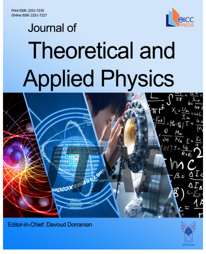

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Kelvin-Helmholtz instability in rotating dusty plasmas with sheared magnetic field and polarization force

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Original Research

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Abstract:
The impact of rotation and dust polarization force on Kelvin-Helmholtz instability (KHI) in a magnetized, velocity-sheared dusty plasma is explored in this study. The investigation involves deriving the general dispersion relation from the linearized perturbation equations. The dispersion relation is solved numerically to analyze how dust polarization force and rotation influence the critical shear and growth rate of the unstable mode emerging in dusty plasma. The findings reveal that an escalation in both dust polarization force and rotation results in an augmented critical shear required to excite the Kelvin-Helmholtz instability. Both dust polarization force and rotation are identified as having a suppressing effect on the growth rate of the K-H instability. These outcomes bear significance in the examination of the stability of planetary and stellar atmospheres, Saturn's E-ring, and laboratory dusty plasmas.

Keywords: Dusty plasma; Magnetic fields; Rotation and instabilities

1. Introduction

Most dusty plasmas in space and laboratories are far from thermodynamic equilibrium. A non-equilibrium plasma is characterized by the presence of unstable collective modes whose amplitudes grow exponentially. The studies of instabilities are of great importance because these are helpful in understanding the origin of enhanced fluctuations as well as dust voids and nonlinear dust oscillations in space and laboratory dusty plasmas. There are different types of hydrodynamic instabilities in dusty plasmas such as Rayleigh-Taylor instability (RTI) and Kelvin-Helmholtz instability (KHI). Rayleigh-Taylor instability arises due to the density gradient. If two fluids have different densities in which the heavy fluid is supported by the lighter one under the influence of gravity, then RTI develops at the interface of two fluids. The growth rate of Rayleigh-Taylor instability in thruster plasma has been investigated by several authors [1–6]. Another instability, Kelvin Helmholtz instability plays a significant role in transport and mixing properties of any medium. It has been discussed in different configurations and different kinds of fluids with velocity shear or with two different velocities at the interface. Due to velocity shear or having a sufficient relative velocity between fluid layers, the K-H instability occurs in fluids as well as in magnetised plasmas. In the course of unstable dynamics, the boundary layers can be disrupted and electric current sheets form in magnetized plasma. K-H instability is applicable to a wide variety of plasma regimes, e.g., fully ionized plasma, plasma-neutral gas systems or partially ionized dense dusty plasma consisting of electrons, ions, neutrals and heavy charged dust grains. The shear flow instability is a well-known phenomenon in fluid mechanics and astrophysics. Shear flows are of great importance for the study of dynamics of plasmas because unstable Kelvin-Helmholtz modes can be induced in boundary layers. Investigations have been focused on the impact of rotation and magnetic field on K-H instability of ideal incompressible fluids. The instability arises in a dusty plasma that flows parallel to the magnetic field with a flow velocity varying in the direction perpendicular to the magnetic field. A large number of research papers have been devoted to the study of this instability in many astrophysical environments such as Earth's aurora, planary magnetospheres protoplanetary disks [7], the magnetopause [8]. Angelo and Song studied KHI due to shear in ion flow in dusty plasma [9]. Rawat and Rao studied KHI driven by sheared dusty fluid [10]. Watson et al. have investigated the Kelvin-Helmholtz instability due to shear flow in a weakly ionized medium [11]. Dolai et al. addressed the KHI dilemma turbulence in uniformly magnetised dusty plasmas [12]. K-H instability may also occur due to the velocity shear in the dusty plasma medium formed by the moving high speed combat aircrafts [13, 14]. There are also examples where nonlinear structures may develop in dusty plasmas under the impact of dust grain density, dust polarity, ion temperature and magnetic field, though instability is expected when the required conditions are not met [15, 16]. Under strong density gradient, the waves have been found to accelerate and reflect in magnetized dusty plasmas where instabilities may also develop [17, 18].

The function of dust polarisation force in studying different waves and instabilities in nonuniform dusty plasmas has been taken into consideration [19]. Due to external electric field and density gradients in nonuniform dusty plasmas the Debye sheath around the dust particulate gets deformed or polarized. The magnitude of polarization force increases with an increase in dust charge, and also depends on the plasma density. The polarization force arises due to the plasma ion polarization around the dust grains. So one can include it in the equation of motion for dust fluid. Dust polarisation plays an essential part in the dust Debye sheath and cannot be avoided. It has been explained how polarisation force affect the growth rates and dispersion characteristics in various laboratory and astrophysical dusty plasma environments. In strongly coupled dusty plasmas, the addition of polarisation force dramatically alters the propagation of dust acoustic waves [20], dust acoustic solitary waves [21], and shock waves in strongly coupled dusty plasma [22]. The effect of polarisation force on the Jeans instability and radiative condensation instability of dusty plasmas destabilises the growth rate of instability [23–25]. Shahmansouri and Alinejad presented a theoretical investigation on the dynamics of nonlinear electrostatic waves in a strongly coupled dusty plasma with strong electrostatic interaction between dust grains in the presence of the polarization force [26]. The effect of generalized polarization force on the propagation of the dust–acoustic waves in dusty plasma has been studied by Shahmansouri and Mamun [27]. The influence of the polarization force on dust-acoustic modes has been discussed by Shahmansouri and Misra [28].

There are several astrophysical systems like accretion disks, one cannot ignore the presence of rotation. The magnetized ions exert a rotational force on the magnetized dust particles due to the collisional momentum transfer to the dust particles. Based upon the equivalence of the magnetic Lorentz force and Coriolis force on the plasma, we assume the nature of this force similar to the Coriolis force. If any part of the dust cloud experiences a rotational force, then the entire dust cloud will rotate rigidly. Dolai and Prajapati analysed the influence of dust cloud rotation on the growth rate of Rayleigh-Taylor instability [29]. The behaviour of the growth rate with respect to the suspended dust particles and components of rotation have been examined by Hoshoudy and Kumar [30]. The effect of rotation with charge fluctuation on the self-gravitational instability of dusty plasma has been studied by Vishal and Pensia [31].

Prajapati and Boro have investigated the effects of the dust polarization force on the suppression of the magnetic shear driven K-H instability in nonuniform magnetized laboratory dusty plasmas [32]. That work dealt with the problem of magnetic field driven shear flow instability without considering ion drag force and generalized expression of the polarization force. Dolai and Prajapati studied the effects of inertial ions [33], dust and ion streaming velocities dust polarization force and ion drag force on the combined K-H and dust-ion two-stream instabilities in subsonic sheared dusty plasmas flows. But they have not studied the effect of rotation along with dust polarization force on the K-H instability arising in the dusty plasma considered. So, in the present problem, we aim to study the effects of rotation and dust polarization force on Kelvin- Helmholtz instability driven by sheared flow in the magnetized dusty plasma. We consider three component magnetized dusty plasma system consisting of electrons and ions having Boltzmann distributions at temperatures T_e and T_i respectively with charged dust particles.

In section 2, we discuss the basic equations governing the plasma motion. The dispersion relation is obtained in section 3. In section 4, dispersion relation is solved numerically and discussed for magnetized dusty plasma in a typical laboratory experiment. Finally, a brief discussion is given in sec. 5.

2. Governing equations

We consider a three-component magnetized low β dusty plasma consisting of electrons, ions and charged dust particles. The dust grains are assumed to be uniform in size with constant charge $Q_d = \pm Z_d e$ ('+' is taken for positively charged grains and '-' is taken for negatively charged grains), where Z_d is the number of electrons residing on the dust grain. The charge neutrality condition $en_i = en_e - Q_d n_d$ is satisfied, where n_e, n_i, n_d are the number densities of electrons, ions and dust respectively. The homogeneous magnetic field $\mathbf{B}(0,\ B_y,\ B_z)$ is in **y** and **z** direction. The temperatures of ions and electrons are T_i and T_e respectively in thermodynamic equilibrium state. In equilibrium, the streaming velocity of the dust particles is

$$\mathbf{v}_{d0} = \mathrm{v}_{0y}\hat{y} + \mathrm{v}_{0z}(x)\hat{z} \tag{1}$$

and dust density is taken to be of the form

$$n_{d0}(x) = \tilde{n}_{d0} e^{-\lambda x}, \tag{2}$$

where λ is the inverse scale length of the density gradient. Hamaguchi and Farouki [34] introduced total force $\mathbf{F}$ exerted on a small charged grain in nonuniform plasmas as $\mathbf{F} = Q_d\mathbf{E} - \mathbf{F}_P$, where $Q_d\mathbf{E}$ is electrostatic force due to external electric field $\mathbf{E}$ and $\mathbf{F}_P$ is polarization force on charged dust grain. Mathematically, polarization force $\mathbf{F}_P$ is given by $(Q_d^2\nabla\lambda_D)/(2\lambda_D^2)$. Thus the total force on dust particle can be expressed as

$$\mathbf{F} = Q_d\mathbf{E} - \frac{Q_d^2 \nabla \lambda_D}{2\lambda_D^2}, \tag{3}$$

where

$$\lambda_D = \frac{\lambda_{De}\lambda_{Di}}{(\lambda_{De}^2 + \lambda_{Di}^2)^{\frac{1}{2}}}$$

is the linearized Debye length and $\lambda_{Di(e)}$ are the ion(electron) Debye radii defined by

$$\lambda_{Di(e)} = \left(\frac{T_{i(e)}}{4\pi e^2 n_{i(e)}}\right)^{\frac{1}{2}}.$$

Taking into account $\mathbf{E} = -\nabla\emptyset$ and $n_e T_i \ll n_i T_e$, Equation (3) reduces to

$$\mathbf{F} = -Q_d \nabla\emptyset(1-\Gamma_p),$$

where

$$\Gamma_p = \frac{1}{4}\left(\frac{|Q_d|e}{\lambda_D T_i}\right) \times \left(1 - \frac{T_i}{T_e}\right)$$

is polarization parameter. In dusty plasmas, the polarisation force acting on a charged dust grain is significant in order to study the dust modified waves and instabilities.

The dynamics of dust particles governed by the continuity equation and the momentum equation modified by polarization force and rotation are as

$$\frac{\partial n_d}{\partial t} + \nabla\cdot(n_d\mathbf{v}_d) = 0, \tag{4}$$

and

$$m_d\left[\frac{\partial \mathbf{v}_d}{\partial t} + (\mathbf{v}_d\cdot\nabla)\mathbf{v}_d\right] = -Q_d\nabla\emptyset(1-\Gamma_p) + \frac{Q_d(\mathbf{v}_d\times\mathbf{B})}{c} + 2m_d(\mathbf{v}_d\times\mathbf{R}), \tag{5}$$

where m_d, $\mathbf{v}_d$ and $\mathbf{R}$ are dust mass, dust fluid velocity, and rotation of dust grains due to coriolis force respectively. Here dust temperature and strong coupling effects are omitted because the dust pressure term is not included.

3. Dispersion relation

In order to discuss the KHI in velocity sheared dusty plasma, each physical quantity can be written as $n_j = n_{j0} + \tilde{n}_j$, j stands for e, i, d, $\mathbf{v}_d = \mathbf{v}_{d0} + \tilde{\mathbf{v}}_d$ and $\emptyset = \emptyset_0 + \tilde{\emptyset}$, where n_{j0}, $\mathbf{v}_{d0}$ and $\emptyset_0$ are equilibrium parts while $\tilde{n}_j$, $\tilde{\mathrm{v}}_d$ and $\tilde{\emptyset}$ are perturbed parts of physical quantities. Here equilibrium velocity $\mathbf{v}_{d0}$ is given by $\mathbf{v}_{d0} = (0,\ v_{0y},\ v_{0z}(x))$ and perturbed velocity $\tilde{\mathbf{v}}_d$ is given by $\tilde{\mathbf{v}}_d = (v_x,\ v_y,\ v_z)$. The electrostatic potential $\emptyset$ varies in x-direction with constant potential gradient i.e $\partial\emptyset/\partial x$ = constant. Quasi neutrality condition in equilibrium and perturbed states are given by

$$\begin{aligned} en_{i0} &= en_{e0} - Q_d n_{d0}, \\ e\tilde{n}_i &= e\tilde{n}_e - Q_d\tilde{n}_d \end{aligned} \tag{6}$$

Following Prajapati and Boro [32], we have considered electrons and ions unmagnetized and dynamics of magnetized dust modified by the polarization force. The perturbed electron (ion) number densities can be expressed as

$$\tilde{n}_{e(i)} = \tilde{n}_{e0(i0)}\left(\pm e\frac{\tilde{\emptyset}}{T_{e(i)}}\right).$$

Using the perturbed electron and ion number densities in perturbed quasineutrality condition, we obtain

$$e^2\tilde{\emptyset} = Q_d\tilde{n}_d\left(\frac{T_iT_e}{n_{i0}T_e + n_{e0}T_i}\right) \tag{7}$$

Linearization of Equations (4) and (5) gives

$$\frac{\partial\sigma}{\partial t} + \nabla\cdot\tilde{\mathbf{v}}_d - \lambda \mathrm{v}_x + \mathbf{v}_d\cdot\nabla\sigma = 0. \tag{8}$$

$$m_d\frac{\partial\tilde{\mathrm{v}}_d}{\partial t} + m_d(\mathbf{v}_{d0}.\nabla)\tilde{\mathrm{v}}_d + m_d(\tilde{\mathrm{v}}_d.\nabla)\mathbf{v}_{d0} = -Q_d\nabla\tilde{\emptyset}(1-\Gamma_p) + \frac{Q_d(\tilde{\mathbf{v}}_d\times\mathbf{B})}{c} + 2m_d(\tilde{\mathrm{v}}_d\times R) \tag{9}$$

where $\sigma = \tilde{n}_d/n_0$ is the relative amplitude of fluctuations in the dust density. Considering the small perturbed quantities to be varied as $\exp(ik_yy + ik_zz - i\omega t$ with a weak dependence in x-direction, we get the following equations from Equations (8) and (9) as

$$i\lambda\mathrm{v}_x + k_y\mathrm{v}_y + k_z\mathrm{v}_z - \Omega\sigma = 0, \tag{10}$$

$$\Omega\mathrm{v}_x - i(\omega_{cd1} + 2R)\mathrm{v}_y + i\omega_{cd2}\mathrm{v}_z = 0, \tag{11}$$

$$i(\omega_{cd1} + 2R)\mathrm{v}_x + \Omega\mathrm{v}_y - \frac{Q_d}{m_d}(1-\Gamma_p)k_y\tilde{\emptyset} = 0, \tag{12}$$

$$i(\frac{\partial\mathrm{v}_{0z}}{\partial x} + \omega_{cd2})\mathrm{v}_x + \Omega\mathrm{v}_z - \frac{Q_d}{m_d}(1-\Gamma_p)k_z\tilde{\emptyset} = 0, \tag{13}$$

where $\Omega = \omega - k_y\mathrm{v}_{0y} - k_z\mathrm{v}_{0z}$ is the Doppler shifted frequency, ω is the perturbation frequency and k_y, k_z are the wavenumbers in y and z–directions. The symbols $\omega_{cd1} = (Q_dB_z)/(Cm_d)$ and $\omega_{cd2} = (Q_dB_y)/(Cm_d)$ are the dust cyclotron frequencies in Gaussian unit.

Substituting the value of $\tilde{\emptyset}$ in Equations (12) and (13), we get

$$i(\omega_{cd1} + 2R)\mathrm{v}_x + \Omega\mathrm{v}_y - C_{DA}^2(1-\Gamma_p)k_y\sigma = 0, \tag{14}$$

$$i(\frac{\partial\mathrm{v}_{0z}}{\partial x} + \omega_{cd2})\mathrm{v}_x + \Omega\mathrm{v}_z - C_{DA}^2(1-\Gamma_p)k_z\sigma = 0, \tag{15}$$

where

$$C_{DA} = \left(\frac{z_d^2 n_{d0}T_iT_e}{m_d(n_{i0}T_e + n_{e0}T_i)}\right)^{\frac{1}{2}}$$

is the phase speed of dust acoustic wave (DAW). Equations (10), (11), (14) and (15) can be written in a matrix form as

$$DX = 0 \tag{16}$$

where D is a 4×4 square matrix and X is a 4×1 column matrix, given by

$$D = \begin{bmatrix} -i\lambda & -k_y & -k_z & \Omega \\ \Omega & -i(\omega_{cd1}+2R) & i\omega_{cd2} & 0 \\ i(\omega_{cd1}+2R) & \Omega & 0 & -Ak_y \\ i(\frac{\partial v_{0z}}{\partial x}+\omega_{cd2}) & 0 & \Omega & -Ak_z \end{bmatrix}$$

and

$$X = \begin{bmatrix} v_x \\ v_y \\ v_z \\ \sigma \end{bmatrix}$$

Non-trivial solution of it can be obtained by putting det $D = 0$, which gives the following dispersion relation:

$$\Omega^4 - \left[AK^2 + (\omega_{cd1}+2R)^2 - \omega_{cd2}\left(\frac{\partial v_{0z}}{\partial x}+\omega_{cd2}\right)\right]\Omega^2 + A\lambda[k_y(\omega_{cd1}+2R) - k_z\omega_{cd2}]\Omega + A\left[k_z^2(\omega_{cd1}+2R)^2 - k_yk_z(\omega_{cd1}+2R)\frac{\partial v_{0z}}{\partial x} - k_y^2\omega_{cd2}\left(\frac{\partial v_{0z}}{\partial x}+\omega_{cd2}\right)\right] = 0 \tag{17}$$

where $A = C_{DA}^2(1-\Gamma_p)$ and $K^2 = k_y^2 + k_z^2$. If $B_y = B\cos\theta$ and $B_z = B\sin\theta$ then $\omega_{cd1} = \omega cd\sin\theta$ and $\omega_{cd2} = \omega cd\cos\theta$, where θ is the angle between the direction of motion and y axis in $y-z$ plane. Now the dispersion relation (17) can be rewritten as

$$\Omega^4 - \left[AK^2 + (\omega_{cd}\sin\theta+2R)^2 - \omega_{cd}\cos\theta\left(\frac{\partial v_{0z}}{\partial x} + \omega_{cd}\cos\theta\right)\right]\Omega^2 + A\lambda[k_y(\omega_{cd}\sin\theta+2R) - k_z\omega_{cd}\cos\theta]\Omega + A\left[k_z^2(\omega_{cd}\sin\theta+2R)^2 - k_yk_z(\omega_{cd}\sin\theta+2R)\frac{\partial v_{0z}}{\partial x} - k_y^2\omega_{cd}\cos\theta\left(\frac{\partial v_{0z}}{\partial x}+\omega_{cd}\cos\theta\right)\right] = 0 \tag{18}$$

We observe from the Equation (18) that rotation of dust cloud, sheared magnetic field, dust polarisation parameter, and sheared flow have impact on the dispersion relation for K-H instability. For the laboratory dusty plasmas, the value of the polarisation interaction parameter is in between 0 and 1 i.e. $0 < \Gamma_p < 1$. If we consider the sheared flow $\partial v_{0z}/\partial x = 0$ then Equation (18) reduces to the dispersion relation which gives the DAW mode in magnetized dusty plasma.

If we consider the approximations $\Omega^2 \ll \omega_{cd}^2$, $K^2C_{DA}^2(1-\Gamma_p) \ll \omega_{cd}^2$ in Equation (18), we obtain

$$\left[\left(\sin\theta+\frac{2R}{\omega_{cd}}\right)^2 - \cos\theta(s+\cos\theta)\right]\varphi^2 - \Lambda(1-\Gamma_p)\left[\beta\left(\sin\theta+\frac{2R}{\omega_{cd}}\right) - \gamma\cos\theta\right]\varphi - (1-\Gamma_p)\left[\gamma^2\left(\sin\theta+\frac{2R}{\omega_{cd}}\right)^2 - \beta\gamma\left(\sin\theta+\frac{2R}{\omega_{cd}}\right)s + \beta^2\cos\theta(s+\cos\theta)\right] = 0 \tag{19}$$

where

$$\varphi = \frac{\Omega}{\omega_{cd}}, \quad \psi = \frac{C_{DA}}{\omega_{cd}}, \quad \Lambda = \lambda\psi$$
$$\beta = k_y\psi, \quad \gamma = k_z\psi, \quad \text{and} \quad s = \frac{1}{\omega_{cd}}\frac{\partial v_{0z}}{\partial x}$$

If $\theta = 90°$, then Equation (19) becomes

$$\left(1+\frac{2R}{\omega_{cd}}\right)\varphi^2 - \Lambda(1-\Gamma_p)\beta\varphi - (1-\Gamma_p)\gamma\times\left[\gamma\left(1+\frac{2R}{\omega_{cd}}\right) - \beta s\right] = 0 \tag{20}$$

In the absence of Rotation ($R = 0$), the dispersion relation (20) reduces to the dispersion relation obtained by Prajapati and Boro [32]. If we take polarization force ($\Gamma_p = 0$) then the dispersion relation (20) reduces to the dispersion relation obtained by D'Angelo and Song [9]. In absence of rotation and polarisation parameters the dispersion relation (20) is also identical to the dispersion relation obtained by Rawat and Rao [10]. Thus, we can say that the presence of rotation in magnetised dusty plasma with polarization force influences the dispersion relation for K-H instability significantly.

The roots of quadratic Equation (20) can be obtained as

$$\varphi_{1,2} = \frac{\Lambda(1-\Gamma_p)\beta}{2(1+\frac{2R}{\omega_{cd}})} \pm \frac{\sqrt{\Lambda^2\beta^2(1-\Gamma_p)^2 + 4(1+\frac{2R}{\omega_{cd}})(1-\Gamma_p)\gamma[\gamma(1+\frac{2R}{\omega_{cd}}) - \beta\, s]}}{2(1+\frac{2R}{\omega_{cd}})} \tag{21}$$

The roots will be complex in nature if the discriminant part is less than zero. Thus critical shear to excite the K-H instability is given by

$$s > \frac{\gamma}{\beta}(1+\frac{2R}{\omega_{cd}})\left[1+\frac{\Lambda^2\beta^2}{4\gamma^2(1+\frac{2R}{\omega_{cd}})^2}(1-\Gamma_p)\right] \tag{22}$$

This is an onset criteria for the K-H instability, which explicitly depends on the rotation (R) and dust polarisation interaction parameter (Γ_p) in magnetised sheared dusty plasmas. Depending on the dust charge and size, the critical value S_{crit} becomes

$$S_{crit} = \frac{\lambda}{2\omega_{cd}}(1+\zeta)\left(\frac{Z_dT_i}{m_d}\frac{\delta}{1+\eta(1-\delta)}\right)^{\frac{1}{2}}\left[1+\frac{1}{(1+\zeta)^2}(1-\Gamma_p)\right] \tag{23}$$

where $\delta = (z_dn_{d0})/n_{i0}$ is the dust number density to ion number density ratio in equilibrium, $\eta = T_i/T_e$ is the ion to electron temperature ratio, $\zeta = 2R/\omega_{cd}$ is the rotation parameter and $\gamma/\beta = \lambda\psi/2$. In many laboratory dusty plasma experiments, generally $T_i \ll T_e$ i.e. $\eta \ll 1$. Here S_{crit} is the minimum value of shear velocity required to excite the K-H instability in rotating sheared dusty plasma with dust polarisation force.

4. Results and discussion

Equation (23) shows that the critical shear S_{crit} depends upon the dust-ion number density ratio (δ), polarisation parameter (Γ_p), dust charge (Z_d), ion/electron temperature ratio (η) and rotation parameter(ζ). We observed that the system is K-H unstable if $S > S_{crit}$. In laboratory experiments, we take the typical dusty plasma parameters as $T_i = 0.03$ eV, $T_e = 3$ eV, $a = 1$ μm, and $z_d = 10^3$, $n_{d0}/n_{i0} = 10^{-4}$, $\Gamma_p = 0.12$, $n_{i0} = 3.7 \times 10^8$ cm^{-3}, $m_d = 10^{-9}$ g, and $B = (1-4) \times 10^4$ G [20, 35, 36]. The above values of parameters satisfy the assumptions $T_e \gg T_i$ and $T_e n_{i0} \gg T_i n_{e0}$. Taking into account these assumptions, the phase speed of dust acoustic wave C_{DA} and normalised critical shear S_{crit} reduce to

$$C_{DA} = z_d \left(\frac{T_i}{m_d}\right)^{\frac{1}{2}} \left(\frac{n_{d0}}{n_{i0}}\right)^{\frac{1}{2}}$$

and

$$S_{crit} = \frac{\lambda C_{DA}}{2\omega_{cd}}(1+\zeta)\left[1 + \frac{1}{(1+\zeta)^2}(1-\Gamma_p)\right]$$

The calculated dust acoustic speed C_{DA} is ≈ 7 cm/s, which is approximately the same as observed in the experimental findings of Barkan et.al. [37]. It has been determined that the dust cyclotron frequency ω_{cd} is approximately 6.4×10^{-4} s^{-1}. If we take $\lambda \sim 1$ (μm)$^{-1}$, we get $S_{crit} \approx 1.2517$, which is greater than the value 1.0281 obtained by Prajapati and Boro [32]. This is due to the inclusion of rotation along with polarization force. It is worth mentioning that the value of S_{crit} in our case is greater than the value of S_{crit} obtained by Prajapati and Boro [32]. To study the effect of rotation on the variation of S_{crit} with inverse scale length λ has been shown in Figures 1, 2, and 3; and the variation of normalized growth rate of instability with normalized perturbation wave number has been shown in Figure 4. In Figure 1, the critical shear S_{crit}, is displayed against the inverse scale length of the density gradient (λ) for polarization parameter $\Gamma_p = 0.0$, 0.3 and 0.6, rotation parameter $\zeta = 0.0$, 0.4 and 1.2, $\omega_{cd} = 6.4 \times 10^{-4}$ s^{-1} and $C_{DA} = 7$ cm/s.

Figure 1 shows that the critical shear increases as λ increases. Critical shear decreases with the increase in dust polarisation parameter and attains larger values in the absence of dust polarisation force. It is observed that inclusion of rotation influences the onset of the K-H instability as the S_{crit} in our case is slightly larger than that obtained in previous study carried out by Prajapati and Boro [32]. The graphs of upper, middle and lower panels of Figure 1 show a slight linear increase in S_{crit} with the increase in rotation parameter ζ and dust polarisation parameter Γ_p. This increment in S_{crit} shows a delay in trigger of K-H instability of the system in comparison to the previous study done by

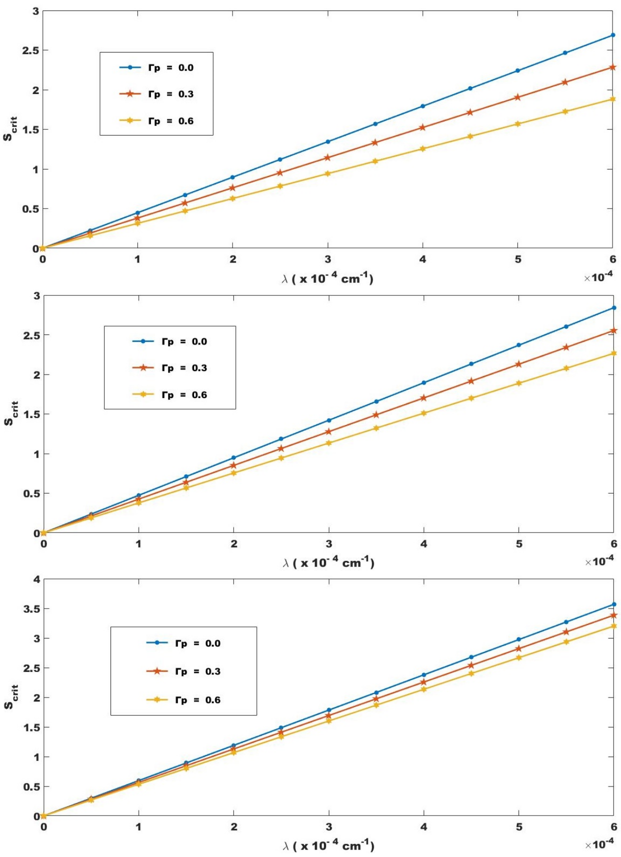

Figure 1. Normalized critical shear (S_{crit}) vs inverse scale length (λ) for $\Gamma_p = 0.0$, 0.3, and 0.6, $C_{DA} = 7$ cm/s, $\omega_{cd} = 6.4 \times 10^{-4}$ s^{-1} and $\zeta = 0.0$ (upper panel), $\zeta = 0.4$ (middle panel) and $\zeta = 1.2$ (lower panel).

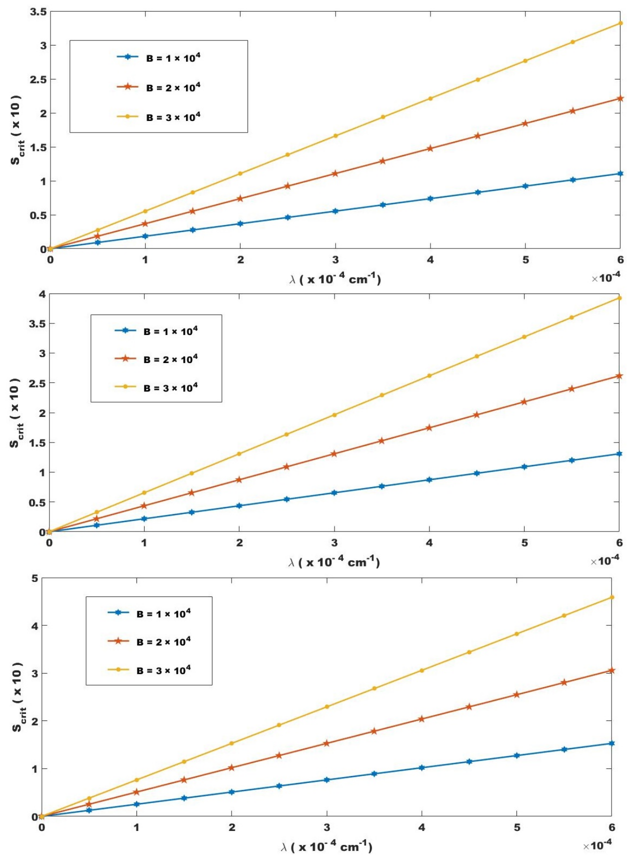

Figure 2. Normalized critical shear (S_{crit}) vs inverse scale length (λ) for magnetic field $B = 1 \times 10^4$, 2×10^4 and 3×10^4 G, $C_{DA} = 7$ cm/s, $\Gamma_p = 0.5$ and $\zeta = 0.4$ (upper panel), $\zeta = 0.8$ (middle panel) and $\zeta = 1.2$ (lower panel).

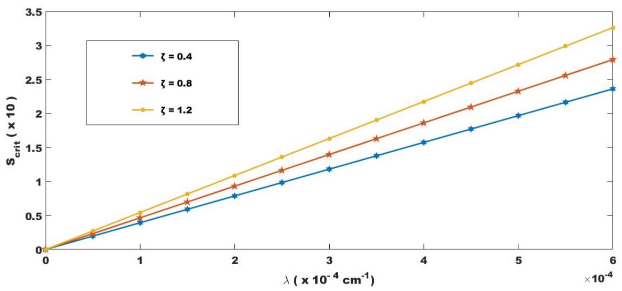

Figure 3. Normalized critical shear (S_{crit}) vs inverse scale length (λ) for rotation parameter $\zeta = 0.4$, 0.8 and 1.2, $C_{DA} = 7$ cm/s, $\Gamma_p = 0.3$ and $\omega_{cd} = 6.4 \times 10^{-4}$ s^{-1}.

Prajapati and Boro [32].

In Figure 2, normalized critical shear S_{crit} is plotted as a function of inverse scale length (λ). The graphs are plotted for different values of magnetic field $B = 1 \times 10^4$, 2×10^4 and 4×10^4 G, $C_{DA} = 7$ cm/s, and $\Gamma_p = 0.5$ by taking different values of rotation parameter $\zeta = 0.4$, 0.8, and 1.2. It is observed from the linear pattern of upper, middle and lower panels of Figure 2 that the critical shear increases with increase in magnetic field as well as rotation parameter.

In Figure 3, the variation of normalized critical shear S_{crit} with inverse scale length λ for different values of rotation parameter $\zeta = 0.4$, 0.8, and 1.2, $C_{DA} = 7$ cm/s, $\Gamma_p = 0.5$ and $\omega_{cd} = 6.4 \times 10^{-4}$ s^{-1} is shown. Fig. 3 shows that critical shear S_{crit} increases with increases in rotation parameter and inverse scale length λ.

Figure 4 shows the normalized growth rate of the K-H instability, $Img(\varphi)$ versus the normalized perturbation wavenumber $\gamma = k_z\psi$ for $\Gamma_p = 0.0$, 0.4, and 0.8 with parameters $S = 1.2517$, $\beta = 0.5$, and $\Lambda = 0.3$. In all three parabolic profile patterns, we observe that with the increase of the polarisation parameter Γ_p from 0.0 to 0.8, the growth rate of the K-H instability first increases to attain its maximum and then gradually decreases to become zero. The upper panel of Figure 4 shows that growth rate of the K-H instability increases up to $\gamma = 0.25$, decreases up to $\gamma = 0.54$, and becomes zero at perturbation wavenumber $\gamma = 0.55$. In the middle panel of Figure 4, the growth rate of the K-H instability increases up to $\gamma = 0.20$, decreases up to $\gamma = 0.42$, and becomes zero at perturbation wavenumber $\gamma = 0.43$, and in lower panel the growth rate of the K-H instability increases up to $\gamma = 0.15$, decreases up to $\gamma = 0.34$, and becomes zero at perturbation wavenumber $\gamma = 0.35$. This phenomena indicates that the system is stable after these critical values of perturbation wavenumber $\gamma = 0.55$, 0.43, 0.35. Furthermore, all the panels of Figure 4 show that the growth rate of K-H instability decreases with an increase in the polarization force at fixed rotation parameter $\zeta = 0.4$, 0.8, and 1.2. Comparing the findings of our analysis with the result of Prajapati and Boro [32], it is important to note that the growth rate of the instability $Img(\varphi)$ decreases in the presence of rotation.

5. Conclusion

In this study, we have delved into the investigation of Kelvin-Helmholtz (K-H) instability in magnetized sheared

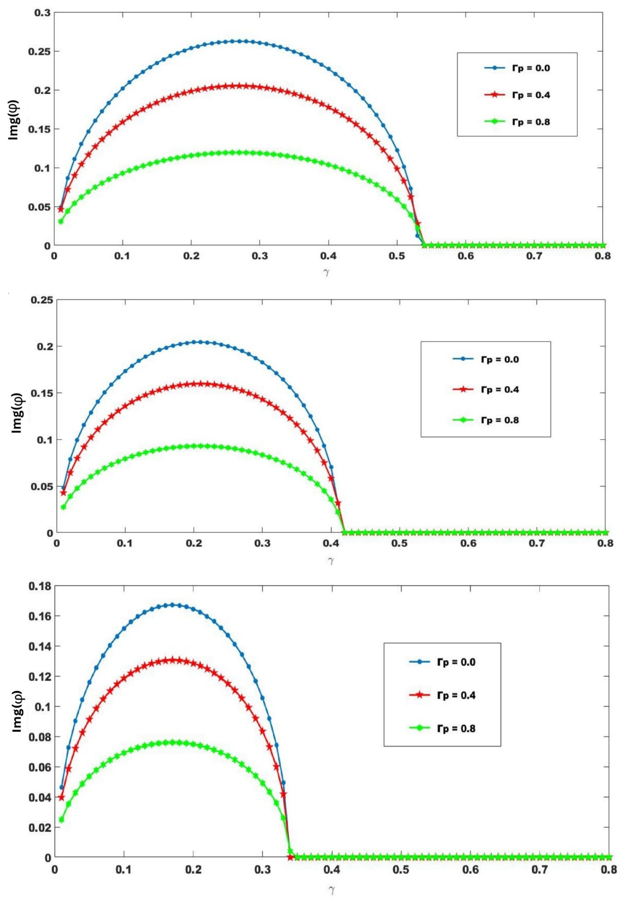

Figure 4. The growth rate of the K-H instability $Img(\varphi)$ versus normalized perturbation wavenumber $\gamma = k_z\psi$ for different values of polarization parameter $\Gamma_p = 0.0$, 0.4 and 0.8, $\beta = 0.5$, $\Lambda = 0.3$, $S = 1.2517$ and $\zeta = 0.4$ (upper panel), $\zeta = 0.8$ (middle panel) and $\zeta = 1.2$ (lower panel).

dusty plasma, taking into account the influence of dust polarization force and rotation. Our findings emphasize the substantial impact of rotation and dust polarization parameters on the critical shear and growth rate of the K-H instability.

It is found that the critical shear S_{crit} decreases with the increase in polarization force for different values of rotation parameter $\zeta = 0.0$, 0.4, and 1.2 whereas it increases with the augmentation in the rotation parameter for different values of polarisation force parameter $\Gamma_p = 0.0$, 0.3 and 0.6.

The critical shear S_{crit} increases with the rise in magnetic field for different values of rotation parameter $\zeta = 0.4$, 0.8,and 1.2. Additionally, it also escalates with the increase in the rotation parameter for different values of magnetic field $B = 1 \times 10^4$ G, 2×10^4 G, and 3×10^4 G.

It is observed that the growth rate of K-H instability decreases with the increase in polarisation force at constant rotation parameters $\zeta = 0.4$, 0.8, and 1.2. From this study, it can be concluded that the growth rate of K-H instability in the presence of rotation is lower than that obtained without rotation, and system tends to stabilize at lower values of the perturbation wavenumber γ for increasing values of the rotation parameter.

The present study might be applicable to study the possible K-H instability arising due to velocity shear in dusty plasma formed by moving aircrafts like Eurofighter Typhoon, Dassault Rafale, F16 and F22 combat aircrafts [13, 14]. Moreover, such instabilities are expected to develop in the plasma plume of a magnetic nozzle where magnetic field structure and velocity shear play a vital role [38–41].

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Authors Contributions
Harendra Kumar Singhal and Nagendra Kumar, played equal roles in conceptualizing, designing the study, drafting and critical revision of the manuscript. Meenakshi Yadav significantly contributed to the conception and design of the research work. The final manuscript has been reviewed and approved by all authors. The order of authorship reflects a collective decision, considering the substantial contributions of each author to the overall research project.

Availability of data and materials
The data and materials will be made available on request.

Conflict of Interests
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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