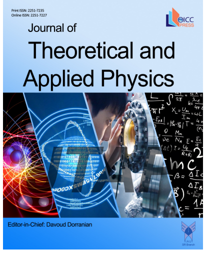

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Surface plasmon polariton dispersion relation with a graded-permittivity plasmonic medium

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Original Research

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Abstract:

In this paper, the Surface Plasmon Polariton (SPP) dispersion relation in the TM polarization for the plane interface between a homogeneous dielectric medium (with constant ε_d) and a graded-permittivity (i.e. nonhomogeneous) plasmonic medium (with dielectric function $\varepsilon_p(z)$) is obtained where 'z' is normal coordinate to the interface. For small variation of $\varepsilon_p(z)$ it is revealed that the dispersion relation is related to the derivative of $\varepsilon_p(z)$ at the interface and when ε_p becomes constant or when its derivative at the interface is zero the well-known dispersion relation for homogeneous plasmonic medium is obtained. For the case where $\varepsilon_p(z)$ depends linearly on z, the dispersion relation is obtained. It is seen that two branches appear. The upper branch has a free-space-like behavior at small wavenumbers. Inclusion of frequency dependence in the $\varepsilon_p(z)$ through Drude model, reveals that the upper branch has negative-value group velocity at intermediate wavenumbers. The results show that non-homogeneity of the plasmonic medium offers a good tunability to control dispersion relation and SPP group velocity.

Keywords: Surface plasmon polariton; Dispersion relation; Nonhomogeneous plasmonic medium; Group velocity

1. Introduction

A surface electromagnetic wave is a nearly two-dimensional wave that is confined at the interface between two different media and is excited and propagated there. Surface Plasmon Polaritons (SPPs), surface Dyakonov waves and surface waves with photonic crystals (e.g. Tamm SPPs) are some of such surface electromagnetic waves [1–3]. Practical uses of surface waves and especially SPPs are very well-known in the fabrication of detectors and sensors, implementation of nonlinear optics experiments and also development of optical information, manipulation and transportation devices [4–7].

Most of the theoretical works regarding SPPs are related to a plane interface between two semi-infinite dielectric and plasmonic media which both are homogeneous and linear [7–10]. In this simple case, it is shown that using a proper polarization (mostly TM-polarized light) with a correct dispersion relation through a suitable phase-matching configuration, the SPP wave can be excited. Inclusion of non-homogeneity, anisotropy, nonlinearity and loss in the dielectric or plasmonic medium can lead to complexity of SPP excitation and its dispersion relation and implementation of phase-matching conditions [1, 11–13]. Many of such works are about the anisotropy of media and one can find few reports regarding the non-homogeneity of the plasmonic or dielectric media. It is known that roughness of the interface (especially engineered corrugation or indentation) can have deep effects on the SPP properties [1, 2, 14]. Similarly, it seems that the non-homogeneity of media permittivity near the interface may have interesting effects on SPP characteristics. The most famous work regarding to this subject is the paper of Kaw and McBride dating back nearly to 55 years ago [15]. For a plasmonic medium with linear dependence of $\varepsilon_p(z)$ on 'z', they calculated the dispersion relation analytically only for two limiting cases of $k_x \to 0$ and $k_x \to \infty$. For the intermediate k_x they couldn't find the relation and used extrapolation to complete dispersion curve. Beside this old paper, there are many new papers that are interested in plasmonic property in nonhomogeneous media. Kruger et al. [16] have reported the dispersion relation of SPP and its characteristics considering a linearly graded permittivity function at a thin (a few tens of nano-meter) transition layer between the dielectric and the plasmonic media. They found that presence of such a sharp (but not abrupt) jump in 'ε' at the interface leads to increased confinement of SPP wave and, hence, increased propagation loss. Similarly, Cada et al. [17] have also reported the dispersion relation of SPP considering a linear non-homogeneity. They assumed that the dielectric function linearly changes form a fixed value at the dielectric medium to another constant value at the plasmonic (a doped semiconductor) side. They showed that such transition layer with a micrometer length can have strong effects on the dispersion relation especially SPP resonance frequency and appearance of negative-value and zero-value SPP group velocities. In both of these out-standing reports, the permittivity changes linearly and continuously from a constant dielectric-side value to a constant plasmonic-side one. In the present paper, no such continuous change is considered. It is assumed that the dielectric medium is homogenous (with constant ε_d) but the plasmonic medium has a z-dependent ('z' is normal coordinate to interface) dielectric function ($\varepsilon_p(z)$). The main difference of our work is that the dielectric function has discontinuity at the interface. With such assumption the dispersion relation of SPP and its characteristics for a general case and also for a Drude model frequency-dependence is considered.

The paper is organized as follows: First, the SPP dispersion relation is obtained. Then, a numerical analysis is applied to the dispersion relation. At the last section, the conclusions are drawn.

2. Derivation of dispersion relation

In order to study SPPs with a nonhomogeneous plasmonic medium, two semi-infinite media one with a real and positive relative dielectric constant ε_d and the other with negative dielectric function $\varepsilon_p(z)$ are considered. Both media are linear, isotropic, lossless and nonmagnetic. A schematic of the media along with the dielectric function profile are shown in Fig. 1.

As is seen form Fig. 1, the interface is indeed the xy plane. To study the SPP modes, it is better to firstly find the form of electromagnetic fields in the nonhomogeneous medium. The wave equations for the z-dependent nonhomogeneous medium for TM polarization (i.e. $H_x = 0$, $E_y = 0$ and $H_z = 0$) have the following forms [18]:

$$\nabla^2\mathbf{E}(x,z) + \frac{\omega^2}{c^2}\varepsilon_p(z)\mathbf{E}(x,z) + \boldsymbol{\nabla}\left[\frac{E_z(x,z)}{\varepsilon_p(z)}\left(\frac{d\varepsilon_p(z)}{dz}\right)\right] = 0 \tag{1}$$

$$\nabla^2\mathbf{H}(x,z) + \frac{\omega^2}{c^2}\varepsilon_p(z)\mathbf{H}(x,z) - i\omega\varepsilon_0\left(\frac{d\varepsilon_p(z)}{dz}\right)E_x(x,y)\hat{y} = 0 \tag{2}$$

Using the technique of variable separation, it not so hard to show that the x-dependent behavior of the above equations is in the form of $e^{(ik_xx)}$. This means that the general solutions of the above equations are of the form $h(z)e^{(ik_xx)}$.

Based on the above treatment and also using the known solutions of the electromagnetic wave equations for a homogeneous medium, the total form of electromagnetic fields can be constructed. Electric and magnetic fields in the dielectric and plasmonic media for the SPP wave propagating

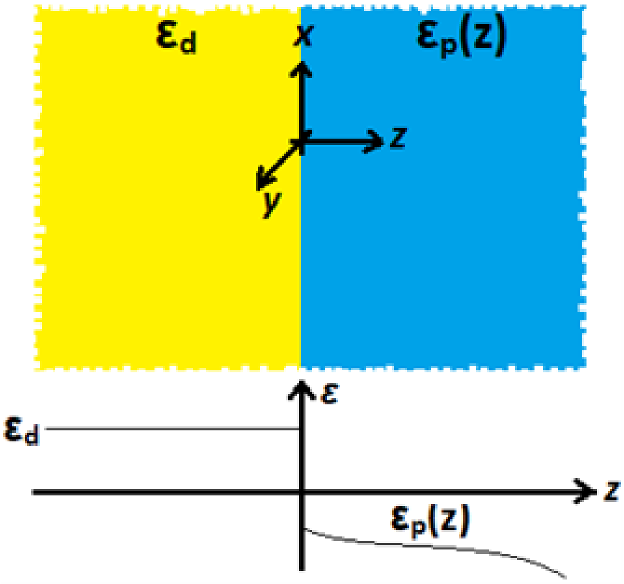

Figure 1. Two semi-infinite media (one is dielectric and the other one is plasmonic) along with the profile of dielectric function.

along positive x-axis are considered as follows:

$$\mathbf{E}(x,z) = \begin{cases} (E_{x0}, 0, E_{z0})e^{ik_xx}e^{\alpha_d z}, & \text{if } z < 0 \\ \left(E_{x0}f_1(z), 0, \frac{\varepsilon_d}{\varepsilon_p(0)}E_{z0}f_2(z)\right)e^{ik_xx}, & \text{if } z > 0 \end{cases} \tag{3}$$

$$\mathbf{H}(x,z) = \begin{cases} (0, H_{y0}, 0)e^{ik_xx}e^{\alpha_d z}, & \text{if } z < 0 \\ (0, H_{y0}, 0)e^{ik_xx}g(z), & \text{if } z > 0 \end{cases} \tag{4}$$

where k_x is the wave-vector along x-axis and α_d is damping constant along negative z-axis. E_{x0}, E_{z0} and H_{y0} are fields amplitudes. $\varepsilon_p(0)$ is permittivity value of the plasmonic medium at the interface and $f_1(z)$, $f_2(z)$ and $g(z)$ are dimensionless unknown descending functions for plasmonic side (i.e. $z > 0$). Continuity of the tangential components of electric and magnetic fields at the interface implies that:

$$f_1(0) = f_2(0) = g(0) = 1 \tag{5}$$

Application of Maxwell's first equation (i.e. the Ampere's law) to related fields of both the media in Eqs. 3 and 4 yields:

$$\alpha_d H_{y0} = i\omega\varepsilon_d\varepsilon_0 E_{x0} \tag{6}$$

$$k_x H_{y0} = -\omega\varepsilon_d\varepsilon_0 E_{z0} \tag{7}$$

$$\frac{dg(z)}{dz} = \frac{\alpha_d\varepsilon_p(z)}{\varepsilon_d}f_1(z) \tag{8}$$

$$g(z) = \frac{\varepsilon_p(z)}{\varepsilon_p(0)}f_2(z) \tag{9}$$

Application of Maxwell's second equation (i.e. Faraday's law) to the fields of plasmonic medium and using Eq. 9 reads:

$$\frac{df_1(z)}{dz} = \frac{\varepsilon_d}{\alpha_d}\left(\frac{k_x^2}{\varepsilon_p(z)} - \frac{\omega^2}{c^2}\right)g(z) \tag{10}$$

where $c = 1/\sqrt{\varepsilon_0\mu_0}$ is the light speed at empty space. Equations 8 and 10 are two coupled differential equations to find $f_1(z)$ and $g(z)$ which their solution is dependent on explicit form of $\varepsilon_p(z)$. Here, our main purpose is to find the dispersion relation and therefore we do not pay attention to finding of $f_1(z)$ and $g(z)$.
Application of Maxwell's fourth equation (i.e. Gauss' law) to the electric field of plasmonic medium in Eq. 3 yields:

$$\frac{df_2(z)}{dz}|_{z=0} = \alpha_d\frac{\varepsilon_p(0)}{\varepsilon_d} - \frac{1}{\varepsilon_p(0)}\frac{d\varepsilon_p(z)}{dz}|_{z=0} \quad (11)$$

Evaluation of Eq. 10 in $z = 0$ and using Eq. 5 leads to:

$$\frac{df_1(z)}{dz}|_{z=0} = \frac{\varepsilon_d}{\alpha_d}\left(\frac{k_x^2}{\varepsilon_p(0)} - \frac{\omega^2}{c^2}\right) \quad (12)$$

For small variations of $\varepsilon_p(z)$, the Eikonal approximation $\frac{df_2(z)}{dz}|_{z=0} \cong \frac{df_1(z)}{dz}|_{z=0}$ can be adopted [18]. In this case, Equations 11 and 12 lead to the following dispersion relation for the plasmonic medium:

$$\alpha_d^2\left(\frac{\varepsilon_p(0)}{\varepsilon_d}\right)^2 - \frac{\alpha_d}{\varepsilon_d}\frac{d\varepsilon_p(z)}{dz}|_{z=0} = k_x^2 - \varepsilon_p(0)\frac{\omega^2}{c^2} \quad (13)$$

On the other hand, the dispersion relation for the homogenous dielectric medium is:

$$\alpha_d^2 = k_x^2 - \varepsilon_d\frac{\omega^2}{c^2} \quad (14)$$

which is directly justifiable by inserting the electric field of Eq. 3 for $z < 0$ in Eq. 1 and letting $\varepsilon_p(z) = \varepsilon_d$. The above relation can also be easily verified from Eq. 13 with $\varepsilon_p(z) = \varepsilon_d$. Elimination of α_d between Eq. 13 and Eq. 14 yields the following final dispersion relation:

$$\omega = \sqrt{c^2k_x^2\frac{\varepsilon_d + \varepsilon_p(0)}{\varepsilon_d\varepsilon_p(0)} + \gamma} \quad (15)$$

where:

$$\gamma = \frac{c\sqrt{c^2k_x^2 - \varepsilon_d\omega^2}}{\varepsilon_p(0)(\varepsilon_d - \varepsilon_p(0))}\frac{d\varepsilon_p(z)}{dz}|_{z=0} \quad (16)$$

For a constant ε_p which has $d\varepsilon_p/dz = 0$ and hence $\gamma = 0$, the dispersion relation 15 is simplified to the familiar plasmonic dispersion relation [1, 2, 8]. Equation 15 says that for a nonhomogeneous plasmonic medium the dispersion relation of SPP is modified by γ. It is very interesting that γ is dependent only on the first derivative of $\varepsilon_p(z)$ at the interface and there is no dependence on the detailed behavior of $\varepsilon_p(z)$ within the body of plasmonic medium (of course with the Eikonal approximation mentioned above). It means that, for a nonhomogeneous plasmonic medium, the modification of dispersion relation depends only on the first derivative of $\varepsilon_p(z)$ at $z = 0$. In other words, media of different $\varepsilon_p(z)$ but with equal first derivative at $z = 0$ all have the same dispersion relations. Perhaps, this dependence on derivative at the interface and not on detailed functionality of $\varepsilon_p(z)$ originates from the fact that a surface wave (i.e. SPP) penetrates a very small length into the plasmonic medium (i.e. skin depth). It means that the explicit form of $\varepsilon_p(z)$ beyond the skin depth does not matter on the dispersion relation. In the other words, the important components of the nonhomogeneous $\varepsilon_p(z)$ must be only its value and its first derivative at the interface as derived above. Rather similar behavior has also been reported for surface waves in half-space gaseous plasmas [15, 19].

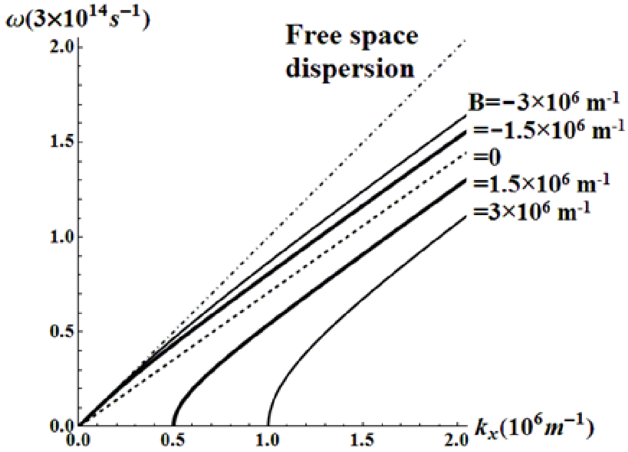

Figure 2. Dispersion plots from Eq. 17 for different values of 'B' in Eq. 18.

Eqs. 15 and 16 yield the following frequency-separated equation:

$$\omega = c\sqrt{\frac{Q_1 + Q_2}{2(\varepsilon_d - \varepsilon_p(0))^2\varepsilon_d\varepsilon_p(0)^2}} \quad (17)$$

where

$$Q_1 = -\acute{\varepsilon}_p(0)^2\varepsilon_d^2 + 2k_x^2(\varepsilon_d - \varepsilon_p(0))^2\varepsilon_p(0)(\varepsilon_d + \varepsilon_p(0)),$$
$$Q_2 = \acute{\varepsilon}_p(0)\varepsilon_d^{\frac{3}{2}}\sqrt{\varepsilon_d\acute{\varepsilon}_p(0)^2 - 4k_x^2(\varepsilon_d - \varepsilon_p(0))^2\varepsilon_p(0)}$$

Where $\acute{\varepsilon}_p(0) = \frac{d\varepsilon_p(z)}{dz}|_{z=0}$. This form of dispersion relation is important in studying of asymptotic behaviors (such as $k_x \to 0$ and $k_x \to \infty$) which will be considered in the next section along with a few numerical examples.

3. A numerical analysis and discussion

As an example, it is assumed that $\varepsilon_d = 1$ and the dielectric function of the plasmonic medium has the following linear dependence on 'z' coordinate:

$$\varepsilon_p(z) = \varepsilon_p(0) + Bz \quad (18)$$

For some values of 'B' and $\varepsilon_p(0) = -2$ the dispersion curves from Eq. 17 are plotted in Fig. 2.
It is clear that the dispersion curve depends on the value of 'B' (i.e. the first derivative of $\varepsilon_p(z)$ at the interface). It is interesting that the dispersion plot is split into two branches depending on the sign of 'B'. One of the branches which is for positive 'B' shows a wave-number cutoff on k_x axis. The dispersion plots show that inhomogeneity of plasmonic medium offers a great flexibility on dispersion characteristics of SPP wave.
From Fig. 2, one can easily see that the upper branch of dispersion plot which occur for a negative 'B' has a bi-value

asymptotic group velocity. It means that for small values of k_x (i.e. $k_x \to 0$) the dispersion plot asymptotically tends to free space dispersion, viz $\omega = ck_x$. The free-space-like behavior in this regime can easily be verified by taking small-k_x Taylor expansion of Eq. 17. The free-space-like behavior of upper branch proposes the idea that such SPPs may be excited without rigorous phase-matching configurations such as Atto or Kreschmann ones [1, 2]. On the other hand, it is seen from the upper branches in Fig. 2 that for high values of k_x (i.e. $k_x \to \infty$ or electrostatic limit [1]) the group velocity tends to that of homogeneous plasmonic medium, namely Eq. 15 with $\gamma = 0$. This behavior can also be verified by taking $k_x \to \infty$ limit of Eq. 17. Such a bi-value nature of SPP group velocity can have potential application is optical switching and modulators [8, 20].

The lower branch dispersion plots have different behavior. As said above, the main characteristic of them is the existence of a cutoff wavenumber. Such a behavior is characteristics of the dielectric medium dispersion similar to Eq. 14. It means that the lower branch has a dielectric-like nature. Near the cutoff wavenumber the group velocity is infinity but the phase velocity is zero. In the limit of $k_x \to \infty$, the lower branch asymptotically leads to dispersion of SPP with homogeneous plasmonic medium (i.e. 'B' = 0) similar to that of upper branch.

In order to include a dispersion property in $\varepsilon_p(z)$, the simple Drude model is assumed. Through the model, the frequency dependence of $\varepsilon_p(z)$ can be supposed to be as:

$$\varepsilon_p(z,\omega) = 1 - \frac{\omega_p^2(z)}{\omega^2} \tag{19}$$

where $\omega_p(z)$ is the z-dependent plasmonic frequency. A linear behavior for $\omega_p^2(z)$ can be considered for further analysis:

$$\omega_p^2(z) = \omega_{p0}^2 + Dz \tag{20}$$

where 'D' is a constant. Due to the assumed models, the value of $\varepsilon_p(z,\omega)$ and its first derivative at $z = 0$ will be:

$$\varepsilon_p(0,\omega) = 1 - \frac{\omega_{p0}^2}{\omega^2} \tag{21}$$

$$\frac{\varepsilon_p(z,\omega)}{dz}|_{z=0} = -\frac{D}{\omega^2} \tag{22}$$

Putting Eq. 19 into the dispersion relation 15 yields:

$$\omega = \sqrt{c^2k_x^2\frac{\omega^2(\varepsilon_d+1)-\omega_{p0}^2}{\varepsilon_d(\omega^2-\omega_{p0}^2)} - DQ_3}, \quad \text{where}$$
$$Q_3 = \frac{c\omega^2\sqrt{c^2k_x^2-\varepsilon_d\omega^2}}{(\omega^2-\omega_{p0}^2)(\omega^2(\varepsilon_d-1)+\omega_{p0}^2)} \tag{23}$$

For an arbitrary value of $\omega_{p0} = 10^{14}\ \mathrm{s}^{-1}$ and the assumption of $\varepsilon_d = 1$, the dispersion curves are plotted in Fig. 3 for some values of 'D'.

The dimensionless parameter $F = cD/\omega_{p0}^2$ and the plasmonic wavenumber $k_{p0} = \omega_{p0}/c$ are used for convenience. It is clear that, depending on the sign of 'F', the SPP dispersion for nonhomogeneous plasmonic medium obeying the

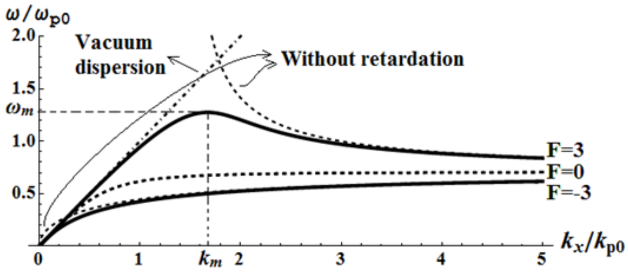

Figure 3. Dispersion plots from Eq. 23 for different values of 'F'. Free space dispersion plot (the dot-dashed curve) is also shown for comparison. Thin dashed plots (indicated by thin arrows) show the non-retarded cases (i.e. $c \to \infty$ limit).

Drude model is put above (for positive 'F') or under (for negative 'F') the dispersion plot of SPP with homogenous plasmonic medium (i.e. $F = 0$). It is interesting that all plots have the same behavior at $k_x \to 0$ and $k_x \to \infty$ regions. These identical asymptotic behaviors can also be verified by taking related limits of Eq. 23. In the small-k_x limit, the second term of Eq. 23 should vanish which leads to:

$$c^2k_x^2 - \varepsilon_d\omega^2 = 0 \to \omega = \frac{c}{\sqrt{\varepsilon_d}}k_x \tag{24}$$

It says that, the behavior of SPP dispersion for long wavelengths (i.e. $k_x \to 0$) is independent on 'F' and is that of the free space. At the opposite limit of $k_x \to \infty$ (i.e. the electrostatic limit) the first term of Eq. 23 grows more rapidly and hence yields the following limiting dispersion relation:

$$\omega = \frac{\omega_{p0}}{\sqrt{\varepsilon_d+1}} \tag{25}$$

This is, in fact, the SPP resonance frequency which can also be found by electrostatic treatments [1, 2, 8]. Eq. 25 implies that for $k_x \to \infty$ limit, the surface plasmon resonance frequency (which is characteristic of homogeneous plasmonic medium) is retrieved with no dependence on 'F' value. Another interesting case is the retardation effect (i.e. the finiteness of light velocity) on the dispersion plots. The limit of $c \to \infty$ is, in fact, the regime of no retardation. One can easily find the analytical form of dispersion relation for this regime by taking the $c \to \infty$ limit of Eq. 23 as follows:

$$\omega^2 = \frac{2k_x\omega_{p0}^2}{2k_x - \varepsilon_d D\omega_{p0}^{-2} + \sqrt{\varepsilon_d}\sqrt{\varepsilon_d D^2\omega_{p0}^{-4} - 4D\omega_{p0}^{-2}k_x + 4\varepsilon_d k_x^2}} \tag{26}$$

which evidently tends to Eq. 25 for $D = 0$. Such non-retarded dispersion plots are shown in Fig. 3. It is seen that the plots are, in fact, the asymptotes of the upper and lower dispersion plots for $k_x \to \infty$ limit [1].

The presence of a local maximum for upper branch at frequency k_m (shown in Fig. 3) is a characteristic difference of the nonhomogeneous plasmonic medium. Interestingly, presence of such a local maximum is also predicted in [1, 15] for sheet electron gases and half space plasmas.

The value of k_m and its frequency (i.e. ω_m in Fig. 3) depends on 'F' and more plots (not shown here) reveals that by increasing the 'F' value, the k_m and ω_m values also increase. The upper branch has an important property with respect to group velocity. It is clear that, the SPP group velocity is positive for $k_x < k_m$ but it has negative group velocity for $k_x > k_m$. The property implies that, for a non-homogeneous plasmonic medium obeying Drude model in SPP configuration, there is a region (i.e. $k_x < k_m$) that a negative-group-velocity SPP can be potentially generated. The group velocity is also evidently zero at k_m. Similar behavior of group velocity is also reported in [17] but due to their assumption of linear change of '$\varepsilon(z)$' from a constant ε_d to a constant ε_p, there are no things such as upper and lower branches in [17].
For negative 'F' values, on the other hand, there is no such zero- or negative-group-velocity region for SPP. It is interesting that the cutoff wavenumber which is seen in the lower branches of Fig. 2 is absent when both the non-homogeneity and the Drude-model frequency dependence of $\varepsilon_p(z,\omega)$ are taken into account. Such an effect on the cutoff wavenumber has also been reported in [16].

4. Conclusion

In this paper, the SPP dispersion relation for the plane interface between a semi-infinite homogeneous dielectric medium and also semi-infinite but non-homogenous plasmonic medium was considered. It was found that for small changes of $\varepsilon_p(z)$ the dispersion relation depends only on the first derivative of $\varepsilon_p(z)$ at the interface, i.e. $\frac{d\varepsilon_p(z)}{dz}|_{z=0}$. It seems that, the lack of deep penetration of SPP electromagnetic wave into the palsmonic medium is the main reason of independence on the detailed form of $\varepsilon_p(z)$ within the body. The main effect of non-homogeneity of plasmonic medium is that the dispersion plot is split into two branches. The upper branch shows a bi-group-velocity behavior for small and high wavenumber limits. For the lower branch, the main characteristic is the presence of a wavenumber cutoff.
Inclusion of frequency-dependent property of plasmonic medium through the Drude model leads to another interesting findings. In this case, the dispersion plot also is split into two branches. The lower branch does not show a cutoff wavenumber anymore. In the upper branch a local maximum is seen with the value of ω_m depending on the optical constants of the media. Wavenumbers higher than the k_m (i.e. the location of ω_m) show a negative-value group velocity. Such interesting behavior can have potential applications in SPP engineering for optics and photonics. It seems that, preparation of non-homogeneity for $\varepsilon_p(z)$ can be realized with semiconductors and level of their doping as mentioned in the introduction. Using gaseous plasmas with z-dependent density or temperature may also provide an experimental test space for the findings of the present paper.

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Ethical approval
This manuscript does not report on or involve the use of any animal or human data or tissue. So the ethical approval is not applicable.

Authors Contributions
All authors contributed equally in design the main sample, measure all the processes and also prepare the text.

Availability of data and materials
Data in this manuscript are available by request from the corresponding author.

Conflict of Interests
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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