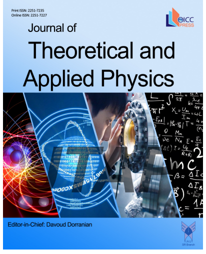

Volume 19, Issue 5, 192552 (1-6)

Journal of Theoretical and Applied Physics (JTAP)

https://doi.org/10.57647/j.jtap.2025.1905.52

The time wave equation with new conformable fractional derivative definition

Abdessamad Ait Brahim[1,*], Abdelmajid El Hajaji[2], Khalid Hilal[1]

[1]*AMSC Laboratory, University of Sciences and Technology , Beni Mellal, Morocco.*
[2]*OEE Departement, ENCGJ, University of Chouaib Doukali, El jadida, Morocco.*

[*]**Corresponding author:** abdessamad191212@gmail.com

Original Research

Received:
26 January 2025
Revised:
15 February 2025
Accepted:
27 June 2025
Published online:
31 October 2025

Abstract:

This study employs a novel approach to solving two and three-dimensional time fractional wave equations by utilizing the new results of fractional derivative definition, with a particular focus on the newly introduced "New conformable fractional derivative" as outlined in [1]:

$$\left(\mathscr{D}^{\beta}N\right)(t) = \lim_{h\longrightarrow 0}\frac{N\left(t+he^{(\beta-1)t}\right)-N(t)}{h},$$

where $N:[0,\infty)\rightarrow\mathbb{R}$ a function and $\beta\in[0,1)$. This definition offers simplicity and high effectiveness in addressing fractional differential equations with complex solutions, surpassing traditional fractional derivative definitions such as Caputo and Riemann-Liouville. Our findings indicate that the new conformable fractional derivative definition is both practical and efficient for resolving higher-dimensional fractional differential equations.

Keywords: Fractional wave equation; New conformable fractional derivative

1. Introduction

Fractional calculus (FC) represents a powerful extension of traditional calculus by allowing for differentiation and integration to be performed at non-integer orders. This concept aims to generalize the idea of derivatives and integrals beyond the integer domain, thus enabling more versatile modeling of physical, biological, and engineering systems. Numerous dynamic systems are better described using fractional-order models, where the governing equations involve derivatives or integrals of non-integer orders. These models provide more accurate representations of systems exhibiting memory effects, long-range interactions, or complex dynamics that integer-order models fail to capture adequately [1–5].

The study of systems governed by fractional-order equations is significantly more intricate than that of integer-order systems. One of the hallmarks of such systems is their inherent memory property, meaning that their future behavior depends not only on their current state but also on their entire history. This makes fractional systems intrinsically more complex and capable of modeling real-world phenomena that cannot be easily described using traditional methods [6–8]. For instance, fractional differential equations are crucial in modeling viscoelastic materials, diffusion processes, anomalous transport, and other physical systems exhibiting non-local behavior or power-law dynamics.

The concept of fractional derivatives, which extends the classical derivative to non-integer orders, can be traced back to the late 17th century. In fact, it was during a correspondence between the mathematicians L'Hopital and Leibniz on September 30, 1695, that the first discussions arose concerning the definition of the operator d^n/dx^n for non-integer n. Over the centuries, fractional derivatives have evolved through various definitions, with the Riemann-Liouville and Caputo derivatives being among the most prominent. The Riemann-Liouville fractional derivative is based on an integral representation, while the Caputo derivative modifies this definition to allow for better initial condition handling in practical applications. Fractional differential equations (FDEs) are often termed as extraordinary differential equations due to their distinctive mathematical structure, which sets them apart from conventional integer-order differential equations. These equations are now widely applied across multiple disciplines of science and engineering, including physics, chemistry, biology, economics, and engineering. The application of fractional calculus to dynamic systems in these fields has proven invaluable, providing insights into complex behaviors such as anomalous diffusion, memory-dependent processes, and fractal-like patterns. In physics, for instance, fractional differential equations are used to describe a variety of phenomena such as wave propagation in heterogeneous media, fluid dynamics in porous materials, and the relaxation processes in complex systems. The use of fractional derivatives allows for a more comprehensive and nuanced understanding of these systems by incorporating the effects of long-range correlations and non-local interactions, which are not accounted for in traditional models.

The challenge of solving fractional differential equations lies in their inherent non-locality and the need for special techniques tailored to handle fractional orders. Over recent years, a wealth of methodologies has been developed to tackle these equations, including analytical, numerical, and hybrid methods. These techniques aim to provide exact or approximate solutions to fractional models, which are often essential for real-world applications. Furthermore, researchers continue to propose new definitions and approaches to fractional derivatives, many of which involve integral representations that make it easier to solve and interpret fractional-order systems [9–11].

One of the key developments in this area has been the growing effort to establish new fractional derivative definitions that offer greater flexibility and applicability to practical problems. Many of these definitions are constructed in an integral form, which allows for a more intuitive understanding of fractional derivatives and simplifies their computation. As such, fractional calculus continues to play a central role in advancing our ability to model and understand complex systems, offering tools that can address a wide range of engineering, scientific, and technological challenges. The ongoing research in this field promises to yield even more efficient methods for solving fractional differential equations, further expanding the scope and impact of fractional calculus in modern science and engineering.

The new method presented in this study offers several advantages in physics, particularly when solving two- and three-dimensional time fractional wave equations. By utilizing the newly introduced "New conformable fractional derivative" [1], this approach provides a range of benefits compared to traditional fractional derivative definitions such as Caputo and Riemann-Liouville. Below are the primary advantages of this method in the context of physics:

Simplicity in Implementation: The new conformable fractional derivative definition simplifies the process of dealing with fractional differential equations. Unlike traditional definitions, which often involve complex procedures and intricate computations, the conformable derivative provides a straightforward and intuitive approach. This makes the mathematical formulation and solution of complex wave equations more accessible and efficient for researchers working in various areas of physics.

Improved Efficiency for Complex Solutions: This method is particularly effective for solving fractional differential equations with complex solutions. Traditional definitions, such as those of Caputo or Riemann-Liouville, often struggle to accurately describe systems where the behavior of the solutions is non-local or involves memory effects. The conformable fractional derivative provides a more effective means of capturing these behaviors, offering clearer and more reliable solutions in systems with anomalous diffusion, non-local interactions, or fractal-like structures.

Better Handling of Higher-Dimensional Equations: The new conformable fractional derivative is particularly useful when dealing with higher-dimensional fractional differential equations, such as those encountered in two- and three-dimensional wave equations. This method allows for the seamless extension of fractional calculus to higher dimensions, which is essential in describing wave phenomena in more complex physical systems, such as multi-dimensional media, heterogeneous materials, and systems with intricate geometries.

Physical Relevance and Applicability: The use of this novel fractional derivative is highly relevant in describing physical systems exhibiting non-local or fractional-order behavior, such as waves propagating in media with memory effects, viscoelastic materials, or anomalous wave propagation in porous structures. It offers an enhanced framework for modeling these phenomena in a way that traditional integer-order differential equations cannot capture adequately. The conformable fractional derivative thus opens up new possibilities for modeling real-world physical systems that were previously difficult to describe using classical methods [12, 13].

Practicality for Real-World Problems: In addition to its mathematical elegance, the conformable fractional derivative offers practical benefits for solving real-world problems. By simplifying the equations and reducing computational complexity, it allows for more efficient numerical simulations and analysis of complex systems in physics, engineering, and applied sciences. This method also lends itself well to experimental verification, as it offers clear and predictive results that align well with observed physical behaviors in systems exhibiting anomalous or fractional dynamics [14–16].

In summary, fractional calculus extends classical calculus to non-integer orders, offering more accurate and effective models for a wide variety of dynamic systems. The study of fractional differential equations presents unique challenges, but it is essential for capturing the complexities of systems with memory and non-local interactions. The continuous advancement of fractional calculus methods holds the promise of further breakthroughs in science and engineering, providing deeper insights into both fundamental and applied phenomena across many disciplines. Among these definitions, two of the most prominent are:

If n is a positive integer and $\beta \in [n-1, n]$ derivative of N is given by

- Riemann-Liouville definition:

$$\mathscr{D}_a^{\beta}(N)(t) = \frac{1}{\Gamma(n-\beta)} \frac{d^n}{dt^n} \int_a^t \frac{N(u)}{(t-u)^{\beta-n+1}} du \tag{1}$$

- Caputo definition:

If n is a positive integer and $\beta \in [n-1,n)$, β derivative of N is given by

$$\mathscr{D}_a^{\beta}(N)(t)=\frac{1}{\Gamma(n-\beta)}\int_a^t \frac{N^{(n)}(u)}{(t-u)^{\beta-n+1}}du. \tag{2}$$

In reference [12], R. Khalil et al. presented an innovative concept of fractional derivative termed the "conformable fractional derivative".

Definition 1 Let $N:[0,\infty)\to\mathbb{R}$ be a function. β order "conformable fractional derivative" of N is defined by

$$\mathscr{D}_{\beta}(N)(t)=\lim_{\varepsilon\to 0}\frac{N\left(t+\varepsilon t^{1-\beta}\right)-N(t)}{\varepsilon} \tag{3}$$

for all $t>0, \beta\in(0,1)$. If N is β-differentiable in $(0,a), a>0$, and $\lim_{t\to 0^+}N^{(\alpha)}(t)$ exists, therefore define

$$N^{(\beta)}(0)=\lim_{t\to 0^+}N^{(\beta)}(t). \tag{4}$$

The properties satisfied by this new definition are outlined in the following theorem [12].

Definition 2 Given a function $N:[0,\infty)\longrightarrow\mathbb{R}$, and therefore the "conformable fractional derivative" of N order α is represented by

$$\left(\mathscr{D}^{\beta}N\right)(t)=\lim_{k\longrightarrow 0}\frac{N\left(t+ke^{(\beta-1)t}\right)-N(t)}{k}, \tag{5}$$

for all $t>0$, and $\beta\in(0,1)$.

If N is β differentiable in $(0,a), a>0$, and $\lim_{t\to 0^+}\left(\mathscr{D}^{\beta}N\right)(t)$ exists, therefore define

$$\left(\mathscr{D}^{\beta}N\right)(0)=\lim_{t\longrightarrow 0^+}\left(\mathscr{D}^{\beta}N\right)(t). \tag{6}$$

Theorem 1 [1] If a function $N:[0,+\infty)\longrightarrow\mathbb{R}$ and β differentiable at $t_0>0$, then N is continuous at t_0.

Theorem 2 If N be β differentiable at a point $t>0$. We gets

1. $\mathscr{D}^{\beta}(aN+bN)=a\left(\mathscr{D}^{\beta}N\right)+b\left(\mathscr{D}^{\beta}N\right)$, for all $a,b\in R$
2. $\mathscr{D}^{\beta}\left(t^n\right)=ne^{(\beta-1)t}t^{p-1}$ for all $n\in R$
3. $\mathscr{D}^{\beta}(\gamma)=0$, for all constant $N(t)=\gamma$.
4. $\left(\mathscr{D}^{\beta}NH\right)=N\left(\mathscr{D}^{\beta}H\right)+H\left(\mathscr{D}^{\beta}N\right)$.
5. $\left(\mathscr{D}^{\beta}(N/H)\right)=\left(N\left(\mathscr{D}^{\beta}H\right)+H\left(\mathscr{D}^{\beta}N\right)\right)/H^2$.
6. If N is differentiable, then $\left(\mathscr{D}^{\beta}N\right)(t)=e^{(\beta-1)t}N'(t)$.

2. The fractional Wave equation (FWE) with "new conformable fractional derivatives" (NCFD)

The wave equation, which describes the propagation of waves in a given medium, is a fundamental partial differential equation in classical physics and engineering. In its standard form, it represents the relationship between the spatial and temporal variations of a wave function. However, when fractional calculus is applied, particularly through the use of conformable fractional derivatives, the traditional wave equation is modified to accommodate non-integer orders of differentiation. This extension allows for a more generalized description of wave propagation, especially in complex systems where classical integer-order calculus may not be sufficient. By introducing these fractional derivatives, we are able to capture anomalous diffusion and memory effects that are often observed in physical systems with non-local properties or fractal structures. The following equation incorporates this modification to the conventional wave equation, where the fractional derivatives are used to represent the more generalized dynamics of wave propagation in both space and time.

To explore the solution of the two-dimensional wave equation incorporating the "new conformable fractional derivatives" [1], we shall begin by expressing the equation itself. The two-dimensional wave equation in rectangular coordinates is as follows:

$$\frac{\delta^{\beta}}{\delta t^{\beta}}\frac{\partial^{\beta}N}{\partial t^{\beta}}=L^2\left(\frac{\partial^2 N}{\partial u^2}+\frac{\partial^2 N}{\partial v^2}\right), 0<u<a, 0<v<b, t>0, \tag{7}$$

in the rectangular domain $\mathrm{D}=\{(u,v):0<u<a,0<v<b,u,v\in R\}$, consider the vibrating membrane or plate.

$$N(0,v,t)=N(a,v,t)=N(u,0,t)=N(u,b,t)=0 \tag{8}$$

$$N(u,v,0)=M(u,v);0\le u\le a,0\le v\le b \tag{9}$$

$$N_t(u,v,0)=P(u,v);0\le u\le a,0\le v\le b \tag{10}$$

where $0<\beta\le 1$.

Let $N(u,v,t)=H(u)Q(v)K(t)$. Hence, we proceed to derive the requisite derivatives in equation (7) and dividing all sides by $L^2H(u)Q(v)K(t)$ we gets,

$$\frac{1}{L^2K(t)}\frac{\delta^{\beta}}{\delta t^{\beta}}\frac{\partial^{\beta}K}{\partial t^{\beta}}=\frac{1}{H(u)}\frac{\partial^2 H}{\partial u^2}+\frac{1}{Q(v)}\frac{\partial^2 Q}{\partial v^2}=-\mu^2. \tag{11}$$

We manipulate the right side of the formula, we obtain:

$$\frac{1}{H(u)}\frac{\partial^2 H}{\partial u^2}=-\frac{1}{Q(v)}\frac{\partial^2 Q}{\partial v^2}-\mu^2. \tag{12}$$

For $0<u<a$ and $0<v<b$ in the right and left side of the formula (12), we gets

$$\frac{1}{H(u)}\frac{\partial^2 H}{\partial u^2}=-\frac{1}{Q(v)}\frac{\partial^2 Q}{\partial v^2}-\mu^2=-v^2 \tag{13}$$

Let v denote the separation constant. Equations (11) and (13) yield the subsequent equalities:

$$\frac{\delta^{\beta}}{\delta t^{\beta}}\frac{\partial^{\beta}K}{\partial t^{\beta}}+L^2\mu^2K(t)=0 \tag{14}$$

$$\frac{\partial^2 H}{\partial u^2}+v^2H=0 \tag{15}$$

$$\frac{\partial^2 Q}{\partial v^2}+\iota^2 Q=0 \tag{16}$$

Utilizing equation (6) from Theorem (1) alongside Eq. (29) in [13], which defines the sequential "conformable fractional derivative", transforms the first equation into:

$$(\beta-1)e^{(2\beta-1)t}K'(t)+e^{(2\beta-2)t}K''(t)+L^2\mu^2K(t)=0. \tag{17}$$

The solution to the mentioned equation can be readily obtained as follows:

$$K(t)=A\cos\left(\frac{L\mu e^{(1-\beta)t}}{1-\beta}\right)+B\sin\left(\frac{L\mu e^{(1-\beta)t}}{1-\beta}\right) \tag{18}$$

In figure 1, we represent the comparative solutions $K(t)$ for several values of β, illustrating how variations in β affect the oscillatory behavior of the system.
Then, solutions for the remaining equations can be obtained as follows:

$$H(u)=C\cos(\vartheta u)+D\sin(\vartheta u) \tag{19}$$

$$Q(v)=E\cos(lv)+F\sin(lv) \tag{20}$$

The conditions referenced as (8) necessitate the fulfillment of these equalities

$$H(u)=\sum_{n=1}^{\infty}D_n\sin\left(\frac{n\pi u}{a}\right), \tag{21}$$

$$Q(v)=\sum_{m=1}^{\infty}F_m\sin\left(\frac{m\pi v}{b}\right). \tag{22}$$

Therefore, based on our assumption, $N(u,v,t)$ can be derived as.

$$N(u,v,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\Phi_{mn}\cos\left(\frac{\gamma_{mn}Le^{(1-\beta)t}}{1-\beta}\right)+\Psi_{mn}\sin\left(\frac{\gamma_{mn}Le^{(1-\beta)t}}{1-\beta}\right)\right]\sin\left(\frac{n\pi u}{a}\right)\sin\left(\frac{m\pi v}{b}\right) \tag{23}$$

with $\gamma_{mn}=\frac{\pi^2n^2}{a^2}+\frac{\pi^2m^2}{b^2}$.
Now, leveraging conditions (9) and (10), we can determine the coefficients in equation (23) as follows:

$$\Phi_{mn}=\frac{4}{ab}\int_0^a\int_0^b M(u,v)\sin\left(\frac{n\pi u}{a}\right)\sin\left(\frac{m\pi v}{b}\right)dvdu, \tag{24}$$

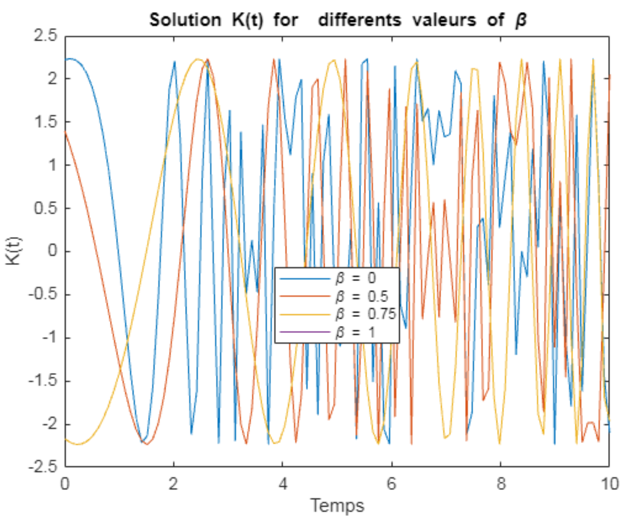

Figure 1. Comparative solutions for serval values of β.

$$\Psi_{mn}=\frac{4}{\gamma_{mn}abL}\int_0^a\int_0^b P(u,v)\sin\left(\frac{n\pi u}{a}\right)\sin\left(\frac{m\pi v}{b}\right)dvdu. \tag{25}$$

Therefore, the solution to equation (7) is derived as follows:

$$N(u,v,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\left[\frac{4}{ab}\int_0^a\int_0^b M(u,v)\sin\left(\frac{n\pi u}{a}\right)\sin\left(\frac{m\pi v}{b}\right)dvdu\right]\cos\left(\frac{\gamma_{mn}Le^{(1-\beta)t}}{1-\beta}\right)+\left[\frac{4}{\gamma_{mn}abL}\int_0^b\int_0^b P(u,v)\sin\left(\frac{n\pi u}{a}\right)\sin\left(\frac{m\pi y}{b}\right)dv\,du\right]\sin\left(\frac{\gamma_{mn}Le^{(1-\beta)t}}{1-\beta}\right)\right]\sin\left(\frac{n\pi u}{a}\right)\sin\left(\frac{m\pi v}{b}\right). \tag{26}$$

3. The three Dimensional Fractional Wave Equation (FWE)

The vibrational motion of an object or gas, in the absence of external influences, follows the principles of wave propagation, where the displacement of points within the medium varies as a function of both spatial position and time. In classical mechanics, this motion is governed by the traditional wave equation, which links the second derivatives of the displacement with respect to space and time. However, in systems exhibiting complex behaviors such as fractality, non-local interactions, or anomalous diffusion, fractional calculus provides a more accurate description of wave phenomena. By extending the wave equation using fractional derivatives, we are able to account for these non-standard effects that cannot be captured by integer-order calculus. The fractional form of the wave equation, which incorporates derivatives of non-integer order, offers a more general framework that can describe wave propagation in materials or media with memory, elasticity, or heterogeneous properties. In particular, the fractional derivatives reflect the non-local behavior of the medium, where the response at a given point may depend not only on the current state of the point itself but also on its historical or spatially distant states. This extended form of the wave equation is crucial for modeling a wide range of real-world phenomena, such as vibrations in materials with complex microstructures or the behavior of gases in confined geometries.
In three-dimensional space, when an object or gas undergoes free vibrational motion within a prism, unaffected by external forces, this motion is described by the three-dimensional wave equation. Expressed in fractional form, this equation can be written as:

$$\frac{\delta^\beta}{\delta t^\beta}\frac{\partial^\beta N}{\partial t^\beta}=L^2\left(\frac{\partial^2N}{\partial u^2}+\frac{\partial^2N}{\partial v^2}+\frac{\partial^2N}{\partial z^2}\right), \quad 0<u<a, 0<v<b, 0<z<d, t>0 \tag{27}$$

where the following conditions:

$$N(0,v,z,t)=N(a,v,z,t)=N(u,0,z,t)=N(u,b,z,t)=N(u,v,0,t)=N(u,v,d,t)=0. \tag{28}$$

$$N(u,v,z,0)=P(u,v);0\leq u\leq a,0\leq v\leq b,0\leq z\leq d. \quad (29)$$

$$N_t(u,v,z,0)=M(u,v);0\leq u\leq a,0\leq v\leq b,0\leq z\leq d. \quad (30)$$

Here, $0<\beta\leq 1$, and the derivative represents a conformable fractional derivative. Let's consider $N(u,v,t)=H(u)Q(v)K(z)J(t)$. Next, by taking the necessary derivatives in equation (27) and dividing both sides by $L^2H(u)Q(v)R(z)J(t)$, we obtain:

$$\frac{1}{L^2J(t)}\frac{\delta^\beta}{\delta t^\beta}\frac{\partial^\beta J}{\partial t^\beta}=\frac{1}{H(u)}\frac{\partial^2 H}{\partial u^2}+\frac{1}{Q(y)}\frac{\partial^2 Q}{\partial v^2}+\frac{1}{K(z)}\frac{\partial^2 K}{\partial z^2}=-\mu^2 \quad (31)$$

By following the procedure outlined in section 2, we derive the following equations:

$$\frac{\delta^\beta}{\delta t^\beta}\frac{\partial^\beta J}{\partial t^\beta}+L^2\mu^2J(t)=0, \quad (32)$$

$$\frac{\partial^2 H}{\partial u^2}+\vartheta^2H=0, \quad (33)$$

$$\frac{\partial^2 Q}{\partial v^2}+\imath^2Q=0, \quad (34)$$

$$\frac{\partial^2 K}{\partial z^2}+\rho^2K=0. \quad (35)$$

Utilizing equation (6) from Theorem (1) alongside Eq. (29) in [13], which defines the sequential conformable fractional derivative, transforms the first equation into:

$$(\beta-1)e^{(2\beta-1)t}J'(t)+e^{(2\beta-2)t}J''(t)+L^2\mu^2J(t)=0 \quad (36)$$

We can be easily get the following solution of above equation:

$$J(t)=A\cos\left(\frac{Lue^{(1-\beta)t}}{1-\beta}\right)+B\sin\left(\frac{Lue^{(1-\beta)t}}{1-\beta}\right). \quad (37)$$

In figure 2, we represent the comparative solutions $J(t)$ for several values of β, illustrating how variations in β affect the oscillatory behavior of the system.

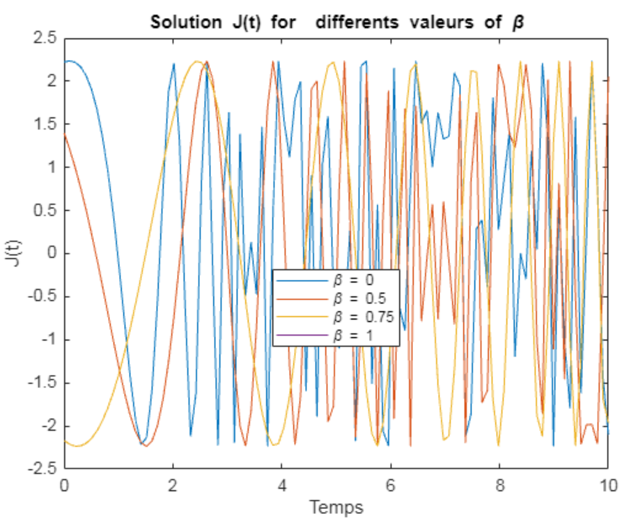

Figure 2. Comparative solutions for serval values of β.

Moreover, solutions to other equations can be derived as follows:

$$H(u)=C\cos(\vartheta u)+D\sin(\vartheta u) \quad (38)$$

$$Q(v)=E\cos(\imath v)+F\sin(\imath v) \quad (39)$$

The conditions described by (28) necessitate these equalities to hold.

$$H(x)=\sum_{n=1}^{\infty}D_n\sin\left(\frac{n\pi u}{a}\right), \quad (40)$$

$$Q(v)=\sum_{m=1}^{\infty}F_m\sin\left(\frac{m\pi v}{b}\right), \quad (41)$$

$$K(z)=\sum_{r=1}^{\infty}K_r\sin\left(\frac{r\pi z}{d}\right). \quad (42)$$

Thus, according to our assumption, we obtain $N(u,v,z,t)$ as follows:

$$N(u,v,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{r=1}^{\infty}\Phi_{mnr}\cos\left(\frac{\gamma_{mnr}Le^{(1-\beta)t}}{1-\beta}\right)\sin\left(\frac{n\pi u}{a}\right)\sin\left(\frac{m\pi v}{b}\right)\sin\left(\frac{r\pi z}{d}\right)+\phi_{mmr}\sin\left(\frac{\gamma_{mnr}Le^{(1-\beta)t}}{1-\beta}\right)\sin\left(\frac{n\pi u}{a}\right)\sin\left(\frac{m\pi v}{b}\right)\sin\left(\frac{r\pi z}{d}\right) \quad (43)$$

Utilizing conditions (29) and (30), we determine the coefficients Φ_{mn} and ϕ_{mnr} as follows:

$$\Phi_{mn}=\frac{8}{abd}\int_0^a\int_0^b\int_0^d P(u,v,z)\sin\left(\frac{n\pi u}{a}\right)\sin\left(\frac{m\pi v}{b}\right)\sin\left(\frac{r\pi z}{d}\right)dzdvdu, \quad (44)$$

$$\phi_{mn}=\frac{8}{\gamma_{mnr}abdL}\int_0^a\int_0^b\int_0^d M(u,v,z)\sin\left(\frac{n\pi u}{a}\right)\sin\left(\frac{m\pi v}{b}\right)\sin\left(\frac{r\pi z}{d}\right)dzdvdu. \quad (45)$$

4. Conclusion

This paper explores the resolution of the "two and three-dimensional" fractional wave formula. Utilizing the new conformable fractional derivative (NCFD) [1], we can seamlessly convert fractional differential equations into familiar classical differential equations. This approach eliminates the necessity for complex methods or definitions, simplifying the attainment of exact solutions. The solution procedure and outcomes demonstrate the practicality and suitability of this definition for addressing "higher-dimensional" partial fractional differential equations.

The key advantage of the NCFD lies in its ability to transform complex fractional problems into more manageable forms, enabling researchers and practitioners to leverage existing analytical techniques and tools traditionally used for classical differential equations. By applying the NCFD, we reduce computational complexity and enhance the efficiency of obtaining precise solutions, which is particularly beneficial in modeling physical phenomena where fractional derivatives play a crucial role.

Our study highlights the robustness of the NCFD in extending the applicability of classical methods to fractional differential equations in higher dimensions. The results obtained not only affirm the validity of the NCFD but also showcase its versatility in various scientific and engineering contexts. For instance, in wave propagation, heat conduction, and other dynamic systems, where fractional calculus provides a more accurate description of underlying processes, the NCFD proves to be a valuable tool.

Furthermore, the straightforward implementation of the NCFD fosters a deeper understanding of fractional dynamics and promotes its integration into mainstream mathematical modeling. This can potentially lead to new insights and advancements in fields such as physics, engineering, finance, and biology, where the behavior of systems often exhibits fractional-order characteristics.

In conclusion, the use of the new conformable fractional derivative presents a significant step forward in solving higher-dimensional partial fractional differential equations. It bridges the gap between classical and fractional calculus, offering a simplified yet powerful method for exact solutions. Future research can build upon this foundation, exploring more complex systems and further validating the broad applicability of the NCFD in diverse scientific domains.

Authors Contribution
All authors equally contributed to the conception, methodology, data analysis, and writing of the manuscript. All authors read and approved the final manuscript.

Availability of data and materials
The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflict of interests
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] A. Kajouni, A. Chafiki, K. Hilal, and M. Oukessou. “A New Conformable Fractional Derivative and Applications.”. *International Journal of Differential Equations*, 2022.
DOI: https://doi.org/10.1155/2021/6245435.

[2] K. S. Miller and B. Ross. “An Introduction to the Fractional Calculus and Fractional Differential Equations.”. *Wiley-Interscience Publication*, 1993.

[3] A. Esen, N. M. Yagmurlu, and O. Tasbozan. “Approximate Analytical Solution to Time-Fractional Damped Burger and Cahn-Allen Equations.”. *Applied Mathematics and Information Sciences*, **7**: 1951–1956, 2013.
DOI: https://doi.org/10.12785/amis/070533.

[4] A. A. Kilbas, H. M. Srivastava, and J. J. Trujillo. “Theory and Applications of Fractional Differential Equations.”. Elsevier, 2006.

[5] H. Wang and B. Zheng. “Exact solutions for fractional partial differential equations by an extended fractional Riccati sub-equation method.”. *WSEAS Transactions on Mathematics*, **13**:952–961, 2014.
DOI: https://doi.org/10.1063/1.4892534.

[6] I. Goychuk and P. Hanggi. “Fractional diffusion modeling of ion channel gating.”. *Phys Rev. E*, **70**(5):1–9, 2004.

[7] M. Abu Hammad and R. Khalil. “Conformable Fractional Heat Differential Equation.”. *International Journal of Pure and Applied Mathematics*, **94**:215–221, 2014.
DOI: https://doi.org/10.12732/ijpam.v94i2.8.

[8] B. K. Dutta and L. K. Arora. “On the existence and uniqueness of solutions of a class of initial value problems of fractional order.”. *Mathematical Sciences*, page 20137, 2013.
DOI: https://doi.org/10.1186/2251-7456-7-17.

[9] Y. Çenesiz and A. Kurt. “The new solution of time fractional wave equation with conformable fractional derivative.”. *Journal of New Theory*, (7):79–85, 2015. URL www.newtheory.org.

[10] E. Juan, N. Valdes, P. M. guzmán, L. M. Lugo, and A. Kashuri. “The local generalized derivative and Mittag-Leffler function.”. *Sigma J Eng, Nat Sci*, **38**:1007–1017, 2020.
DOI: https://doi.org/10.33044/revuma.

[11] V. C. Miguel, A. Fleitas, P. M. Guzmán, J. E. Nápoles, and J. J. Rosales. “Newton’s law of cooling with generalized conformable derivatives.”. *Symmetry*, **13**:1093, 2021.
DOI: https://doi.org/10.3390/sym13061093.

[12] R. Khalil, M. Al Horani, A. Yousef, and M. Sababheh. “A new definition of fractional derivative.”. *Journal of Computational and Applied Mathematics*, **264**:65–70, 2014.
DOI: https://doi.org/10.1016/j.cam.2014.01.002.

[13] T. Abdeljawad. “On conformable fractional calculus.”. *Journal of Computational and Applied Mathematics*, **279**:57–66, 2015.
DOI: https://doi.org/10.1016/j.cam.2014.10.016.

[14] K. S. Miller. “An Introduction to Fractional Calculus and Fractional Differential Equations.”. J. Wiley and Sons, 2006.
DOI: https://doi.org/10.12691/ajma-3-3-1.

[15] A. Kilbas, H. Srivastava, and J. Trujillo. “Theory and applications of fractional differential equations.”. Math. Studies, 2006.
DOI: https://doi.org/10.1016/S0304-0208(06)80001-0.

[16] I. Podlubny. “Fractional Differential Equations.”. Academic Press, 1999.