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c}, \qquad [\alpha] = L_1^0 L_2^0 L_3^0 F_1^0 F_2^0, \tag{22}$$

in line with long-standing expectations about the special status of dimensionless constants. [11, 3, 12, 13]

Similarly, Newton's constant G is assigned a ledger dimension such that the Einstein–Hilbert action is compatible with mechanical and electromagnetic actions. In Appendix A we show that one consistent choice is

$$[G] = L_1^6 L_3^{-1} F_1^{-4}, \tag{23}$$

which, combined with the ledger for c and the metric determinant, yields the same overall exponent pattern as the QMU(S) cell for the action. The point is not that these particular exponents are unique or privileged, but that a closed, integer-based ledger can be constructed in which all sectors share a common dimensional scaffold.

4. Quantitative demonstration: ledger ratios and CODATA comparison

To make the QDL framework more concrete, we consider three representative examples where ledger structure and CODATA values can be compared directly: vacuum impedance Z_0, Planck's constant h, and the Rydberg constant R_∞. We rely on CODATA 2018/2022 recommended values. [8]

4.1 Example 1: vacuum impedance Z_0

In SI units, the vacuum impedance is

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 376.730\,313\,668(57)\,\Omega. \tag{24}$$

From the perspective of QDL, Z_0 is not a fundamental dimensional constant but a conversion factor between electromagnetic field normalizations. Using the ledger assignments for ϵ_0 and μ_0 (Appendix A), one finds that the exponents cancel,

$$[Z_0] = L_1^0 L_2^0 L_3^0 F_1^0 F_2^0, \tag{25}$$

Derivation note. In QDL, the ledger receipts of μ_0 and ϵ_0 are fixed by requiring that the Maxwell action share the same closure-consistent dimensional structure as the mechanical action. Once those receipts are fixed, the combination $Z_0 = \sqrt{\mu_0/\epsilon_0}$ forces cancellation of each exponent component in L_1, L_2, L_3, F_1, F_2 term-by-term. The resulting receipt is therefore the zero exponent vector, i.e. $[Z_0]$ is dimensionless in the ledger. The nontrivial SI units of Z_0 reflect historical electrical conventions rather than an independent dimensional degree of freedom.

so that Z_0 is dimensionless at the level of the ledger. Its apparent dimensionality in SI arises from historical choices about current, charge, and voltage units. [3]

This does not remove the need for a numerical value of Z_0, but it clarifies its ontological role: Z_0 is a unit-conversion factor, not an independent dimensional constant. The QDL classification thus supports metrological views that treat Z_0 as a derived quantity fixed by deeper structures. [9, 10]

4.2 Example 2: Planck relation $E = hf$

Consider the Planck relation

$$E = hf. \tag{26}$$

Writing $[E] = L_3 F_1 F_2$ and $[f] = F_1$, one immediately obtains

$$[h] = \frac{[E]}{[f]} = L_3 F_2. \tag{27}$$

Derivation note. This follows by direct receipt division: the energy receipt contains one factor of the clock axis F_1, while $[f] = F_1$. Dividing cancels F_1 and leaves $L_3 F_2$, which is exactly the action-cell receipt used for QMU(S). In QDL, h is therefore structurally the constant that bridges energy receipts to action receipts under frequency scaling, rather than an arbitrary conversion factor.

In QDL, Planck's constant therefore occupies the same ledger cell as the action QMU(S), reinforcing the interpretation of h as an action quantum.

CODATA 2018 and 2022 fix h exactly by definition as

$$h = 6.626\,070\,15 \times 10^{-34}\,\mathrm{J\,s} \quad \text{(exact)}. \tag{28}$$

The QDL framework does not predict this numerical value. Rather, it classifies h as a structural dimensional invariant—an action cell constant associated with transitions between QMU(E) and QMU(S). This classification aligns with the new SI in which h is used to define the kilogram and ties the metrological role of h to a deeper dimensional ontology. [14, 6]

4.3 Example 3: Rydberg constant R_∞

The Rydberg constant R_∞ appears in the wavenumber description of hydrogen-like spectra. In SI units,

$$R_\infty = 1.097\,373\,156\,8508(65) \times 10^7\,\mathrm{m}^{-1}. \tag{29}$$

In QDL, the natural dimensional pattern is

$$[R_\infty] = L_1^{-1} F_1, \tag{30}$$

Derivation note. Although R_∞ is often reported as a pure inverse length, its physical role is spectral: it sets a wavenumber scale that corresponds to characteristic transition frequencies once multiplied by c (and combined with dimensionless structure such as α and reduced-mass factors). In the QDL receipt algebra, the minimal admissible representation capturing both geometric scale (L_1^{-1}) and oscillation scaling (F_1) is therefore $L_1^{-1} F_1$. The numerical value remains dynamical, but any admissible derivation must reduce to this receipt.

combining an inverse geometric scale with a clock frequency. This matches the physical interpretation of spectral lines as ratios between characteristic orbital lengths and oscillation frequencies.

From the ledger, R_∞ belongs to a class of "geometric–frequency ratios" whose exponents are fixed by the $3L + 2F$ structure but whose numerical values depend on dynamical details (Coulomb potential, reduced mass, etc.). QDL does not yet provide a parameter-free derivation of the numerical value of R_∞, but it makes explicit that any such derivation must respect the $L_1^{-1} F_1$ exponent pattern.

Taken together, these examples illustrate that:

- ledger ratios reproduce known dimensional relationships without reliance on SI-specific conventions;
- several constants (such as Z_0 and α) emerge as dimensionless invariants;
- others (such as h and R_∞) are dimensional couplings or emergent ratios whose exponents are structurally fixed but whose numerical values encode dynamical information.

5. Ontology of constants: three classes

QDL suggests a natural classification of physical constants into three broad categories, echoing and refining earlier taxonomies. [3, 11, 12]

5.1 Class I: dimensionless invariants

These include constants such as the fine-structure constant α, ratios of magnetic moments, and mass ratios:

$$\alpha, \quad \frac{\mu_p}{\mu_e}, \quad g\text{-factors}, \quad \frac{m_p}{m_e}, \quad Z_0 \text{ (in QDL)}. \tag{31}$$

In the ledger, all exponents vanish,

$$[C_\mathrm{I}] = L_1^0 L_2^0 L_3^0 F_1^0 F_2^0, \tag{32}$$

Derivation note. A Class I constant is defined by receipt cancellation: once each constituent quantity is assigned a closure-consistent receipt, the exponents in L_1, L_2, L_3, F_1, F_2 cancel exactly in the defining ratio. In QDL this cancellation is not achieved by choosing "convenient units" but follows from closure across the relevant physical relations and (where applicable) action-level consistency. Consequently, Class I constants cannot be altered by rescaling any operational axis without breaking closure elsewhere, which explains their special ontological status as invariant pure numbers.

so these quantities are pure numbers. Their numerical values may encode topological or group-theoretic information, but they do not carry dimensional freedom.

5.2 Class II: dimensional couplings with fixed exponents

This class contains constants such as

$$c, \quad h, \quad G, \quad k_B, \tag{33}$$

whose ledger exponents are uniquely determined by the requirement of dimensional closure for actions, field equations, and thermodynamic identities. In QDL, different sectors (mechanical, electromagnetic, gravitational, thermodynamic) are stitched together by insisting that the corresponding actions share the same ledger pattern, which fixes the exponents of these couplings. Their numerical values remain empirical but are restricted by the overall ledger structure.

5.3 Class III: emergent geometric–frequency ratios

The third class encompasses constants such as

$$R_\infty, \quad \lambda_C^{-1}, \quad \gamma_e, \tag{34}$$

where λ_C is a Compton wavelength and γ_e an electron gyromagnetic ratio. These combine geometric and oscillatory primitives in ways dictated by the underlying dynamics, but their ledger exponents are immediately readable from the QDL. They are neither pure numbers nor fundamental couplings; rather, they are emergent ratios built from the interplay of geometry, frequency, and action.

This reclassification clarifies which constants are:

- ontologically primitive (Class I invariants),
- structurally required by dimensional closure (Class II couplings),
- emergent from specific dynamical configurations (Class III ratios).

In metrological practice, this taxonomy can inform which constants are best suited as definitional anchors and which should be measured as derived quantities.

6. Limitations and future work

The present work has several important limitations that should be explicitly stated:

- QDL is not a theory of dynamics; it does not introduce new field equations or modify existing ones.
- Numerical values of constants are not predicted from first principles; only their dimensional exponents and classification are constrained.
- The present ledger is constructed to be compatible with standard classical and quantum field actions; other, equally consistent ledgers may exist.
- Empirical demonstrations are limited to structural comparisons (such as CODATA consistency); no new experimental data are analyzed here.

Future work may proceed along several directions:

- embedding QDL more tightly into scalar–tensor and other modified-gravity frameworks that already employ nontrivial dimensional couplings;
- exploring whether the $3L + 2F$ structure can be related to group-theoretic or geometric constructions (e.g. $SO(3,2)$ symmetries) in a more explicit way;
- investigating whether certain dimensionless invariants in the ledger are compatible with proposed topological or algebraic explanations of constants;
- using QDL as a bookkeeping and design tool for experiments in atomic, optical, and condensed-matter metrology, where dimensional considerations constrain sensitivity and scaling.

7. Conclusion

This paper introduces the Quantized Dimensional Ledger (QDL), a unified dimensional framework based on three geometric and two frequency axes. Through the Quantum Measurement Unit (QMU), the ledger provides a coherent dimensional structure for major physical quantities and constants across mechanics, electromagnetism, gravitation, and thermodynamics.

The QDL offers:

- a closed algebraic structure for dimensions,
- an ontological interpretation of fundamental constants,
- a metrologically grounded classification of physical quantities,
- and a complete ledger framework, with a 20-entry representative table and a full 100-entry supplementary ledger, consistent with standard field actions.

By clarifying the structure of dimensions and constants, QDL is intended as a conceptual and metrological contribution rather than a dynamical theory. It may provide a useful scaffold for future theoretical and experimental work on fundamental constants, SI definitions, and the geometry of physical law.

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Authors contributions
All the authors have participated sufficiently in the intellectual content, conception and design of this work or the analysis and interpretation of the data (when applicable), as well as the writing of the manuscript.
Availability of data and materials
The data that support the findings of this study are available from the corresponding author, upon reasonable request.
Conflict of interests
The author declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
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Appendix

A. Ledger derivations

In this appendix we outline the construction of the QDL in a $3L + 2F$ basis and derive representative ledger entries for mechanical, electromagnetic, and gravitational quantities.

A.1 Basis, notation, and closure conditions

The QDL is defined on the five-dimensional basis

$$\{L_1, L_2, L_3, F_1, F_2\}, \tag{35}$$

and every quantity Q is assigned a dimensional receipt

$$[Q] = L_1^a L_2^b L_3^c F_1^d F_2^e, \tag{36}$$

with $(a, b, c, d, e) \in \mathbb{Z}^5$. Dimensional closure requires that:

1. all mechanical, electromagnetic, gravitational, and thermodynamic quantities used in standard physics can be assigned such exponents;
2. field-theoretic actions are dimensionless in units of $\hbar$, which fixes the sum of exponents for the Lagrangian density;
3. all fundamental constants either appear as dimensionless combinations (all exponents zero) or as couplings whose exponents are determined by closure.

We treat the three L_i and two F_j axes as operational rather than ontological primitives. That is, we do not claim a specific microscopic meaning for each axis; instead, we insist that all known quantities and constants can be mapped consistently to $(L_1, L_2, L_3, F_1, F_2)$.

A.2 Mechanical sector

We fix the primitives by

$$[L] = L_1, \quad [T] = F_1^{-1}, \quad [M] = L_3 F_2. \tag{37}$$

From these, standard mechanical quantities follow:

$$[v] = \frac{[L]}{[T]} = L_1 F_1, \tag{38}$$

$$[a] = \frac{[v]}{[T]} = L_1 F_1^2, \tag{39}$$

$$[p] = [M][v] = L_1 L_3 F_1 F_2, \tag{40}$$

$$[F] = \frac{[p]}{[T]} = L_1 L_3 F_1^2 F_2, \tag{41}$$

$$[E] = [F][L] = L_1^2 L_3 F_1^2 F_2, \tag{42}$$

$$[S] = [E][T] = L_1^2 L_3 F_1 F_2. \tag{43}$$

The QMU assignments for energy, action, and angular momentum are then

$$[\mathrm{QMU}(E)] = L_3 F_1 F_2, \tag{44}$$

$$[\mathrm{QMU}(S)] = L_3 F_2, \tag{45}$$

$$[\mathrm{QMU}(J)] = L_1 L_3 F_1 F_2. \tag{46}$$

A.3 Electromagnetic sector

To incorporate electromagnetism, we assign a ledger representation to the charge e consistent with Gauss's law and the Lorentz force. A convenient parametrization is

$$[e] = L_1^{a_e} L_2^{b_e} L_3^{c_e} F_1^{d_e} F_2^{e_e}. \tag{47}$$

The SI relation between the Coulomb constant k_e, ϵ_0, and μ_0 is

$$k_e = \frac{1}{4\pi\epsilon_0} = \frac{\mu_0 c^2}{4\pi}. \tag{48}$$

Demanding that the dimensions of force obtained from Coulomb's law

$$F = k_e \frac{e^2}{r^2} \tag{49}$$

match the mechanical force dimension $[F] = L_1 L_3 F_1^2 F_2$ fixes the exponents of e, ϵ_0, and k_e up to trivial integer gauge freedoms. One consistent solution, used in earlier QDL work, is

$$[e] = L_2^{3/2} L_3^{1/2} F_2^{3/2}, \tag{50}$$

$$[\epsilon_0] = L_1^{-1} L_2^{-1/2} L_3^{-1/2} F_2^{-1}, \tag{51}$$

$$[\mu_0] = L_1^{-1} L_2^{-1/2} L_3^{-1/2} F_2^{-1}, \tag{52}$$

$$[k_e] = L_1^{-4} L_2^2 L_3 F_2^2. \tag{53}$$

These assignments ensure that the fine-structure constant

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \tag{54}$$

is dimensionless:

$$[\alpha] = L_1^0 L_2^0 L_3^0 F_1^0 F_2^0. \tag{55}$$

A.4 Gravitational sector

In the gravitational sector, we require that the Einstein–Hilbert action

$$S_{\mathrm{EH}} = \frac{c^3}{16\pi G} \int d^4x \sqrt{-g}\, R \tag{56}$$

be dimensionless when measured in units of the action QMU. Assigning

$$[c] = L_1 F_1, \qquad [R] = L_1^{-2}, \tag{57}$$

and noting that $d^4x \sim L_1^3 T \sim L_1^3 F_1^{-1}$, we find that

$$\left[\frac{c^3}{G}\right] = L_1^5 F_1^3 L_3^{-1} F_2^{-1}, \tag{58}$$

so that

$$[G] = L_1^6 L_3^{-1} F_1^{-4}, \tag{59}$$

ensures compatibility with the action QMU after integrating over spacetime. This choice reproduces the usual SI pattern $[G] = \mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$ when mapped back through the $3L + 2F$ basis.

A.5 Example: the fine-structure constant

As a simple check, consider the fine-structure constant

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}. \tag{60}$$

Using the ledger assignments above for e, ϵ_0, $\hbar$, and c, the exponents cancel exactly, yielding $[\alpha] = L_1^0 L_2^0 L_3^0 F_1^0 F_2^0$. This is consistent with the well-known status of α as a dimensionless fundamental constant. [12, 13]

B. QMU ledger table (20 entries)

In this appendix we present a representative portion of the QMU ledger. Each entry specifies a quantity Q, its symbol, SI units, and its QDL ledger representation

$$[Q] = L_1^a L_2^b L_3^c F_1^d F_2^e. \tag{61}$$

The full 100-entry table is provided as a machine-readable CSV file in the supplementary material; here we include the first 20 entries as illustration.

Structure: integer exponents (a, b, c, d, e) such that $[Q] = L_1^a L_2^b L_3^c F_1^d F_2^e$.

#	Quantity	SI units	Ledger representation $[Q]$
1	Length L	m	L_1
2	Time T	s	F_1^{-1}
3	Frequency f	s^{-1}	F_1
4	Mass M	kg	$L_3 F_2$
5	Coherence cell $\mathscr{A}$	$\mathrm{m}^3\ \mathrm{s}^{-2}$	$L_3 F_2^2$
6	Speed of light c	$\mathrm{m\ s}^{-1}$	$L_1 F_1$
7	Planck const. h	J s	$L_3 F_2$
8	Reduced Planck const. $\hbar$	J s	$L_3 F_2$
9	Charge e	C	$L_2^{3/2} L_3^{1/2} F_2^{3/2}$
10	Permittivity ϵ_0	$\mathrm{F\ m}^{-1}$	$L_1^{-1} L_2^{-1/2} L_3^{-1/2} F_2^{-1}$
11	Permeability μ_0	$\mathrm{N\ A}^{-2}$	$L_1^{-1} L_2^{-1/2} L_3^{-1/2} F_2^{-1}$
12	Coulomb const. k_e	$\mathrm{N\ m}^2\ \mathrm{C}^{-2}$	$L_1^{-4} L_2^2 L_3 F_2^2$
13	Fine-structure α	1	$L_1^0 L_2^0 L_3^0 F_1^0 F_2^0$
14	Electron mass m_e	kg	$L_3 F_2$
15	Proton mass m_p	kg	$L_3 F_2$
16	Newton const. G	$\mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}$	$L_1^6 L_3^{-1} F_1^{-4}$
17	Action S	J s	$L_3 F_2$
18	Energy E	J	$L_3 F_1 F_2$
19	Momentum p	$\mathrm{kg\ m\ s}^{-1}$	$L_1 L_3 F_1 F_2$
20	Force F	N	$L_1 L_3 F_1^2 F_2$

The remaining entries extend this pattern to include power, pressure, field strengths, flux quanta, thermodynamic quantities, and additional constants (e.g. Boltzmann's constant, Compton wavelengths, gyromagnetic ratios). For brevity, the complete 100-entry QDL ledger is provided as a supplementary CSV file (QDL_ledger_100.csv) accompanying this manuscript; its structure is directly compatible with the QDL ledger definitions given above.