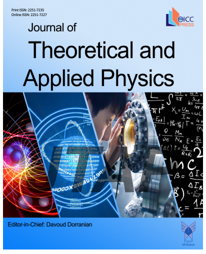

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Calculate the nuclear structure for the ^{72,74,76}Kr isotopes by using NuShellX@MSU code

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Abstract:

Effective f5pvh interactions have been identified, and the NuShellX@MSU code within the fp-shell was employed to calculate the energy levels and electromagnetic transition probabilities of the ^{72,74,76}Kr nucleus. The use of the nuclear shell model utilising the f5pvh interaction within the fp-shell demonstrates success, as evidenced by the concordance with both experimental and theoretical data.

Keywords: Nuclear structure; NuShellX@MSU; Shell model; Energy level; Transition probability

1. Introduction

Mayer and Jensen developed the shell model, which was effectively utilised to assess the nuclear structural characteristics of relatively light and near-closed shell nuclei, including rotation, parity, magnetic moment, and other properties [1]. It is considered the origin of macroscopic nuclear models and is one of the fundamental microscopic nuclear models [2]. The Cohen-Kurath and USD interactions are two prevalent effective interactions for light nuclei, namely for p and sd-shells. In modern times, the study of sd-shell nuclei abundant in neutrons has garnered scientists' attention as it reveals new aspects of nuclear composition [3, 4]. The nuclear shell model codes like Oxbash [5], Antoine [6], NuShell [7], and NuShellXetc [8], were utilized extensively with shell model calculations in the fp-shell, sd-shell, and finally in the fp-shell [9]. The study of sd-shell nuclei abundant in neutrons has garnered scientists' attention as it reveals new aspects of nuclear composition. In this study, we employed the entire f5p as a single-particle space (SPS) [10]. Furthermore, applying The nucleic structure code NuShellX@MSU is employed for particular computations [11]. It possesses the following shells: $1\mathrm{f}_{7/2}$, $2\mathrm{p}_{3/2}$, $1\mathrm{f}_{5/2}$, and $2\mathrm{p}_{1/2}$ for protons, as well as $1f_{7/2}$, $2\mathrm{p}_{3/2}$, $1\mathrm{f}_{5/2}$, and $2\mathrm{p}_{1/2}$ for neutrons. Complete $\pi 2\mathrm{p}_{3/2}$ and $\nu 2\mathrm{p}_{3/2}$ shells are utilized in the calculations. The SPEs were acquired for specific shells from Grawe et al. In the mean-field approximation, protons and neutrons operate independently inside a shared potential, elucidating numerous nuclear characteristics of the nucleus [11]. The interaction of nucleons produces electric and magnetic fields [12]. Bill Rae developed a suite of computer tools called NuShellX for calculations employing the shell model Hamiltonian matrix on an exceptionally large scale [13]. It can accommodate J-scheme matrix size up to 100 million and use a J-coupled proton-neutron basis. The wrapper scripts referred to as NuShellX@MSU were created by Alex Brown and generate Hamiltonian data files and model space to serve as input for NuShellX [8]. The nuclear shell model posits that the energy levels of nuclei are characterized by their radial quantum number (n), orbital angular momentum (ℓ), and total angular momentum of a single particle (j). In the nuclear shell model, this type of level is designated as the single-particle level. It can be inferred that under conditions of high density and strong forces, nucleons will clash incessantly, rendering the orbit of an individual particle unsustainable. Nevertheless, the initial stage in advancing the "shell model" involves the selection of more accurate potentials, such as the foundational premise of the shell model and the influence of interactions among nucleons through a single-particle potential. Nevertheless, the typical shell model potential aligns with "Pauli's exclusion principle," which confines nucleons to a limited array of allowable orbitals as follows [14]:

$$V(r)=\frac{-V_0}{1+e^{\frac{(r-R)}{a}}} \tag{1}$$

The mean radius, nucleus thickness, and voltage depth are represented by R, a, and V_0, with the coefficients defined as "$V_0=57$ MeV", "$R\approx 1.25$ Afm," and "$a\approx 0.65$ fm, respectively". Furthermore, it facilitates modifications to the well's depth due to the similarity energy of disparate quantities of protons and neutrons. A neutron can interact with a proton in various manners: n-n and p-p, denoting Coulomb repulsion, and the final interaction between a proton and a neutron (n-p), which is more potent than both (n-n) and (p-p). It is feasible to calculate the boundary conditions for energy levels of a spherically symmetric potential $V(r)$ utilizing the Schrödinger equation for the central potential $V(r)$. Utilizing the following mathematical expression as [15]:

$$-\frac{\hbar}{2m}\nabla\Psi(r)+\left\{(\frac{\ell(\ell+1)\hbar^2}{2mr^2}V(r)\right\}\Psi(r)=E\Psi(r) \tag{2}$$

The eigenvalue of energy can be represented by (E).

2. Theoretical framework

Without taking into account the central interaction rate, the force that results from the collision of two nucleons is the interaction component of the residual trace that remains between them [16]. In this way, the simple independent particle model is cancelled out and the level disintegration, a property of the shell model, is eliminated by the residual trace between other nucleons [17, 18]. Furthermore, outside of the locked core $V(\mathbf{r},\mathbf{r})$, the residual impact disrupts the Hamilton trace, which is represented by (the energy potential of the two nucleons). And this equation is used to remove Hamilton for the perturbation situation [19, 20].

$$H=H_{\text{core}}+\sum_{i=1}^{2}H_i+V(\mathbf{r}_1,\mathbf{r}_2) \tag{3}$$

The energy equation derived from this Eq. will be:

$$E=E_{\text{core}}+\sum_{i=1}^{2}\varepsilon_i+\langle j_1 j_2|V(\mathbf{r}_1,\mathbf{r}_2)|j_1 j_2\rangle \tag{4}$$

On the other hand, E_{core}, the energy of locked core bindings, represents εi, or energy for single particle (SPE) of orbit, and may be calculated using the equation from the region around "the closed shell" that has a mass number more than one nucleon than the closed core [20].

$$\varepsilon_i=BE(\text{core}+1)-BE(\text{core}) \tag{5}$$

When discussing the impact interaction between nucleons and orbits outside of the locked core, "Two-Body Matrix Elements" (TBME) are represented by the equation $\langle j_1 j_2|V(\mathbf{r}_1,\mathbf{r}_2)|j_1 j_2\rangle$.The element can be described by the particles in the outer shell, or outside the closed shell, for the Hamiltonian matrix, and the equation can be used to represent the configurations of state j [21, 22].

$$H_{ij}=(\varepsilon_i+\varepsilon_j)\delta_{ij}+\langle j_1 j_2|V(\mathbf{r}_1,\mathbf{r}_2)|j_3 j_4\rangle_{J,T} \tag{6}$$

The main source of knowledge about "total angular momentum and parity" in the study of electromagnetic transfers is electromagnetic interaction, which is a recognized phenomenon in contrast to nuclear interactions [20]. Nuclear breakdown, internal transformation, and interactions are currently associated with gamma radiation emission study. The rate at which gamma ray photon emission is broken down by the multipolar transition from the first state J_i to the final state J_f is established by [20, 21]:

$$T(\sigma\lambda;J_i\rightarrow J_f)=\frac{8\pi(\lambda+1)}{\hbar\lambda[(2\lambda+1)!!]^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1}B(\sigma\lambda,J_i\rightarrow J_f) \tag{7}$$

where, J_i and J_f are the starting and final states, E_γ represent to the photon energy of the gamma emission in the MeV unit and $B(\sigma\lambda,J_i\rightarrow J_f)$ represent to the multipolar transition probability. There σ represent to a multipolar type and λ imultipolar arrangement. The disintegration rate can be related to half-life [23].

$$T(\sigma\lambda;J_i\rightarrow J_f)=\frac{\ln 2}{T_{1/2}(1+\alpha)} \tag{8}$$

In this equation, $T_{1/2}$ is the half-life of the starting state, and α is the total conversion coefficient of the gamma rays. Equations (11) and (10) enable the computation of $B(\sigma\lambda,J_i\rightarrow J_f)$ [24, 25].

$$B(\sigma\lambda,J_i\rightarrow J_f)=\frac{\lambda[(2\lambda+1)!!]^2}{8\pi(\lambda+1)}\frac{\hbar\ln 2}{T_{1/2}(1+\alpha)}\left(\frac{\hbar c}{E_\gamma}\right)^{2\lambda+1} \tag{9}$$

The likelihood of decreased transmission for electrical transitions is expressed in terms of e2fm2λ, while for magnetic transitions, it is expressed as (μN = e$\hbar$2Mpc2fm2λ-2). the experimental values for the transition probability can frequently expressed in "Weisskopf units". These estimations based mainly on a model with the assumptions [26].

One active particle and an inert core comprise the nucleus. There is a transition between the phases $J_i=\ell\pm\frac{1}{2}$, $J_f=\frac{1}{2}$ [27].

The radiation component represents the initial and final states in the nucleus core.

It is linked to the reduced transition probability and the mono-particle electromagnetic effect outside [26].

$$B(\sigma\lambda;J_i\rightarrow J_f)=\frac{1}{2J_i+1}|\langle J_f\alpha_f||O^{\sigma\lambda}||J_i\alpha_i\rangle|^2 \tag{10}$$

Given that the residual interactions matrix for the two particles is similar to $O^{\sigma\lambda}$, which represents the multipolar electromagnetic operation for the configuration (λ). The likelihood of transitioning to (BE2) can be articulated by the subsequent equation [23]:

$$B(E2;J_i \to J_f) = \frac{1}{2J_i+1}|\langle J_f\alpha_f||O^E||J_i\alpha_i\rangle|^2 \tag{11}$$

3. Results and discussion

Utilizing the nuclear shell model, theoretical calculations of the energy levels and electromagnetic transitions in the (^{72,74,76}Kr) nucleus are conducted.

To demonstrate the energy levels and B(M1), B(E2) for the ^{72,74,76}Kr isotopes utilizing the f5p model space, calculations were performed in the model space ($1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, and $1g_{9/2}$) above the closed shells of ^{56}Fe, employing the f5pvh interaction.

3.1 Energy levels ^{72}Kr isotope

According to the nuclear shell model, the ground state of the ^{72}Kr nucleus is a ^{56}Fe nucleus closed with (Np = Nn = 28), comprising 16 nucleons distributed as (P and N) in the fp-shell, with $J^{\pi} = 0^+$ and $T = 0$. A comparison between the existing theoretical results and the experimental results for the ^{72}Kr isotope employing f5pvh interaction is shown in Table 1 [28].

The theoretical calculations show that the ^{72}Kr nucleus has sixteen nucleons outside the closed core, which is consistent with the expected results for parity and the total angular momentum scenario in Table 1.

Upon juxtaposing the experimental data for this isotope presented in the aforementioned table with our theoretical results derived from the f5pvh interaction, the following observations can be made:

1. A comparison with the existing empirical data indicated that the ground state parity and total angular momentum of the 0^+ level were identical.
2. By juxtaposing the theoretically calculated energies (0.709 MeV; 2^+) and (1.321; 4^+) with the existing experimental data, we achieved a commendable concordance regarding the angular momentums.
3. The angular momentum and parity of the practical energies were determined (3.797, 4.756, 6.048, 7.164, 8.608, 10.040, 4.713, 4.504). MeV Angular momentum is associated with positive parity values of 4, 6, 7, 7, 10 and 10. This reflects the degree of alignment between the practical value and our theoretical value.
4. Given that the actual value in close agreement with our theoretical value, total angular momentum of the experimentally uncertain energy (8.526, 8.745, 8.820, 10.558) MeV, corresponding to the angular momentum $9^+_4, 10^+_3, 10^+_4, 10^+_{10}$ is confirmed. Positive symmetry is verified.
5. Given that the actual value in close agreement with our theoretical value, total angular momentum of the experimentally uncertain energy (2.455, 3.265, 4.282, 5.497, 6.891, 8.447 and 10.141) MeV is confirmed but symmetry is negative., corresponding to the angular momentum $4^-_2, 5^-_1, 3^-_3, 0^-_9, 4^-_2, 8^-_5, 10^-_1, 10^-_9$.
6. According to our calculations, the maximum experimental energy value is 16.337 MeV, and the greater predicted energy is theoretically 14.888 MeV.
7. Through the theoretical calculations, we have (109) state with the total angular momentum and parity that have not symmetry by another practical value thus far.

3.2 Electromagnetic transition probability B(E2), B(M1)

Gamma rays can be regarded as a type of electromagnetic radiation characterized by fluctuations in the electric field, which consequently induce variations in the magnetic field. An oscillating charge or a varying magnetic field due to alterations in current or magnetic moment can both generate radiation. The electromagnetic transition probability for the ^{72}Kr isotope has been computed inside the nuclear shell model utilizing the f5pvh interaction. For each transition, the harmonic oscillator potential (HO, b) was employed, where $b > 0$.

We juxtaposed our findings about the electrical transitions of the interaction (f5pvh) with the experimental results and observed concordance for the transitions. B(E2) $2^+_1 \to 0^+_1$. Our computations uncovered novel transitions with Table 2 previously unobserved experimental values. There are no magnetic transitions for this isotope in this space.

3.3 Energy levels ^{74}Kr isotope

According to the nuclear shell model, the ground state for the ^{74}Kr nucleus is a ^{56}Fe nucleus closed with (28) (Np = Nn = 28), where $J^{\pi} = 0^+$ and $T = 1$ with 18 nucleons distributed as P and N in the fp–shell. The theoretical results for the ^{74}Se isotope are compared in Table 3. utilising the f5pvh interaction with the existing experimental findings [29]. This 18 nucleons are found outside the closed core by the (^{74}Kr) nucleus in the theoretical calculations, which is consistent with the results shown in Table 3 for the symmetry and total angular momentum case.

Our theoretical results utilising the f5pvh interaction can be compared with the experimental data for this isotope in the above table to see the following:

1. A comparison with the existing empirical data indicated that the ground state parity and total angular momentum of the 0^+ level were identical.
2. By comparing the theoretically computed energies (0.455 MeV; 2^+_1) with the available experimental data, we were able to get good agreement for the angular momentums and pairty.
3. The practical energies' angular momentum and parity were established (4.469 MeV; 5^+, 5.655; 5^+). This is a result of how well the practical value matches our theoretical value.
4. Given that the actual value in close agreement with our theoretical value, total angular momentum of the experimentally uncertain energy (1.742, 2.112, 3.452, 3.761, 4.244, 4.556, 5.570, 6.853,

Table 1. Shows the comparison between the excitation energy predictions for the ^{72}Kr isotope using the f5pvh interaction measured and the experimental energies [28].

Theoretical values		Experimental values	
J^{π}	E (MeV)	E (MeV)	J^{π}
0_1	0.000	0.0	0^+
2_1	0.897	0.709	2^+
2_2	1.538	——-	——-
3_1	2.073	——-	——-
4_1	2.207	1.321	4^+
4_2	2.407	2.455	(5^-)
2_3	2.967	——	——
1_1	3.027	——	——
0_2	3.064	——	——
5_1	3.247	3.265	(7^-)
2_4	3.394	——	——
4_3	3.474	——	——
0_3	3.694	——	——
3_2	3.719	——	——
4_4	3.757	3.797	——
6_1	3.892	——	——
1_2	3.895	——	——
2_5	3.984	——	——
0_4	4.101	——	——
2_6	4.147	——	——
3_3	4.256	4.282	(9^-)
0_5	4.306	——	——
2_7	4.337	——	——
4_5	4.361	——	——
2_8	4.401	——	——
1_3	4.421	——	——
3_4	4.424	——	——
4_6	4.543	——	——
3_5	4.596	——	——
6_2	4.601	——	——
5_2	4.621	——	——
4_7	4.631	——	——
3_6	4.652	——	——
2_9	4.660	——	——
1_4	4.684	——	——
2_{10}	4.728	——	——
6_3	4.751	4.756	——
0_6	4.766	——	——
1_5	4.871	——	——
1_6	4.884	——	——
1_7	4.890	——	——
4_8	4.913	——	——
5_3	4.925	——	——
3_7	4.958	——	——
1_8	5.000	——	——
0_7	5.001	——	——
5_4	5.053	——	——
4_9	5.105	——	——
6_4	5.111	——	——
3_8	5.116	——	——
4_{10}	5.139	——	——
5_5	5.186	——	——
1_9	5.246	——	——
3_9	5.250	——	——
3_{10}	5.269	——	——
1_{10}	5.346	——	——
0_8	5.373	——	——
6_5	5.458	——	——
0_9	5.490	5.497	(11^-)
7_1	5.575	——	——
5_6	5.621	——	——
0_{10}	5.756	——	——
5_7	5.763	——	——
6_6	5.846	——	——
6_7	5.864	——	——
5_8	5.932	——	——
5_9	5.936	——	——
7_2	5.941	6.048	——-
5_{10}	6.051	——	——
6_8	6.106	——	——
6_9	6.155	——	——
6_{10}	6.186	——	——
7_3	6.196	——	——
8_1	6.258	——	——
8_2	6.336	——	——
8_3	6.465	——	——
8_4	6.574	——	——
7_4	6.774	——	——
7_5	6.837	——	——
8_5	6.838	6.891	(13^-)
7_6	7.027	——	——
8_6	7.116	——	——
7_7	7.141	——	——
7_8	7.170	7.164	——-
8_7	7.214	——	——
7_9	7.242	——	——
7_{10}	7.283	——	——
9_1	7.495	——	——
9_2	7.730	——	——
8_8	7.750	——	——
8_9	7.822	——	——
9_3	7.839	——	——
8_{10}	7.923	——	——
10_1	8.294	8.447	(15^-)
9_4	8.500	8.526	(16^+)
10_2	8.568	8.608	——-
10_3	8.755	8.745	(16^+)
10_4	8.811	8.820	(16^+)
9_5	8.831	——	——
9_6	8.887	——	——
9_7	8.983	——	——
10_5	9.131	——	——
9_8	9.152	——	——
9_9	9.177	——	——
9_{10}	9.321	——	——
10_6	9.767	——	——
10_7	9.864	——	——
11_1	9.964	——	——
10_8	10.056	10.040	——
10_9	10.102	10.141	(17^-)
12_1	10.351	——	——
11_2	10.427	——	——
10_{10}	10.572	10.558	(18^+)
11_3	11.185	——	——
12_2	11.242	——	——
11_4	11.326	——	——
11_5	11.351	——	——
12_3	11.820	——	——
11_6	11.832	——	——
11_7	11.854	——	——
11_8	11.954	——	——
11_9	12.117	——	——
12_4	12.469	——	——
11_{10}	12.589	——	——
12_5	13.015	——	——
12_6	13.372	——	——
12_7	14.440	——	——
12_8	14.790	——	——
12_9	14.888	——	——

Table 2. Comparison of the B(E2)and B(M1) results by using) f5pvh (interaction in units e^{2}fm^4 and μ_N^2 respectively for the ^{72}Kr isotope with the experimental data [28].

J_i^+	→	J_f	B(M1) (μ_N^2)		B(E2) e^{2}fm^4	
			Theory	Exp.	Theory	Exp.
2_1	→	0_1	0.0000	——	310.3000	1014.1557
2_2	→	0_1	0.0000	——	3.2110	——-
2_2	→	2_1	0.0000	——	292.5000	——-
3_1	→	2_1	0.0000	——	6.2460	——-
3_1	→	2_2	0.0000	——	554.1000	——-
4_1	→	2_1	0.0000	——	417.7000	——-
4_1	→	2_2	0.0000	——	2.9260	——-
4_1	→	3_1	0.0000	——	172.7000	——-
4_2	→	2_1	0.0000	——	1.7480	——-
4_2	→	2_2	0.0000	——	116.6000	——-

8.412) MeV, corresponding to the angular momentum $2^+,4^+,1^+,2^+,4^+,1^+,3^+,0^+,7^+,9^+$, is confirmed. Positive symmetry is verified.

5. Given that the actual value in close agreement with our theoretical value, total angular momentum of the experimentally uncertain energy (2.655, 2.811, 3.005, 3.139, 3.366, 3.698, 3.840, 4.132, 4.592, 4.721, 5.086, 5.658, 5.764, 6.210, 6.874, 6.967, 7.487, 8.219, 8.898, 9.684) MeV is confirmed but symmetry is negative., corresponding to the angular momentum $4^+,0^+,2^+,4^+,5^+,3^+,6^+,0^+,1^+,6^+,8^+,7^+,7^+,8^+, 8^+,9^+,9^+,9^+$.

6. According to our calculations, the maximum experimental energy value is 14.828 MeV while the highest predicted energy value is theoretically 10.596 MeV.

7. Through the theoretical calculations, we have (73) state with the total angular momentum and parity that have not parity by another practical value thus far.

3.4 Electromagnetic transition probability B(E2), B(M1)

The electromagnetic transition probability for the ^{74}Kr isotope in the nuclear shell model has been calculated using the f5pvh interaction. For every transition, “The harmonic oscillator potential (HO, b)” was used, where b $>$ 0 (Table 4).

We observed a reasonable agreement between the available experimental data and the theoretical data for the electrical transitions B(E2) $2_1^+ \rightarrow 0_1^+$ and B(E2) $4_1^+ \rightarrow 2_2^+$ for using the (f5pvh) interaction. New transitions that had previously no experimental values were found by our computations. There are no magnetic transitions for this isotope in this space.

3.5 Energy levels ^{76}Kr isotope

Table 5 delineates a comparison between the experimental outcomes for the ^{76}Kr isotope utilizing the f5pvh interaction and the presently accessible theoretical conclusions.

Our theoretical results utilising the f5pvh interaction can be compared with the experimental data for this isotope in the above table to see the following:

1. A comparison with the available empirical data demonstrated that the ground state parity and total angular momentum of the 0^+ level were identical.

2. By comparing the theoretically computed energies (1.687 MeV; 2_1^+) with the available experimental data, we were able to get good agreement for the angular momentums, and parity , and This energy (2.926 MeV; 1) exhibits agood correlation with angular momentum, but with negative parity.

3. A favourable correlation between energy (3.636, 4.026) MeV and angular momentum was achieved, affirming positive parity.

4. The angular momentum of the values (2.970 MeV; 2, 3.672 MeV; 1) has been confirmed and their positive parity has been established.

5. According to our calculations, the maximum experimental energy value is 33.90 MeV while the highest predicted energy value is theoretically 9.595 MeV.

6. Through the theoretical calculations, we have (30) state with the total angular momentum and parity that have not parity by another practical value thus far.

7. Given that the actual value in close agreement with our theoretical value, total angular momentum of the experimentally uncertain energy (2.140, 2.601, 2.816, 2.845, 2.970, 3.242, 3.275, 3.421, 4.380, 4.403,

Table 3. Comparison of excitation energy predictions for the ^{74}Se isotope utilizing the f5pvh interaction and observed experimental energies [29].

Theoretical values		Experimental values	
J^π	E (MeV)	E (MeV)	J^π
0_1	0.000	0.0	0^+
2_1	0.957	0.455	2^+
2_2	1.837	1.742	(2^+)
4_1	2.263	2.112	(4^+)
2_3	2.272	——	——
3_1	2.322	——	——
2_4	2.487	——	——
1_1	2.570	——	——
4_2	2.582	2.655	(4^-)
0_2	2.757	——	——
0_3	2.876	2.811	(5^-)
2_5	2.879	——	——
4_3	2.887	——	——
1_2	2.934	——	——
4_4	2.990	——	——
2_6	3.029	3.005	(5^-)
3_2	3.047	——	——
4_5	3.143	3.139	(6^-)
3_3	3.235	——	——
2_7	3.269	——	——
3_4	3.279	——	——
4_6	3.354	——	——
5_1	3.389	3.366	(7^-)
1_3	3.422	3.452	(7^+)
1_4	3.515	——	——
0_4	3.574	——	——
0_5	3.646	——	——
2_8	3.651	——	——
3_5	3.692	3.698	(7^-)
2_9	3.741	3.761	(8^+)
2_{10}	3.844	——	——
6_1	3.852	3.840	(8^-)
3_6	3.864	——	——
1_5	3.936	——	——
3_7	3.950	——	——
3_8	3.997	——	——
1_6	4.047	——	——
4_7	4.063	——	——
6_2	4.085	——	——
5_2	4.133	——	——
6_3	4.139	4.132	(9^-)
0_6	4.228	——	——
4_8	4.241	——	——
5_3	4.265	——	——
1_7	4.282	4.244	(1^+)
3_9	4.309	——	——
0_7	4.395	——	——
1_8	4.413	——	——
5_4	4.467	4.469	——
3_{10}	4.572	4.556	(10^+)
0_8	4.607	4.592	(9^-)
4_9	4.608	——	——
4_{10}	4.668	——	——
1_9	4.730	4.721	(10^-)
5_5	4.852	——	——
0_9	4.977	——	——
1_{10}	4.978	——	——
6_4	4.997	——	——
6_5	5.097	5.086	(11^-)
5_6	5.109	——	——
5_7	5.130	——	——
7_1	5.163	——	——
5_8	5.256	——	——
5_9	5.416	——	——
6_6	5.499	——	——
0_{10}	5.517	5.570	(12^+)
6_7	5.619	——	——
5_{10}	5.627	5.655	——
6_8	5.631	5.658	(11^-)
8_1	5.740	5.764	(12^-)
7_2	5.841	——	——
6_9	5.916	——	——
7_3	6.164	——	——
6_{10}	6.165	——	——
7_4	6.235	6.210	(13^-)
8_2	6.335	——	——
8_3	6.398	——	——
7_5	6.453	——	——
8_4	6.508	——	——
7_6	6.612	——	——
7_7	6.709	——	——
7_8	6.800	6.853	(14^+)
7_9	6.846	6.874	(13^-)
8_5	6.955	6.967	(14^-)
7_{10}	7.004	——	——
8_6	7.290	——	——
8_7	7.483	7.487	(15^-)
9_1	7.659	——	——
8_8	7.676	——	——
9_2	7.791	——	——
8_9	7.827	——	——
10_1	7.865	——	——
8_{10}	7.888	——	——
9_3	8.269	8.219	(15^-)
9_4	8.423	8.412	(16^+)
9_5	8.577	——	——
10_2	8.663	——	——
10_3	8.745	——	——
10_4	8.793	——	——
9_6	8.879	8.898	(17^-)
9_7	8.973	——	——
9_8	9.128	——	——
9_9	9.277	——	——
9_{10}	9.602	9.684	(17^-)
10_5	10.312	——	——
10_6	10.596	——	——

Table 4. B(E2)and B(M1) results by using) f5pvh (interaction in units e^{2}fm^4 and μ_N^2 respectively for the ^{74}Se isotope and comparing with the experimental data [29].

J_i	$\rightarrow$	J_f	B(M1) (μ_N^2)		B(E2) e^{2}fm^4	
			Theory	Exp.	Theory	Exp.
2_1	$\rightarrow$	0_1	0.0000	——-	244.4000	1236.4319
2_2	$\rightarrow$	0_1	0.0000	——-	22.6400	——-
2_2	$\rightarrow$	2_1	0.0462	——-	117.6000	——-
4_1	$\rightarrow$	2_1	0.0000	——-	327.7000	2897.3106
4_1	$\rightarrow$	2_2	0.0000	——-	0.3188	——-
2_3	$\rightarrow$	0_1	0.0000	——-	14.6300	——-
2_3	$\rightarrow$	2_1	0.0462	——-	14.3300	——-
2_3	$\rightarrow$	2_2	0.0442	——-	32.6700	——-
2_3	$\rightarrow$	4_1	0.0000	——-	4.5950	——-

Table 5. Excitation energy predictions for the ^{76}Kr isotope using f5pvh interaction and observed experimental energies comparison [30].

Theoretical values		Experimental values	
J^+	E (MeV)	E (MeV)	J^π
0_1	0.000	0.0	0^+
2_1	1.495	1.687	2^+
2_2	2.285	2.140	$(1,2^+)$
4_1	2.579	2.601	$(3^-,4^+)$
3_1	2.715	2.742	(4^-)
2_3	2.846	2.816	$(1,2^+)$
4_2	2.869	2.845	(4^+)
1_1	2.907	2.926	$0^-,1^-,2^-$
2_4	2.979	2.970	$(0^+,1,2)$
0_2	3.009	——	——
4_3	3.097	3.024	$(2)^-$
3_2	3.135	——	——
2_5	3.384	3.242	$(1,2^+)$
1_2	3.433	3.275	$(1^+,2)$
0_3	3.466	3.421	$(0^+,1,2)$
4_4	3.633	——	——
5_1	3.757	3.573	(7^-)
3_3	3.758	——	——
2_6	3.773	3.636	$1,2^{(+)}$
1_3	3.944	3.672	$(0,1,2)$
0_4	4.067	——	——
2_7	4.127	4.026	$1,2^{(+)}$
3_4	4.293	——	——
1_4	4.298	4.289	$(0,1,2)^-$
4_5	4.345	——	——
6_1	4.358	4.380	(9^+)
3_5	4.413	4.403	(9^+)
2_8	4.435	4.433	(10^+)
6_2	4.485	4.469	(9^-)
0_5	4.511	——	——
2_9	4.606	——	——
5_2	4.607	——	——
5_3	4.720	4.700	(9^+)
2_{10}	4.770	——	——
4_6	4.841	4.806	(10^-)
1_5	4.910	——	——
3_6	5.122	5.106	(10^-)
4_7	5.141	——	——
3_7	5.182	——	——
1_6	5.314	——	——
4_8	5.371	——	——
0_6	5.615	——	——
3_8	5.645	——	——
5_4	5.709	——	——
4_9	5.746	——	——
4_{10}	5.799	——	——
3_9	5.877	5.873	(12^-)
1_7	5.894	——	——
6_3	5.970	——	——
3_{10}	6.125	——	——
5_5	6.197	——	——
1_8	6.422	——	——
0_7	6.505	——	——
5_6	6.731	——	——
1_9	6.940	6.937	(13^+)
0_8	7.496	7.435	(14^-)
1_{10}	7.799	——	——
0_9	9.595	——	——

Table 6. B(E2)and B(M1) results by using) f5pvh (interaction in units e^2fm^4 and μ_N^2 respectively for the ^{76}Kr isotope and comparing with the experimental data [30].

J_i^+	→	J_f	B(M1) (μ_N^2)		B(E2) e^2fm^4	
			Theory	Exp.	Theory	Exp.
2_1	→	01	0.0000	——	207.8000	1516.3906
2_2	→	01	0.0000	——	0.0635	——
2_2	→	21	0.0625	——	168.3000	——
4_1	→	21	0.0000	——	4.6830	2447.6419
4_1	→	22	0.0000	——	74.9400	——
3_1	→	21	0.0190	——	2.3360	——
3_1	→	22	0.0008	——	186.4000	——
3_1	→	41	0.3314	——	34.9400	——
2_3	→	01	0.0000	——	3.0650	——
2_3	→	21	0.0006	——	53.8600	11.6645

4.700, 6.937) MeV, corresponding to the angular momentum $2^+,4^+,2^+,4^+,2^+,1^+,0^+,6^+,3^+,2^+,5^+,1^+$ is confirmed. Positive parity is verified.

8. Given that the actual value in close agreement with our theoretical value, total angular momentum of the experimentally uncertain energy (2.742, 3.024, 3.573, 4.289, 4.469, 4.806, 5.106, 5.873, 7.435, 4.289) MeV is confirmed but parity is negative., corresponding to the angular momentum $3^+,4^+,1^+,4^+,6^+,4^+,3^+,1^+,0^+,1^+$.

3.6 Electromagnetic transition probability B(E2), B(M1)

The electromagnetic transition probability for the ^{76}Kr isotope in the nuclear shell model has been calculated using the f5pvh interaction. For every transition, “The harmonic oscillator potential (HO, b)” was used, where b > 0 (Table 6).
We observed a reasonable agreement between the available experimental data and the theoretical data for the electrical transitions B(E2) $2_1^+ \to 0_1^+$, B(E2) $4_1^+ \to 2_1^+$, B(E2) and B(E2) $2_3^+ \to 2_1^+$ for using the (f5pvh) interaction. New transitions that had previously no experimental values were found by our computations. There are no magnetic transitions for this isotope in this space.

4. Conclusion

In this work, the f5pvh interaction with a closed core (^{56}Fe) was used to compute the energy levels and electromagnetic transition probabilities within a shell model. The results were demonstrated to be somewhat consistent with the existing experimental data. Through contact, several energy levels have been confirmed, and our computations have yielded more energy levels. There was some agreement between the B(E2) and B(M1) results and the experimental results.

Authors Contribution
Authors have contributed equally in preparing and writing the manuscript.

Availability of data and materials
The data that support the findings of this study are available from the corresponding author, upon reasonable request.

Conflict of interests
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] M. G. Mayer. “On closed shells in nuclei. II.”. *Phys. Rev.*, 75:1969, 1949.

[2] R. Casten. *“Nuclear structure from a simple perspective”*, volume 23. Oxford university press, 2000.

[3] D. J. Dean, T. Engeland, M. Hjorth-Jensen, M. P. Kartamyshev, and E. Osnes. “Effective interactions and the nuclear shell-model.”. *Prog. Part. Nucl. Phys.*, 53:419–500, 2004.

[4] A. K. Hasan and A. N. Rahim. “Study of nuclear structure of 24, 26Na isotopes by using USDB interaction.”. *Int. J. Sci. Res. Publ*, 8: 169, 2018.

[5] B. A. Brown et al. “Oxbash for windows PC.”. *MSU-NSCL Rep.*, 1289, 2004.

[6] E. Caurier and F. Nowacki. “Present status of shell model techniques.”. *Acta Phys. Pol. B*, 30:705, 1999.

[7] E. Caurier and F. Nowacki. “Present status of shell model techniques.”. *Acta Physica Polonica B*, 30:705, 1999.

[8] B. A. Brown and W. D. M. Rae. “The shell-model code NuShellX@MSU.”. *Nucl. Data Sheets*, 120:115–118, 2014.

[9] A. D. Salman, S. A. Al-Ramahi, and M. H. Oleiwi. “Inelastic electron-nucleus scattering form factors for 64, 66, 68Zn isotopes.”. In *AIP Conference Proceedings*. AIP Publishing, 2019.

[10] B. H. Wildenthal. “Empirical strengths of spin operators in nuclei.”. *Prog. Part. Nucl. Phys.*, 11:5–51, 1984.

[11] A. K. Hasan. “Shell model calculations for 18, 19, 20O isotopes by using USDA and USDB interactions.”. *Ukr. J. Phys.*, 63:189, 2018.

[12] H. De Vries, C. W. De Jager, and C. De Vries. “Nuclear charge-density-distribution parameters from elastic electron scattering.”. *At. data Nucl. data tables*, 36:495–536, 1987.

[13] A. K. Hasan. “STUDY OF THE NUCLEAR STRUCTURE FOR THE 18O BY USING NuShellX@MSU CODE.”. *Nucl. Phys.*, 24: 219–224, 2023.

[14] H. H. Abed. “Study of the Nuclear Structure for Nuclei of Different States by Applying the Nuclear Shell Model Using Different Interactions.”. 2024.

[15] F. A. Berezin and M. Shubin. *“The Schrödinger Equation”*, volume 66. Springer Science & Business Media, 2012.

[16] A. Jasielska and S. Wiktor. “Application of shell model with the modified surface delta interaction to 42 Ca and 42 Sc nuclei.”. *Institute of Nuclear Physics*, 1975.

[17] E. Caurier, K. Langanke, G. Martinez-Pinedo, F. Nowacki, and P. Vogel. “Shell model description of isotope shifts in calcium.”. *Phys. Lett. B*, 522:240–244, 2001.

[18] L. Coraggio, A. Covello, A. Gargano, N. Itaco, and T. T. S. Kuo. “Shell-model calculations and realistic effective interactions.”. *Prog. Part. Nucl. Phys.*, 62:135–182, 2009.

[19] A. Gargano, L. Coraggio, A. Covello, and N. Itaco. “Realistic shell-model calculations and exotic nuclei.”. In *Journal of Physics: Conference Series*, page 12004. IOP Publishing, 2014.

[20] R. D. Lawson. *“Theory of the nuclear shell model.”*. 1980.

[21] E. M. Raheem, R. O. Kadhim, and N. A. Salman. “The effects of core polarisation on some even–even sd-shell nuclei using Michigan three-range Yukawa and modified surface delta interactions.”. *Pramana*, 92:1–11, 2019.

[22] A. Bor and B. Mottelson. *“Nuclear structure, vol. 1.”*. WA Benjamin Inc., New-York, 1969.

[23] F. Ertuğral, E. Guliyev, and A. Kuliev. “Quadrupole Moments and Deformation Parameters of the Î66-180Hf, Î80-186W and Î52-168Sm Isotopes.”. *Acta Phys. Pol. A*, 128, 2015.

[24] N. Michel, W. Nazarewicz, and M. Płoszajczak. “Continuum coupling and single-nucleon overlap integrals.”. *Nucl. Phys. A*, 794: 29–46, 2007.

[25] P. J. Brussaard and P. W. M. Glauddemans. *“Shell-Model Applications in Nuclear Spectroscopy North-Holland.”*, volume 977. Amsterdam, 1977.

[26] M. O. Waheed and F. I. Sharrad. “Description of the deformation properties of even-even 102- 106Pd isotopes.”. *Ukr. J. Phys.*, 62:757, 2017.

[27] T. Otsuka, T. Suzuki, R. Fujimoto, H. Grawe, and Y. Akaishi. “Evolution of nuclear shells due to the tensor force.”. *Phys. Rev. Lett.*, 95: 232502, 2005.

[28] D. Abriola and A. A. Sonzogni. “Nuclear Data Sheets for A = 72.”. *Nucl. Data Sheets*, 111:1–140, 2010.
DOI: https://doi.org/10.1016/j.nds.2009.12.001.

[29] B. Singh and A. R. Farhan. “Nuclear Data Sheets for A = 74.”. *Nucl. Data Sheets*, 107:1923–2102, 2006.
DOI: https://doi.org/10.1016/j.nds.2006.05.006.

[30] B. Singh. “Nuclear data sheets update for A= 76.”. *Nucl. Data Sheets*, 74:63–164, 1995.