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Research Article

Self-Focusing of Gaussian Laser Beam in Unmagnetized Plasma Described by Kappa Distribution

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Abstract

Empirical kappa distributions have become increasingly widespread across space and plasma physics. In the present paper, logical straightforward extension of dielectric function of plasma described by Kappa parameter is employed to study the interaction of Gaussian laser beam with plasma. The nonlinear differential equation describing evolution of beam-width parameter is established by using parabolic wave equation approach under WKB and paraxial approximations. The effects of Kappa parameter and ratio of the thermal velocity of plasma electrons to the velocity of light on the propagation dynamics of Gaussian laser beam in non-Maxwellian plasma are specifically inspected. It is shown that the larger Kappa parameter and thermal velocity of electrons become more significant for stronger self-focusing of Gaussian beam in plasma. This enables the need of more exploration of physical mechanisms involved in the field of interaction of lasers with non-Maxwellian plasma.

Keywords: Gaussian beam, Self-focusing, Kappa distribution, Non-Maxwellian plasma, Paraxial

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1. Introduction

The phenomenon of self-focusing is an induced lens effect [1]. It results from wavefront distortion inflicted on the beam while traversing a nonlinear medium. It is similar to the distortion caused by a positive lens. Self-focusing is a typical type of nonlinear phenomenon that depends significantly on the transverse profile of the beam. It sometimes plays vital role to affect other nonlinear phenomena. Self-focusing arises due to nonlinear response of the medium to the field of incident laser beam, leading to modification of its dielectric constant in such a way that the medium starts behaving like a converging lens. The beam thus acquires a minimum radius at some point, beyond which it diverges. As beam diameter increases, axial intensity decreases and hence self-focusing starts again. Thus a periodic focusing is realized. It is possible to choose different intensity profiles for which self-focusing and diffraction effects are evenly balanced and thus beam propagates as a spatial soliton [2]. Review of literature highlights the fact that, till date, most of the investigations on self-focusing of laser beams in plasmas have been specifically tackled to the plasma described by Maxwellian distribution [3,4].

However, various environments of space plasmas are always far from thermal equilibrium. As a result, such plasmas cannot be described by the Maxwellian distribution very well and hence non-Maxwellian distribution of electrons is required for the study of space plasmas. The non-Maxwellian distribution of plasma particles is applied for the analysis of various phenomena in environments of space plasma such as solar corona [5], solar winds [6], solar flares [7] and planetary magnetosphere [8]. Such distribution is also applied to analyze the physical properties of the plasmas with super-thermal particles [9]. In addition, various nonlinear optical effects are also of significant interest in space plasma physics. The existence of non-Maxwellian distribution functions can have profound effects on laser-plasma interaction [10]. It is found that in non-Maxwellian plasma, the Kappa distribution is successfully applied in describing plasma in variety of locations. Nevertheless, the physical explanation of the generation of non-Maxwellian distribution is still under discussion. The generalized plasma dispersion function with Kappa-Maxwellian velocity distribution is already studied [11]. Certain mathematical and physical properties of the Kappa velocity distribution are extensively elaborated [12]. It is well known that the ponderomotive force on plasma electrons is important in laser phenomena where it leads to the appearance of ponderomotive self-focusing [13]. Due to the extremely intense laser production technology, electrons oscillating in the intense laser field become relativistic. This makes possible to explore entirely new optical effects such as relativistic self-focusing [14,15]. In addition, the inclusion of ponderomotive forces and relativistic effects in plasmas can act as a generator of slowly varying fields of laser. These effects have been studied for quantum plasmas [16-19] due to its importance in laser-plasma interaction and dense plasma of astrophysical environments. On the other hand, it has been observed that in near-Earth space plasma, the particle velocity distributions exhibit super-thermal tails that are well described by the family of Kappa distributions [20]. These distributions depend on the spectral index κ usually termed as Kappa parameter which can be understood as a power-law generalization from which the usual Maxwellian distribution is recovered as a limiting case when κ tends to infinity. The detailed analysis of ponderomotive forces in unmagnetized plasmas described by Kappa distribution function has been investigated by Espinoza-Troni et al. [21]. They have appreciated the thermal effects in analyzing effects of Kappa distribution in dispersion relations of unmagnetized plasma. The ponderomotive force is not only depends on the properties of the medium but also its interaction with incident electromagnetic field described by dielectric function. Hence, plasma described by Kappa distribution can significantly affect ponderomotive force and hence the self-focusing effect associated with it.

Literature reveals that prior research on laser beam self-focusing in different scenarios of plasma with non-Maxwellian distributions have been extremely limited. It is important to note that non-Maxwellian electron distributions are demonstrated experimentally from direct laser acceleration in near-critical plasmas [22]. Javan [23] investigated self-focusing of a weakly relativistic laser propagating through non-Maxwellian plasma with kappa distribution by using source dependent expansion method. It has been observed that Kappa of electrons in different astrophysical and laboratory events can be varied in large range from 1.5 to infinity. Therefore an effect of variations of Kappa on the physical phenomena in non-Maxwellian plasma is very important. Abedi-Varaki [24] extended the same method to explore an effect of the wiggler magnetic field strength on the self-focusing of an intense laser beam propagating through magnetized non-Maxwellian plasma. Results showed that in the right-hand polarization, the existence of wiggler magnetic field causes improvement in the self-focusing quality. Asgharnejad et al.[25] studied the nonlinear self-focusing of an intense laser beam propagating along a non-uniform axial magnetic field direction in a non-Maxwellian plasma described by Kappa distribution by using source dependent expansion method. Their results indicate the improvement of self-focusing of the laser beam in the presence of an external non-uniform magnetic field inside non-Maxwellian plasma in comparison with Maxwellian plasma. Generally speaking, in studies related to self-focusing of laser beams in non-Maxwellian plasma, Kappa parameter can affect the evolution of the laser spot-size. Therefore, Kappa parameter can be exploited in the phenomenon of self-focusing of laser beams in non-Maxwellian plasma, which, however, to the best of our knowledge has not been attempted extensively in the literature at present.

In previous studies, the effects of the non-uniform magnetic field and temperature on laser spot-size were carried out by using source dependent expansion method in non-Maxwellian plasma. In this paper, however, we have investigated the self-focusing of Gaussian laser beam under parabolic equation approach in non-Maxwellian plasma. Basically, the parabolic equation is used extensively for laser beam propagation because they simplify complex wave equations into a much more manageable evolution equation. This approach reduces the governing wave equation (derived from Maxwell's equations) to an evolution equation with simplified mathematical complexity and tractability. The entire theoretical formulation is based on WKB and paraxial approximations. Usually, the paraxial approximation assumes the laser beam travelling along a central axis with only small angular deviations, making it valid for the relatively narrow beams in most laser systems. This simplification allows us for the straightforward use of the Gaussian beam and provides a simple yet accurate representation for many applications related with laser-plasma interaction processes. But such paraxial approximation may not be valid for some modified Gaussian laser beam profiles like super Gaussian beams as verified by Devi and Malik [26]. We have highlighted the Kappa parameter and the ratio between the thermal velocity of plasma electrons and velocity of light in the transverse dielectric function of non-Maxwellian plasma and analyze further their influence on self-focusing of Gaussian laser beam. We have established the nonlinear differential equation for beam-width parameter of laser in non-Maxwellian plasma and compared it with Maxwellian case of reference.

2. Dielectric function

In this study, we consider the simplest form of Kappa distribution function for isotropic plasma as [11,12]:

$$f_{\kappa s}(v) = \frac{n_s}{(\pi\alpha_s^2\kappa)^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}\left(1+\frac{v^2}{\kappa\alpha_s^2}\right)^{-(\kappa+1)} \quad (1)$$

where $f_{\kappa s}$ is the Kappa distribution for the species s with κ as Kappa parameter, Species s indicate indices i and e that denote ion and electron, respectively, n_s is their number density, $\alpha_s = (2k_BT_s/m_s)^{1/2}$ is their thermal velocity, k_B is the Boltzmann constant, m_s is the mass, T_s is the temperature and Γ is the Gamma function.

In following, we employ the dispersion relation for high frequency waves propagating through plasma described by a Kappa distribution function given by Eq.(1) with no background magnetic field. The dispersion relation for transverse waves in unmagnetized plasma with respect to the direction of propagation is given by [21]:

$$\frac{k^2c^2}{\omega^2} = 1 + \sum_s \frac{\omega_{ps}^2}{\omega k\alpha_s} Z_{\kappa M}\left(\frac{\omega}{k\alpha_s}\right) \quad (2)$$

where ω is the frequency, k is the wave number, c is the velocity of light, $Z_{\kappa M}$ is the generalized plasma dispersion function [11] and $\omega_{ps} = (4\pi n_s q_s^2/m_s)^{1/2}$ is the plasma frequency for the species s with q_s as the charge on species. By considering only motion of electron species with a static background of ions in achieving quasi-neutrality and neglecting the contribution of the ions, Eq.(2) can be written as [21]:

$$\varepsilon = 1 - \frac{\omega_{pe}^2}{\omega^2}\left(1+\frac{1}{2}\left(\frac{\alpha_e^2}{c^2}\right)\left(\frac{\kappa}{\kappa-3/2}\right)\frac{k^2c^2}{\omega^2}\right) \quad (3)$$

It can be seen from Eq.(3) that the dielectric function contains the Kappa parameter κ and the velocity ratio α_e/c. It is clear that for positive κ values, $\kappa > 3/2$ is required for the temperature to be physical [27]. This means that thermal effects are not significant unless we have a plasma far from thermal equilibrium with $\kappa \sim 3/2$. The purpose of this work is to study how thermal effects in non-Maxwellian plasma impact in its interaction with laser field. In the interaction of laser with plasmas, the quiver velocity of electrons become comparable to that of velocity of light in free space due to high electric field associated with the laser beam. Hence, the effective mass m_e in the plasma electron frequency ω_{pe} in the presence of the laser beam get replaced by γm_0, where m_0 is the rest mass of electron and γ is the Lorentz relativistic factor. Following Sharma and Kourakis [28], we can write $\gamma = (1 + \beta EE^*)^{1/2}$, where $\beta = e^2/m_0^2\omega^2c^2$ is the coefficient of relativistic nonlinearity. The self-focusing is actually a relativistic phenomenon typical for intense laser propagating through underdense plasmas. In reality, the laser beam has finite and well-defined radial profile; therefore, in a fluid picture of plasma electrons, the relativistic factor γ of a electron quivering is a function of radial coordinate r via the laser intensity profile. For sake of simplicity we are limiting our considerations to a radial-symmetric physical system. In most of practical and realistic cases the laser intensity profile (Gaussian beam considered in the present work) is peaked at $r = 0$ and decreases for $r > 0$. This means that on-axis plasma electrons are heavier than the off-axis electrons if looking at the relativistic factor γ. Hence Eq.(3) for dielectric function of unmagnetized, non-Maxwellian plasma reduces to:

$$\varepsilon = 1 - \frac{\omega_{p0}^2}{\gamma\omega^2}\left(1+\frac{1}{2}\left(\frac{\alpha_e^2}{\gamma c^2}\right)\left(\frac{\kappa}{\kappa-3/2}\right)\left(1-\frac{\omega_{p0}^2}{\gamma\omega^2}\right)\right) \quad (4)$$

where $\omega_{p0} = (4\pi n_e e^2/m_0)^{1/2}$ and $\alpha_{e0} = (2k_BT_e/m_0)^{1/2}$ are the plasma frequency and thermal velocity of electrons, respectively in the absence of the laser beam. In writing Eq.(4), we have considered the situation of underdense plasma for which $\omega_{p0} < \omega$. Eq.(4) also confirms well known expression of dielectric function $\varepsilon = 1 - \frac{\omega_{p0}^2}{\gamma\omega^2}$ for usual classical case of Maxwellian plasma by Sharma and Kourakis [28] in the limit $\kappa \to \infty$ and velocity ratio $\alpha_{e0}/c < 1$. In fact the plasma electron profile at the laser front will acts like a focusing lens. This focusing effect might be effective as far as the laser intensity is maintained so high that relativistic quiver velocities are provided to the plasma electrons at the front part.

Following Sodha et al. [3], the dielectric function ε of plasma can be written as:

$$\varepsilon = \varepsilon_0 + \phi(EE^*) \quad (5)$$

where $\varepsilon_0 = 1 - \omega_{p0}^2/\omega^2$ is the linear part of dielectric function and nonlinear part of dielectric function can be written as:

$$\phi(EE^*) = \frac{\omega_{p0}^2}{\omega^2}$$

$$\left(1 - \frac{1}{\gamma}\left(1 + \frac{1}{2}\left(\frac{\alpha_{e0}^2}{\gamma c^2}\right)\left(\frac{\kappa}{\kappa - 3/2}\right)\left(1 - \frac{\omega_{p0}^2}{\gamma\omega^2}\right)\right)\right) \quad (6)$$

It is interesting to see that the nonlinear part ϕ of dielectric function depends on Kappa parameter κ and velocity ratio α_{e0}/c in non-Maxwellian plasma.

3. Nonlinear evolution equation

Using Maxwell's equations, the wave equation governing the propagation of the laser beam may be written as:

$$\nabla^2 E + \frac{\omega^2}{c^2}\varepsilon E + \nabla\left(\frac{E \cdot \nabla\varepsilon}{\varepsilon}\right) = 0 \quad (7)$$

The last term in Eq.(7) is negligible provided $\frac{c^2}{\omega^2}\left|\frac{1}{\varepsilon}\nabla^2 \ln\varepsilon\right| \ll 1$ under the assumption that transverse gradient of dielectric function is small as compared to the laser wavelength. Thus:

$$\nabla^2 E + \frac{\omega^2}{c^2}\varepsilon E = 0 \quad (8)$$

This equation is solved by employing WKB approximation. For mathematical convenience, we express the solution in the cylindrical coordinate system as:

$$E = A(r,z)\, exp\big(i(\omega t - kz)\big) \quad (9)$$

Neglecting $\partial^2 A/\partial z^2$ which implies that the characteristic distance of intensity variation is much greater than wavelength. Thus Eq.(8) reduces to:

$$2ik\frac{\partial A}{\partial z} = \nabla_\perp^2 A + \frac{\omega^2}{c^2}\phi(AA^*)A \quad (10)$$

where $\nabla_\perp^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$ is the transverse Laplacian.

To solve Eq.(10) we express A as:

$$A = A_0(r,z)\exp\big(-ikS(r,z)\big) \quad (11)$$

where both amplitude A_0 and eikonal S are real quantities. Substituting for A from Eq.(11) in Eq.(10) and equating real and imaginary parts, we get:

$$2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{\nabla_\perp^2 A_0}{k^2 A_0} + \frac{\phi(A_0^2)}{\varepsilon_0} \quad (12)$$

and

$$\frac{\partial A_0^2}{\partial z^2} + \frac{\partial S}{\partial r}\frac{\partial A_0^2}{\partial r} + A_0^2\,\nabla_\perp^2 S = 0 \quad (13)$$

We anticipate a solutions for Eqs.(12) and (13) for Gaussian laser beam in the form:

$$S = \frac{r^2}{2f}\frac{df}{dz} + \varphi(z) \quad (14)$$

and

$$A_0^2 = \frac{E_0^2}{f^2}\, exp\left(-\frac{r^2}{r_0^2 f^2}\right) \quad (15)$$

where, r_0 is the initial beam radius, $\varphi(z)$ is the arbitrary function of z, f is the dimensionless beam-width parameter which determines width of the beam during propagation.

Adopting the paraxial approach [3], the differential equation for beam-width parameter f is obtained as:

$$\frac{d^2 f}{d\xi^2} = \frac{1}{f^3} - \left(\frac{r_0\omega}{c}\right)^2\left(\frac{\omega_{p0}}{\omega}\right)^2\frac{\beta E_0^2}{f^3}\left(1 + \frac{\beta E_0^2}{f^2}\right)^{-3/2} G \quad (16)$$

where $G = \left[\kappa\left(2 - \frac{3\delta^2\omega_{p0}^2}{\omega^2}\right) - 3\right] + \frac{\beta E_0^2}{f^2}(2\kappa - 3) + 2\left[\delta^2\kappa + \frac{\omega_{p0}^2(3-2\kappa)}{\omega^2}\left(1 + \frac{\beta E_0^2}{f^2}\right)^{1/2}\right]$. Here, $\delta = \alpha_{e0}/c$ is the ratio between thermal velocity of plasma electrons and velocity of light, and $\xi = z/kr_0^2$ is the dimensionless distance of propagation.

4. Numerical results and discussion

Eq. (16) is the evolution equation governing variation beam-width parameter f with dimensionless distance of propagation ξ in unmagnetized plasma described by Kappa distribution. The first term on the right-hand side of Eq.(16) gives the diffraction effect and second term related to the nonlinear focusing effect gives existence of non-Maxwellian distribution correction. If the first term on right-hand-side of (16) dominates over the second term, the beam diverges, while opposite is true (self-focusing) when the second term exceeds the first one. It should be noted that for Kappa parameter $\kappa \to \infty$ and velocity ratio $\alpha_e/c < 1$, Eq.(16) reduces to Eq.(14) of Singh et. al. [29] for results of classical Maxwellian case of reference for nonlinear dynamics of Gaussian laser beam in plasma. When two terms on the right-hand side of (16) balance each other, the beam propagates without convergence or divergence, which is referred as self-trapping of the beam. For an initial plane wavefront of the beam the initial conditions on f are $f(\xi = 0) = 1$ and $(df/d\xi)_{\xi=0} = 0$.

As both the terms on right-hand side of Eq.(16) cancel each other hence at $\xi = 0$, $(d^2 f/d\xi^2)_{\xi=0} = 0$; since $df/d\xi$ is also zero and $f = 1$ for all values of ξ. Therefore the condition for self-trapping is:

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$$\frac{r_0\omega}{c} = \frac{\omega}{\omega_{p0}}\left(\frac{2(1+\beta E_0^2)^{3/2}}{\beta E_0^2\, G_0}\right)^{1/2} \quad (17)$$

where $G_0 = \left[\kappa\left(2 - \frac{3\delta^2\omega_{p0}^2}{\omega^2}\right) - 3\right] + \beta E_0^2(2\kappa - 3) + 2\left[\delta^2\kappa + \frac{\omega_{p0}^2(3-2\kappa)}{\omega^2}(1+\beta E_0^2)^{1/2}\right]$. This is called critical condition and variation of dimensionless initial beam-radius $r_0\omega/c$ with intensity parameter βE_0^2 is generally known as critical curve. The obtained evolution equation for beam-width parameter [Eq. (16)] is often nonlinear and cannot be solved analytically due to its non-integrability. This requires a reliable numerical technique for an approximate solution. The fourth-order Runge Kutta method is a popular approximate choice because it achieves a high degree of accuracy and efficiency, yielding more stability for the solution of the problem with a relatively small number of steps compared to other approximate methods. This advantage allows us to employ this method in the present work. Therefore, we have solved nonlinear differential Eq. (16) numerically with appropriate boundary conditions stated above with the help of Runge Kutta fourth-order method for typical values of laser-plasma parameters. Initially, it should be better to mention that the order of parameters has been selected in accordance with the usual range of parameters used in the laser-plasma interaction experiments. For example see the performed experiment by Toncin et al. [22] in which non-Maxwellian peaked electron distribution with high electron density in the laser propagation direction has been considered from direct laser acceleration in near-critical plasmas. Such interaction of laser beam with highly localized target leads to the generation of short channel and further self-focusing of laser beam. In all analyzed cases, we have supposed a CO_2 laser system with frequency $\omega = 1.778\times10^{14}\ rad/s$ (that corresponds to the laser center wavelength $\lambda = 10.6\ \mu m$) and initial beam radius $r_0 = 20\ \mu m$, $\beta E_0^2 = 0.1$ (that is equivalent to the laser intensity $\sim10^{19}\ W/cm^2$). We have considered electron density $n_e\sim10^{18}\ cm^{-3}$, velocity ratio $\delta = 0.1 - 0.4$ and Kappa parameter $\frac{3}{2} < \kappa < \infty$ as usual.

Figure 1 presents the variation beam-width parameter f with dimensionless distance of propagation ξ for Kappa parameter $\kappa = 2,\ 4,\ 6,\ \infty$ with $\delta = 0.1$. In Figure 1, strong self-focusing is observed with increase in κ values. This is due to the fact the Kappa distribution describes the plasmas with a Maxwellian-like core and super-thermal power law tails, which is often accurate than the Maxwellian distribution for interaction of laser with space plasma environments. In the phenomenon self-focusing of Gaussian laser beam with plasma described by Kappa distribution, the spatial evolution of electric field of the laser is strongly affected by super-thermal particles.

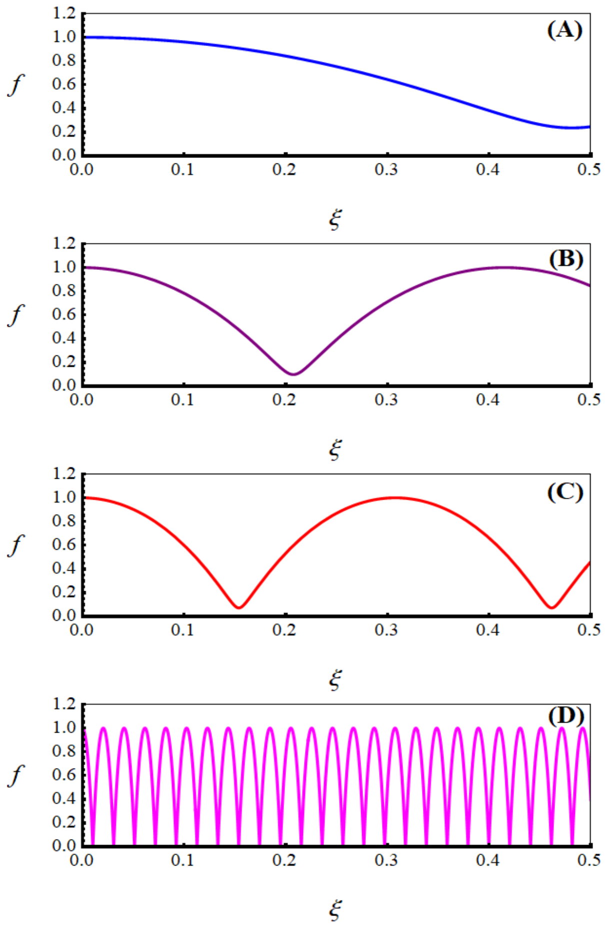

Figure 1. Variation of beam-width parameter f with dimensionless distance of propagation ξ for different Kappa parameters; (A) $\kappa = 2$, (B) $\kappa = 4$, (C) $\kappa = 6$ and (D) $\kappa \to \infty$. Other numerical values of laser-plasma parameters are: $r_0 = 20\ \mu m$, $\omega = 1.778\times10^{14}\ rad/s$, $n_0 = 10^{18}\ cm^{-3}$, $\beta E_0^2 = 1$ and $\delta = 0.1$

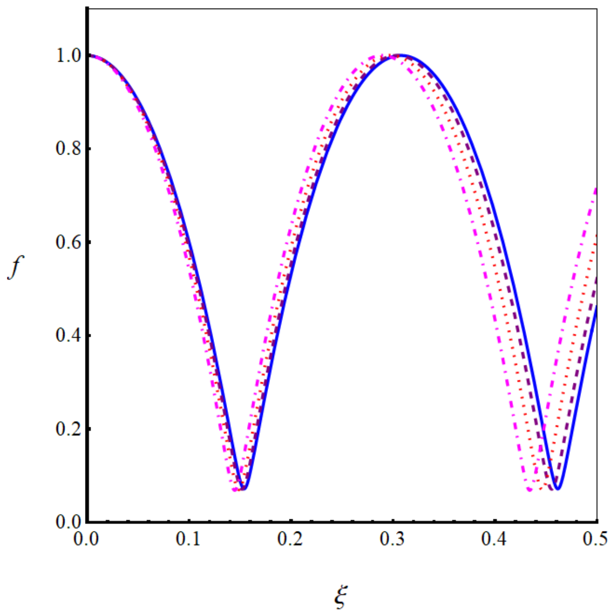

Figure 2. Variation of beam-width parameter f with dimensionless distance of propagation ξ for different values of velocity ratio δ; $\delta = 0{,}1$ (solid curve) $\delta = 0{,}2$ (dashed curve), $\delta = 0{,}3$ (dotted curve) and $\delta = 0{,}4$ (dot-dashed curve). Here, Kappa parameter $\kappa = 6$ and other values of laser-plasma parameters are same as given in Figure 1

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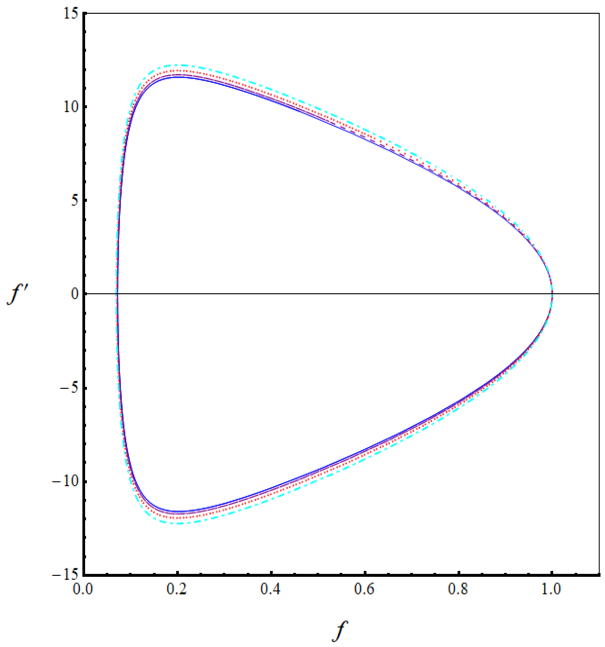

Figure 3. Phase space plots for $\kappa = 6$ with different values of velocity ratio δ; $\delta = 0.1$ (solid curve) $\delta = 0.2$ (dashed curve), $\delta = 0.3$ (dotted curve) and $\delta = 0.4$ (dot-dashed curve). Other values of laser-plasma parameters are same as given in Figure 1

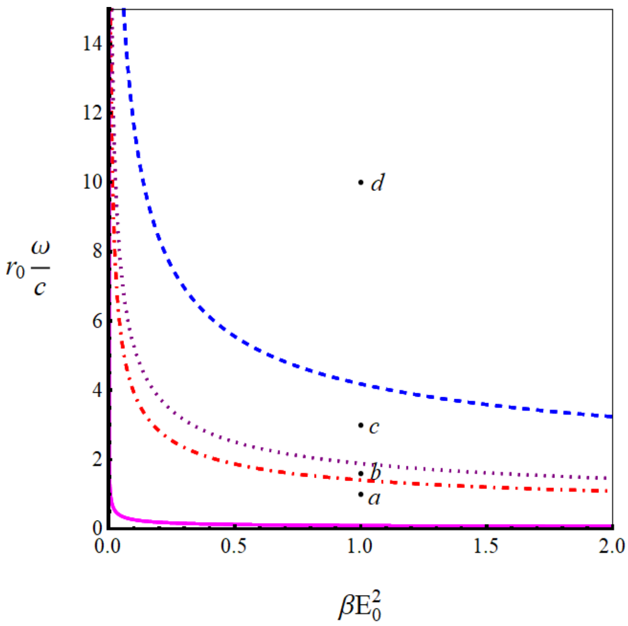

Figure 4. Critical curves for different Kappa parameters; $\kappa = 2$ (dashed curve), $\kappa = 4$ (dotted curve), $\kappa = 6$ (dot-dashed curve) and $\kappa \to \infty$ (solid curve). Other values of laser-plasma parameters are same as given in Figure 1. Coordinates $\left(\beta E_0^2, \frac{r_0\omega}{c}\right)$ corresponding to four representative points; a $(1,1)$, b $(1,1.6)$, c $(1,3)$ and d $(1,10)$ refer to typical points for which $f - \xi$ variation is further given in Figure 5. The critical curves for different Kappa values in figure divides the parameter space into two regions, above and below the curves which correspond to the laser focusing and defocusing, respectively that are specifically illustrated in Figure 5

It is well known that the ponderomotive force arises from nonlinear effects of laser-plasma interaction and the kinetic pressure force arises from the super-thermal electrons. It can be understood that for lower κ values, the pressure force due to the existence of super-thermal electrons quenches the effects of ponderomotive force. This decreases the nonlinearity of the plasma medium leading to reduction in self-focusing effect in plasma with for lower κ values.

On the other side, most of the electrons are present in the Maxwellian-like core for higher κ-values providing stronger self-focusing. Thus, as obvious, self-focusing quality is weaker in non-Maxwellian plasma in comparison with the Maxwellian ($\kappa \to \infty$) case of reference.

Figure 2 illustrate the effect of velocity ratio $\delta = \alpha_{e0}/c$ on dependence of beam-width parameter f with dimensionless distance of propagation ξ. In this figure, we have depicted f as a function of ξ for $\delta = 0.1, 0.2, 0.3, 0.4$ with $\kappa = 6$.

The periodic nature of f is observed with ξ in Figure 3 where increase in δ-values leads to the slight increase in self-focusing.

This is due to the fact that the non-thermal effects on the ponderomotive force increases with the thermal velocity scaled with the velocity of light, i.e. $\delta = \alpha_{e0}/c$.

For instance, increase in δ-value might be more susceptible temperature gradients leading to modification in plasma dielectric function, and, consequently, the self-focusing of the laser beam.

At this point it is worthy to highlight the propagation dynamics of laser beam with the help of phase trajectories. The phase space plots corresponding to Figure 2 are given in Figure 3 for same δ-values, i.e. $\delta = 0.1, 0.2, 0.3, 0.4$ with $\kappa = 6$. The complete spiral trajectories in phase space of Figure 3 signify the periodic focusing nature of the beam-width parameter f in Figure 2. It is also observed that with increase in the value of δ, there is slight increase in area of phase space. This is due to slight increase in extent of self-focusing of the laser beam with increase in δ-values. To further elucidate the results delineating the propagation dynamics of Gaussian laser beam in non-Maxwellian plasma, we have solved Eq.(17) and presented critical curves in Figure 4 for different values of Kappa parameter κ. Figure 4 reveals the dependence of dimensionless initial beam radius $r_0\omega/c$ with intensity parameter βE_0^2 for $\kappa = 2,\ 4,\ 6,\ \infty$. From this figure it is obvious that critical curves divide $(\beta E_0^2, r_0\omega/c)$ plane into two distinct regions corresponding to focusing and defocusing behaviors of f with ξ for respective κ-values. The region above the relevant critical curve will exhibit self-focusing behavior while steady or oscillatory defocusing will be displayed for region below it.

The self-trapped mode of propagation will be apparent for any point on the critical curve. It is to be noted that self-focusing area increases with increase in κ-values.

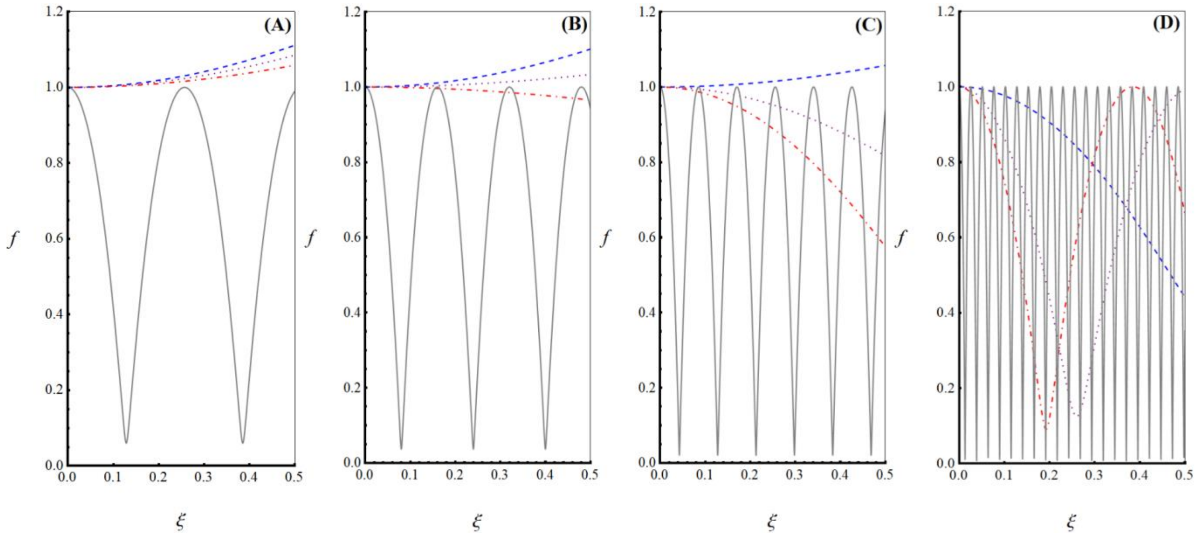

Figure 5. Variation of beam-width parameter f with dimensionless distance of propagation ξ for four representative points a, b, c and d given in Figure 4; (A) $a\ (1,1)$, (B) b (1,1.6), (C) $c\ (1,3)$, (D) $d\ (1,10)$ in which Kappa parameters; $\kappa = 2$ (dashed curve), $\kappa = 4$ (dotted curve), $\kappa = 6$ (dot-dashed curve) and $\kappa \to \infty$ (solid curve). Other values of laser-plasma parameters are same as given in Figure 1

To clarify the variation of f-parameter with ξ in Figure 5, we have chosen four representative points, $a\ (1,1)$, b (1,1.6), $c\ (1,3)$ and $d\ (1,10)$ in the $\beta E_0^2 - r_0\omega/c$ space. It is evident that the behaviour of f with ξ is in conformance with the critical curves as discussed above in Figure 4 Thus, laser beam display self-focusing and defocusing characters during propagation through non-Maxwellian plasma.

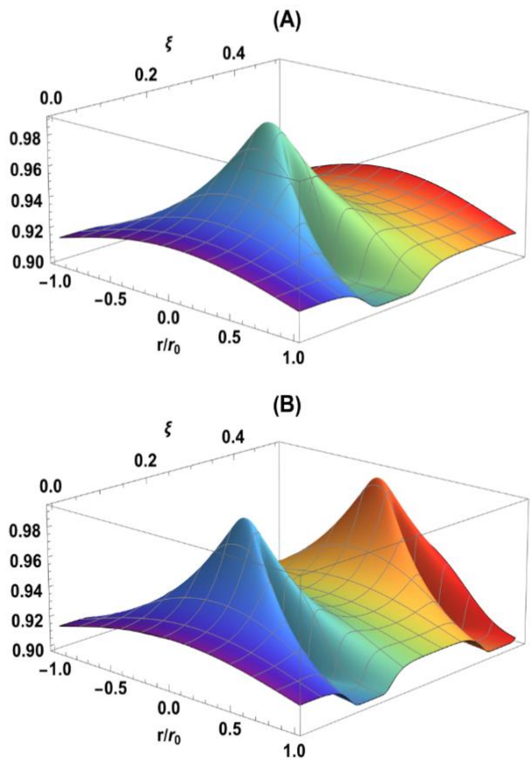

Figure 6. Variation of effective dielectric function ε as function of normalized values of radial distance r/r_0 and propagation distance ξ with $\delta = 0.1$; (A) $\kappa = 4$ and (B) $\kappa = 6$. Other values of laser-plasma parameters are same as given in Figure 1

To have a better understanding of the phenomena and numerical appreciation of the results, the variation of effective dielectric function ε is given with respect to normalized radial distance r/r_0 and ξ in Figure 6 for velocity ratio $\kappa = 4$ and $\kappa = 6$ with $\delta = 0.1$.

The plots show oscillatory character of ε with ξ. It is found that ε is maximum at the axial region and decreases radially away from the axis $(r = 0)$. The oscillation peaks in Figure 6 confirm the minima of oscillatory self-focusing behaviour of f with ξ displayed in relevant Figure 1 for $\kappa = 4$ and $\kappa = 6$. Based on the above analyses, it is found that the propagation characteristics of Gaussian laser beam in non-Maxwellian plasma are greatly related to the Kappa parameter κ and velocity ratio δ.

5. Conclusions

In conclusion, we investigated the self-focusing of Gaussian laser beam with spatial variation of electric field in un-magnetized underdense plasma described by Kappa distribution function.

Particularly, Kappa parameter and ratio between thermal velocity of plasma electrons and velocity of light on propagation dynamics is inspected. We derived the nonlinear differential equation governing evolution of beam-width parameter of Gaussian beam in non-Maxwellian plasma by using parabolic wave equation approach under WKB and paraxial approximations. Our numerical results provide following important conclusions:

- Self-focusing of Gaussian beam is more for higher values of Kappa parameter.
- Self-focusing of Gaussian beam increases significantly with increase in ratio of thermal velocity of plasma electrons to the velocity of light.

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- Quality of self-focusing is weaker in non-Maxwellian plasma in comparison with the classical Maxwellian case.

The above results will provide useful information for the analysis and interpretation of other nonlinear phenomena associated with the interaction of lasers with plasmas described by kappa distribution. It will also provide a useful basis to extend the analysis to the case of magnetized non-Maxwellian plasma. Present study is relevant to the various nonlinear phenomena correlated with astrophysical plasma environments where impact of super-thermal particles and magnetic field plays crucial role. This work can be extended to analyze other versions of non-Maxwellian distributions to tackle various nonlinear phenomena associated with plasmas encountered in space.

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Declarations

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Authors Contribution
All the authors have participated sufficiently in the intellectual content, conception and design of this work or the analysis and interpretation of the data (when applicable), as well as the writing of the manuscript.

Availability of data and materials
The data that supports the findings of this study are available upon reasonable request to corresponding author.

Conflict of interests
The authors declare no conflict of interest.

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