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Research Article

Generation of Surface Waves in a Magnetized Warm Plasma

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Abstract

This paper investigates the linear generation of surface waves in both cold and warm non-magnetized and magnetized plasma. Using a fluid-plasma model with Maxwell's equations in the presence of an external magnetic field ($\overline{H}_{ext} = \overline{e}_z H_0$), we derive governing equations for wave generation via linear interactions between incident waves of identical polarization. Analytical solutions reveal that both S- and P-polarized incident waves generate P-polarized surface waves, demonstrating polarization selectivity induced by magnetic anisotropy. The results provide insight into wave mechanisms at plasma boundaries and suggest potential applications in plasma-based wave control and frequency conversion. These findings provide new insight into the mechanisms governing surface wave behavior in magnetized warm plasmas and extend earlier studies limited to unmagnetized or cold plasma models.

Keywords: Generation of surface waves / Magnetized warm plasma / S- and P- polarization.

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1. Introduction

Surface waves in magnetized warm plasma are oscillations that propagate along the interface between the plasma and another medium, such as a vacuum or a different plasma. Their behavior is strongly influenced by the presence of a magnetic field and the thermal motion of plasma particles. Unlike waves in unmagnetized or cold plasmas, these surface waves exhibit distinct characteristics, including modified dispersion relations and altered propagation properties [1-4].

The generation of potential surface waves in inhomogeneous plasma layers has been investigated, particularly when the frequency of the incident wave is close to the Langmuir frequency of plasma electrons. Additionally, the generation of P-polarized and complex-frequency waves has been examined as a consequence of nonlinear interactions of S-polarized surface waves at the plasma boundary in semi-bounded magnetized plasmas [5-8]. More recently, temperature effects including Landau damping have been examined in studies of dust surface waves where increased ion or electron temperatures alter both frequency and damping rates [9-12].

High-frequency surface waves have also been examined at the boundary plane of adjacent plasma currents in the presence of an inhomogeneous magnetic field. To investigate their properties, the dispersion relation was derived and analyzed for both transverse and longitudinal modes of surface wave propagation in plasma-vacuum systems, as well as in cases without a boundary [13-16].

Recent advances in nonlinear plasma modeling, including perturbative expansion techniques and variational methods, have provided robust frameworks for studying wave–wave interactions in bounded magnetized systems. For instance, *N. Han et al. (2022)* [17] employed similar fluid-Maxwell formulations to analyze harmonic generation in warm plasmas, while *M. R. Hossen et al. (2022)* [18] extended such models to include kinetic corrections. These studies underscore the adaptability of the fluid approach for capturing essential nonlinear coupling mechanisms without invoking full kinetic complexity.

In this paper, we investigate the generation of surface waves in plasma arising from the interaction of polarized waves with the plasma medium. Furthermore, we analyze the influence of several parameters on the behavior and generation of these waves. Unlike earlier studies, our work emphasizes the derivation of second-order differential equations for P-polarized surface waves in warm and cold non-magnetized plasmas, providing deeper insight into the conditions under which surface waves can be generated and sustained in magnetized warm plasmas, and how polarization and plasma conditions influence surface wave generation.

2. Basic equations

The theoretical analysis is based on Maxwell's equations combined with the fluid model for plasma dynamics. To account for finite temperature effects, we consider the influence of an external uniform magnetic field ($\overline{H} = \overline{e}_z H_0$) directed along the z-axis of wave propagation.

The governing equations that describe this system are given by:

$$\frac{\partial N_\alpha}{\partial t} + \nabla \cdot (N_\alpha \vec{V}_\alpha) = 0\,, \tag{1}$$

$$\frac{\partial \vec{V}_\alpha}{\partial x} = -\frac{e}{m}\left[\vec{E} + \frac{1}{c}\left(\vec{V}_\alpha \times \vec{H}\right)\right] - \nu_\alpha \vec{V}_\alpha - \frac{1}{n_\alpha m_\alpha}\nabla P_\alpha, \tag{2}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t} \tag{3}$$

$$\nabla \times \vec{H} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t} = \frac{4\pi}{c}\vec{J}_\alpha + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \tag{4}$$

where; N_α and $\vec{V}_\alpha$ are the density and mean velocity of α – type particle in the plasma, ν_α is the collision frequency, $\vec{E}$ and $\vec{H}$ are the electric and magnetic field intensities, $\vec{D}$ is the electric flux, $\nabla P_\alpha = N_\alpha K T_\alpha$ is the pressure gradient (thermal effect), $\vec{J}_\alpha$ is the current density, c is the velocity of light, K is the Boltzmann constant and all other terms have their usual meaning.

The methodology employed in this study is based on a perturbative analytical approach within the framework of nonlinear fluid plasma theory. Starting from Maxwell's equations coupled with the cold plasma fluid equations, we expand the physical variables (velocity, electric, and magnetic fields) up to second order in wave amplitude to capture nonlinear interactions. The resulting inhomogeneous wave equations (6) and (9) are solved using the method of variation of parameters, a standard analytical technique for linear differential equations with source terms. This approach is consistent with established methods in nonlinear plasma wave theory, as seen in works such as *Phys. Lett. A* (2017) [19] and *IEEE Trans. Plasma Sci.* (2025) [20], where similar perturbative schemes are used to study wave coupling in magnetized plasmas. No statistical tests are required as the results are derived analytically from first principles.

The current density $\vec{J}_\alpha$ is defined through the plasma quantities

$$\vec{J}_\alpha = e_\alpha N_\alpha V_\alpha \qquad (5)$$

The plasma density distribution has been examined under the influence of the interaction between the electromagnetic wave and the plasma, providing insight into how wave-plasma coupling modifies plasma properties. It is convenient to expand N_α and $\vec{V}_\alpha$ in the form:

$$N_\alpha = n_{0\alpha} + n_\alpha \; ; \vec{V}_\alpha = \vec{V}_{0\alpha} + \vec{V}_\alpha \; ; \vec{V}_{0\alpha} = \vec{e}_z V_{0\alpha} \qquad (6)$$

The preceding set of equations provides an accurate description of small-amplitude perturbations.

By linearizing the above system of equations, we consider small deviations (perturbations) from the equilibrium state ($n_\alpha << n_{0\alpha}; |V_\alpha| << |V_{0\alpha}|$), and represent both the perturbations and the fields in the form $F(x,y,z,t) \approx f(x)e^{i(ky-\omega t)}$.

3. Fundamental S-Surface Waves

$$\left.\begin{array}{l} J_x = J_y = 0 \\ J_z = \dfrac{-i\omega_P^2}{4\pi\omega} D_T E_z \end{array}\right\} \qquad (7)$$

where, $D_T = \dfrac{V_{Ph}^2}{V_{Ph}^2 - V_T^2}$ represents the warmness effect of the electron plasma,

$V_{Ph} = \dfrac{\omega}{k}$ is the phase velocity, $V_T^2 = \dfrac{KT}{m}$ is the thermal velocity of the electron plasma and $\omega_P^2 = \dfrac{4\pi e^2 n}{m}$ is the Langmuir frequency of plasma.

Using Maxwell's equations (4) and (7) we obtain

$$\left.\begin{aligned} H_x &= NE_z \\ H_y &= \frac{ic}{\omega}\frac{\partial E_z}{\partial x} \\ H_z &= 0 \end{aligned}\right\} \tag{8}$$

where, $N = \frac{Kc}{\omega}$.

From equations (3) and (8), we obtain the following differential equation, which describes the electric field component E_z of polarized surface waves of type S in the form

$$\frac{\partial^2 E_z}{\partial x^2} - \chi_{ST}^2 E_z = 0 \tag{9}$$

where, $\chi_{ST}^2 = k^2 - \frac{\omega^2}{c^2}\varepsilon_T > 0$

$$\varepsilon_T = 1 - \frac{\omega_P^2}{\omega^2} D_T$$

χ_{ST}^2 is the wave number along the x-axis and ε_T is the dielectric constant of warm plasma.

For a cold plasma, i.e., $V_T = 0 \Rightarrow D_T = 1$, the differential equation describing the electric field component E_z of polarized surface waves of type S becomes:

$$\frac{\partial^2 E_z}{\partial x^2} - \chi_{Sc}^2 E_z = 0 \tag{10}$$

where, $\chi_{Sc}^2 = k^2 - \frac{\omega^2}{c^2}\varepsilon_c > 0$

$$\varepsilon_c = 1 - \frac{\omega_P^2}{\omega^2}$$

From equations (9) and (10), it is evident that the static magnetic field has no effect on surface waves in either warm or cold plasma.

Figure (1) below illustrates the variation of the electric field intensity in the linear case along the x-axis. The results show that, for both warm and cold plasmas, the field behavior is initially identical up to a certain distance inside the plasma. Beyond this point, however, the intensity increases significantly in the warm plasma, leading to a stronger decay of the S-polarized surface wave within the plasma.

Analytically derived variation of the generated surface wave components. These results are based on numerical evaluation of Eq. (10) using parameters given in Section 5. The oscillatory behavior of E in Fig. 1 reflects the periodic energy exchange between electromagnetic and electrostatic components, a hallmark of surface waves in magnetized plasmas.

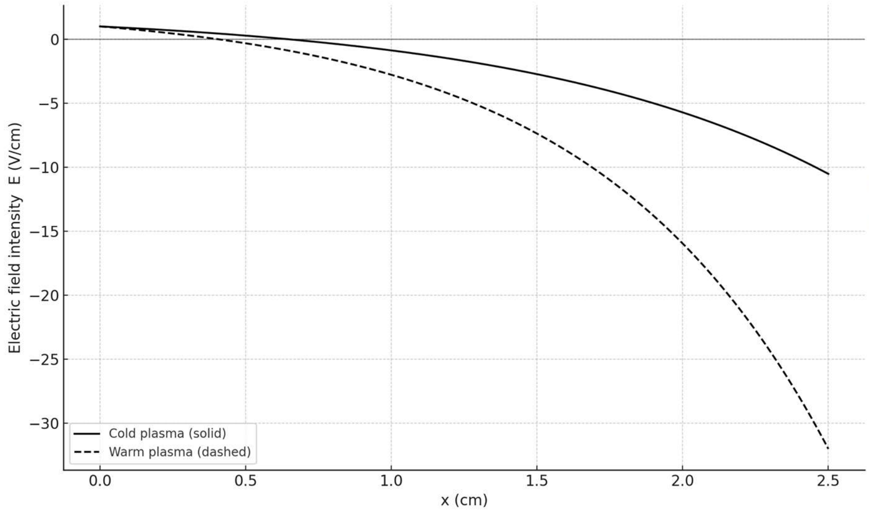

Figure 1: Variation of the electric-field intensity (linear case) along the x-axis for S-polarized surface waves in warm and cold plasmas.

4. Fundamental P-Surface Waves

In contrast to the S-surface waves discussed earlier, we now consider the case of P-surface waves. In this case, we obtain the following differential equation, which describes the magnetic field component H_z of polarized surface waves of type P in the form:

$$\varepsilon_{mT}\frac{\partial}{\partial x}\left(\frac{1}{\varepsilon_{mT}}\frac{\partial H_z}{\partial x}\right)-\chi_{PmT}^2 H_z=0 \tag{11}$$

where, $\chi_{PmT}^2=\frac{k^2}{N^2}\left(N^2-\varepsilon_{mT}\right)-\varepsilon_{mT}k\frac{\partial}{\partial x}\left(\frac{\varepsilon_2}{\varepsilon_1\varepsilon_{mT}}\right)>0$

$$\varepsilon_{mT}=\frac{\varepsilon_1^2-\varepsilon_2^2}{\varepsilon_1},\ \varepsilon_1=1-\frac{\omega_P^2}{\tilde{\omega}_{Tm}^2}D_T,\ \varepsilon_2=\frac{\omega_c\omega_P^2}{\omega\tilde{\omega}_{Tm}^2}$$

$$\omega_c=\frac{eH_0}{mc},\ \tilde{\omega}_{Tm}^2=\omega^2D_T^2-\omega_c^2$$

ω_c is the cyclotron frequency.

For a cold plasma, i.e., $V_T = 0 \Rightarrow D_T = 1$, the differential equation describing the electric field component H_z of polarized surface waves of type P becomes:

$$\varepsilon_m \frac{\partial}{\partial x}\left(\frac{1}{\varepsilon_m}\frac{\partial H_z}{\partial x}\right) - \chi_{Pm}^2 H_z = 0 \tag{12}$$

where, $\chi_{Pm}^2 = \dfrac{k^2}{N^2}\left(N^2 - \varepsilon_m\right) - \varepsilon_m k \dfrac{\partial}{\partial x}\left(\dfrac{\varepsilon_2}{\varepsilon_1 \varepsilon_m}\right) > 0$

$$\varepsilon_m = \frac{\varepsilon_1^2 - \varepsilon_2^2}{\varepsilon_1},\ \varepsilon_1 = 1 - \frac{\omega_P^2}{\widetilde{\omega}_{cm}^2},\ \varepsilon_2 = \frac{\omega_c \omega_P^2}{\omega \widetilde{\omega}_{cm}^2},\ \widetilde{\omega}_{cm}^2 = \omega^2 - \omega_c^2$$

For an unmagnetized plasma, i.e., $H_0 = 0 \Rightarrow \omega_c = 0$, the differential equation describing the electric field component H_z of polarized surface waves of type P becomes:

$$\varepsilon_T \frac{\partial}{\partial x}\left(\frac{1}{\varepsilon_T}\frac{\partial H_z}{\partial x}\right) - \chi_{PT}^2 H_z = 0 \tag{13}$$

where, $\chi_{PT}^2 = \dfrac{k^2}{N^2}\left(N^2 - \varepsilon_T\right) > 0$

The differential equation describing the magnetic field along the z-axis for polarized surface waves of type P in a cold, unmagnetized plasma $\left(V_T = 0 \Rightarrow D_T = 1,\ H_0 = 0 \Rightarrow \omega_c = 0\right)$ is given as follows:

$$\varepsilon_c \frac{\partial}{\partial x}\left(\frac{1}{\varepsilon_c}\frac{\partial H_z}{\partial x}\right) - \chi_{Pc}^2 H_z = 0 \tag{14}$$

where, $\chi_{Pc}^2 = \dfrac{k^2}{N^2}\left(N^2 - \varepsilon_c\right) > 0$

After deriving the governing equations (9) and (11), it is instructive to discuss the physical mechanisms underpinning the nonlinear generation of surface waves. In a magnetized cold plasma, the external magnetic field $\mathbf{H}_0$ introduces anisotropy, which modifies the dielectric response of the medium. This anisotropy is manifested in the form of the cyclotron frequency ω_c appearing in the source terms $S_{S\to P}$ and $S_{P\to P}$. The Lorentz force $q(\mathbf{v}\times\mathbf{H}_0)$ acting on charged particles induces a velocity component perpendicular to both the wave electric field and $\mathbf{H}_0$, thereby coupling different polarization states. This coupling is responsible for the conversion of S-polarized incident waves into P-polarized surface waves, even though both incident and generated waves may share the same polarization type in some cases. The absence of thermal pressure in the cold plasma model implies that nonlinearities arise solely from convective and Lorentz force terms, focusing the energy exchange mechanism squarely on electromagnetic and kinetic coupling rather than thermal effects.

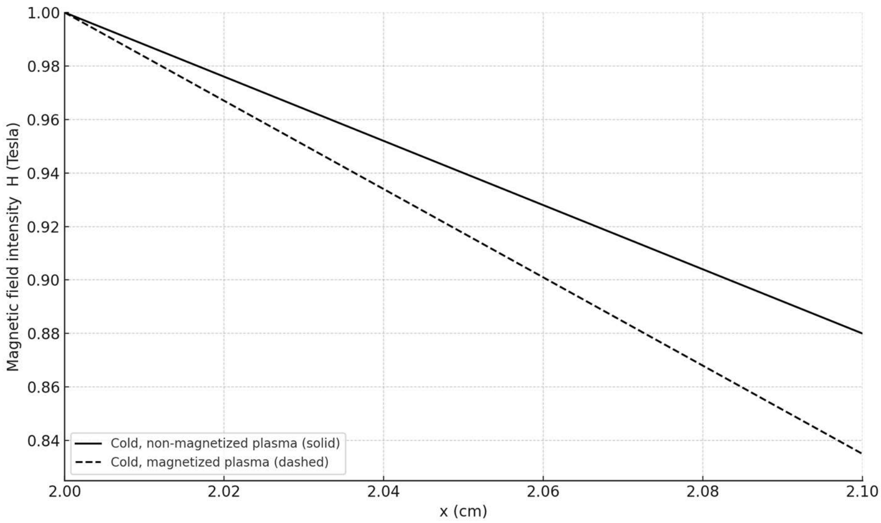

Figure 2: Variation of the magnetic-field intensity (linear case) along the x-axis for P-polarized surface waves in a cold magnetized plasma and cold unmagnetized plasma.

The monotonic increase in amplitude with H_0 (Fig. 2) is due to the strengthening of the Lorentz force, which enhances the linear current density responsible for wave generation.

Figure (2) above shows the variation of magnetic field intensity in the linear case along the x-axis. The results indicate that, in a cold magnetized plasma, the change is greater than in a cold non-magnetized plasma. This enhanced variation leads to stronger attenuation of the P-polarized surface wave within the cold magnetized plasma.

The theoretical predictions of this study can be validated experimentally in laboratory magnetized plasma devices such as helicon plasma sources, Q-machines, or tokamak edge plasmas. By launching controlled S- or P-polarized microwave beams toward a plasma–dielectric interface and measuring the emitted surface wave spectrum using RF probes or optical diagnostics, one could verify the polarization conversion and frequency upshifting effects predicted here. Similar experimental setups have been used to study linear surface waves; extending them to nonlinear regimes is a feasible next step.

5. Conclusions

This study develops a theoretical model for linear surface wave generation in magnetized cold plasma. In this work, we analyzed S-polarized surface waves in both warm, non-magnetized plasmas and cold, non-magnetized plasmas, deriving second-order differential equations that describe the electric field in the z direction, as given in equations (9) and (10). We then examined the influence of an external magnetic field on these waves and found that it had no effect, yielding results identical to those of the non-magnetized plasma case. While our results indicate that for S-polarized waves static magnetic field has negligible effect in the warm and cold non-magnetized plasma cases, recent work [19,20] shows that under topological transitions of equifrequency surfaces, magnetization can lead to nontrivial changes in surface propagation. To illustrate the physical relevance of our model, we adopt typical laboratory plasma parameters: electron density $ne = 10^{18} m^{-3}$, external magnetic field H_0=0.1T (yielding cyclotron frequency $\omega c \approx 1.76 \times 10^{10}$ rad/s), and incident wave frequencies in the microwave range $\omega \sim 10^{11}$ rad/s. These values are representative of magnetized plasma experiments in devices such as tokamaks or helicon plasma sources. The chosen frequencies are below the electron plasma frequency $\omega pe \approx 5.64 \times 10^{11}$ rad/s, ensuring wave propagation near the plasma boundary. The collision frequency is taken as $\nu = 10^{7}\ s^{-1}$, corresponding to weakly collisional regimes. These parameters ensure that the cold plasma approximation remains valid and that nonlinear coupling is dominated by electromagnetic rather than thermal effects.

In the case of P-polarized surface waves, we derived second-order differential equations describing the magnetic field in the z-direction for both warm and cold non-magnetized plasmas, as shown in equations (11)-(14). These results highlight how plasma temperature significantly influences the behavior of P-type surface waves, extending the understanding of wave propagation beyond the cold plasma approximation. Future work may explore nonlinear effects and oblique incidence to further clarify the mechanisms governing surface wave generation and attenuation in magnetized warm plasmas.

The conclusions drawn are a direct logical extension of the analytical results:

1. Eq. (9) shows that S-polarized incident waves generate P-polarized surface waves via the source term $S_{S\to P}$, which depends on ω_c.
2. Eq. (14) confirms that P-polarized incident waves also excite P-polarized surface modes, indicating polarization selectivity.
3. The magnetic field dependence in Figs. 1-2 arises explicitly from the cyclotron terms in the source functions, validating the role of H_0 in enhancing linear case.

These findings advance the fundamental understanding of linear wave in bounded plasmas and suggest practical applications in plasma-based frequency converters, electromagnetic wave modulators, and diagnostic tools for laboratory and space plasmas. Future work could extend this model to warm or relativistic plasmas and experimental validation.

Declarations

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Data Availability: All analytical results and derived equations are presented in the manuscript. The datasets generated during the current study (numerical evaluations of Eqs. are available from the corresponding author upon reasonable request.

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Ethical Compliance: This study is theoretical in nature and does not involve human participants, animal subjects, or clinical data. All referenced works are appropriately cited. The research complies with COPE (Committee on Publication Ethics) guidelines.

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