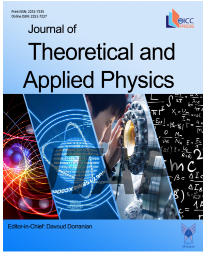

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Unveiling the cosmic tapestry: from quantum threads to emergent spacetime and beyond

Reza Mahdavi, Reza Saffari*

Department of Physics, University of Guilan, Rasht, Iran.

*Corresponding author: rsk@guilan.ac.ir

Original Research

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Abstract:
This article delves into the intricate interplay between quantum information, black hole dynamics, and spacetime emergence. We explore the fundamental phenomenon of quantum entanglement, which challenges traditional distance boundary concepts. The AdS/CFT correspondence is examined, revealing a profound connection between gravitational theories and quantum fields. It also emphasizes the holographic nature of our universe. The enigmatic nature of black holes is investigated, particularly their impact on locality and information preservation. We delve into the intricacies of black hole evaporation, a process guided by entangled qubits, shedding light on information's dynamic nature. Using the concept of the thermofield double, we track the profound link between quantum information and cosmos geometry. Our exploration concludes by charting potential future research directions to unravel the complex symphony underlying our universe's fabric.

Keywords: Quantum information; Black hole dynamics; Spacetime; Quantum entanglement; Holographic nature; Locality; Information paradox

1. Introduction

The natural world manifests diverse particle interactions, including mechanical, electromagnetic, and gravitational waves [1, 2]. According to known theoretical rules and speed limits, these waves transfer information within their respective mediums. These phenomena propagate within our four-dimensional (4D) spacetime, or at least within the 4D realm accessible to our observation. However, entanglement, a peculiar means of transferring information, cannot be explained through 4D spacetime as a medium. While entanglement is limited in its capacity to transmit data between entangled particles, it enables us to determine a distant particle's state instantly. This breaks the light speed limit [3].
This observation implies the existence of an information transfer mechanism that does not conform to our current spacetime understanding. The collapse of the wave function in entanglement due to our measurement of one-half of a mixed state instantaneously affects the other half without transferring any information [4]. The relationship between the maximum information of particle collections and the state of each particle at the time of measurement raises the question of whether there is an information transfer mechanism beyond spacetime.
To address this problem, we propose an approach that explores the possibility of a different type of medium: in this framework, we posit the necessity of at least one additional dimension beyond the familiar four, with the fifth dimension serving as a foundational starting point for the exploration of our hypothesis. This stance is not to suggest that the fifth dimension represents the upper limit of dimensional space, but rather to articulate that our current theoretical model requires this additional dimension to proceed, and it lays the groundwork for the potential integration and consideration of even more dimensions. While this approach may offer some insights into entanglement, we wonder whether it is sufficient to fully understand this peculiar way of transferring information. Further research and innovative theoretical frameworks may be necessary to fully comprehend the fundamental principles governing the natural world, including the mechanisms underlying entanglement.
Observing the similarities between wormhole characteristics and entanglement [5–7] raises the question of whether there is a fundamental correlation between these two topics. Quantum mechanics and general relativity challenge

locality through entanglement and wormholes. However, both of these topics need to be revised when questioning the concept of locality, which significantly affects their observability and investigation. In quantum entanglement, there is no possibility of direct information transfer at a speed faster than light [3]. At the same time, in Einstein-Rosen bridges (wormholes), we face Lorentzian wormholes that pose a challenge to cross them [7]. According to existing theories, when two black holes are entangled, they create a connection through a wormhole. This leads to the destruction of the black hole's internal smoothness [7]. The internal modes of the black hole become indefinable, which is the same as what happens in the phenomenon of entanglement (of course, at much smaller scales): the tiniest information about the subsystem and the highest knowledge about the system [4]. Thus, this study investigates the hypothesis of whether the wormhole connection between two black holes, which have a macroscopic form, can be considered an accumulation of microscopic entanglement connections in their microstates or not. Is the wormhole a specific macroscopic shape of entanglement or not?

The concept of entangled black holes, and the potential for their connection via a wormhole in higher dimensions, has long captured physicists' imaginations [5–8]. In a recent exploration of this idea, researchers considered the entanglement of pairs of particles within a black hole [7]. They proposed a scenario in which a quantum supercomputer outside the black hole could collect Hawking radiation emitted from the black hole without disturbing it. This would result in two equally sized black holes, all of whose particles are entangled one-to-one, leading to a wormhole connection between them [7]. The proposed mechanism offers an intriguing possibility for understanding black holes' nature and their interconnectedness.

The inner regions of two black holes in a Schwarzschild black hole scenario are connected by a wormhole. However, their outer areas remain disconnected from each other [7]. Observers near the event horizons of black holes experience two types of distance from each other: the first being the four-dimensional spacetime distance that separates them infinitely far apart, and the second being the five-dimensional communication bridge inside the black hole that brings them closer together. However, due to cosmic censorship conjecture and the absence of naked singularities [9], the close distance between the observers remains hidden due to the presence of the event horizon. This concept of hidden distances can be extended to quantum entanglement, where particles can appear to be separated by a considerable distance in four-dimensional spacetime but are closer together in the context of quantum entanglement. This can be interpreted as information censorship, analogous to quantum entanglement's hidden information. Therefore, invisible distances can offer insights into the interface between physical reality and the quantum world. It can also provide insights into how this boundary can be obscured or delineated. This provides a fascinating avenue for exploring the fundamental nature of reality and the limitations of our ability to perceive it.

In Dirac's view, entanglement cannot be detected by applying an operator to all functions since an entangled state cannot be written as a linear combination of non-entangled systems. Instead, each specific entanglement has a dedicated operator and detection method [10].

Interestingly, the analogy between entanglement and wormholes suggests that different entanglements must correspond to various wormholes [7]. The Wheeler-DeWitt equation, which relates general relativity and quantum mechanics, identifies conditions under which other states have various communication channels [11].

When two black holes are entangled, an inaccessible wormhole is situated within the event horizon region of two black holes [7]. The inaccessibility of the wormhole to the outside of the event horizon is equivalent to the inability to transmit information through entanglement. Upon entering their respective black holes, Alice and Bob witness the contraction of the wormhole entrance sphere as they approach the singularity [7]. Consequently, stretching the bridge at high speed and increasing the time needed to traverse it results in conditions under which sending signals through it is no longer feasible [7]. Hence, a dilated horizon exists on either side. As we see in both cases (wormholes and entanglement), information transmission is impossible, at least in a form familiar to us.

Therefore, the $ER = EPR$ correspondence is based on similarities between these two which are:

1. Both challenge information localization.
2. However, neither allows signal propagation from outside of their connection.
3. In the case of entanglement, no local operator can have a non-local effect on another subsystem. In the case of wormholes, according to the Penrose diagram of a two-sided black hole, no signal can be transmitted from the outside region on the right to the outside area on the left.

The presence of such a phenomenon raises the possibility that information existing in our spacetime may actually represent events occurring in higher dimensional space. Due to the limitations of our four-dimensional spacetime structure, displaying all of this information may not be possible. In critical situations, such as the very close interaction of two particles in the phenomenon of entanglement, or the extreme gravitational pressure on particles in a black hole, a significant portion of information appears to be emitted from four-dimensional spacetime. At the same time, the result remains in our observable universe.

As a result, a similarity can be observed between uncertainty (at the macroscopic scale) and the hiding of some information in a place beyond the event horizon of a black hole (at the microscopic scale). Can it be inferred that the effect observed in the form of a black hole is a consequence of a highly dense inherent lack of information present in small portions in each particle, which is beyond the reach of observers (even for each particle)? What emerges as values in four-dimensional spacetime is a projection of information in higher dimensional space, and this higher dimensional space only reveals its unique properties in specific situations.

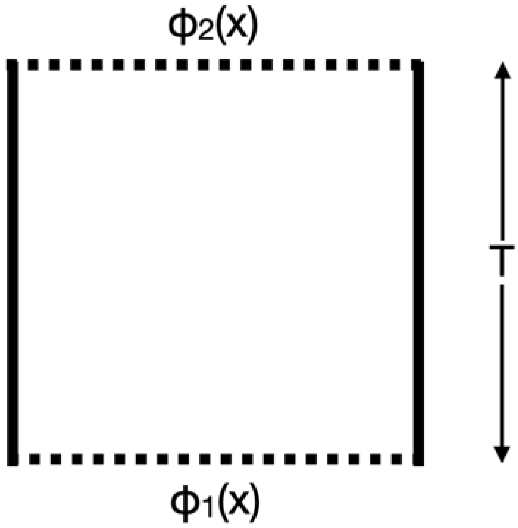

Figure 1. This picture shows the path integral approach from point $\varphi_1(\mathbf{x})$ to point $\varphi_2(\mathbf{x})$. In quantum mechanics and statistical physics, the path integral approach calculates the probability of transitioning from one point to another. The path integral process considers all possible trajectories from $\varphi_1(\mathbf{x})$ to $\varphi_2(\mathbf{x})$ rather than focusing on a specific direction. Each trajectory is assigned a weight based on the action, representing the energy difference between the initial and final states. The path integral summarizes these weighted trajectories to determine the overall probability of transitioning from $\varphi_1(\mathbf{x})$ to $\varphi_2(\mathbf{x})$.

2. Foundations of quantum mechanics and action principle

This section will examine the fundamentals of quantum mechanics and their relationship to spacetime. We explore critical topics, including the meaning and significance of actions and path integrals, the representation and analysis of quantum states using the density matrix and its trace, the relationship between quantum mechanics and spacetime within the context of Rindler spacetime, and the role of the density matrix in describing entangled states and the purification of mixed conditions. By investigating these fundamental concepts, we gain a deeper understanding of the mathematical foundations and physical implications that underlie the intricate connection between quantum mechanics and spacetime.

2.1 Actions and path integral

Actions serve as fundamental mathematical constructs in quantum mechanics, concisely describing the underlying principles governing quantum phenomena [12]. In this subsection, we explore the meaning and significance of actions in quantum theory.
We use path integrals as a powerful tool to calculate quantum probabilities and amplitudes. This is done by summing all possible paths of a particle or system [11].
Examining actions and path integrals establishes a solid foundation for understanding the interplay between mathematical representations and physical phenomena in the quantum realm.
When the time evolution operator (e^{-iHT}) acts on state x_1, it generates state x_2:

$$\langle x_2|e^{-iHT}|x_1\rangle \quad (1)$$

The time evolution operator encompasses all possible paths from the initial state to the final state. Another approach to achieve this is by utilizing the concept of path integrals, which consider the contributions of all paths:

$$\langle x_2|e^{-iHT}|x_1\rangle = \sum_{All\ paths} e^{-iS\frac{x(t)}{\hbar}} \quad (2)$$

In this formulation, S is the action associated with state x at time t; the system is transferred to the last moment through the time evolution operator or considering all interactions from the initial to the final point [11].
In quantum field theory, the initial and final states, instead of being a single point in spacetime, are the sums of fields at that point:

$$\langle\varphi_2(\mathbf{x})|e^{-iHT}|\varphi_1(\mathbf{x})\rangle \quad (3)$$

Again, one can employ the path integral instead of the time evolution operator in a similar manner [12] (Fig. 1):

$$\langle\varphi_2(\mathbf{x})|e^{-iHT}|\varphi_1(\mathbf{x})\rangle = \int_{\varphi(t=0)=\varphi_1}^{\varphi(t=T)=\varphi_2} D\varphi, e^{iS[\varphi]} \quad (4)$$

In this formulation, we are situated in Lorentzian spacetime. The integral boundaries span from the initial to the final spacetime. A transition can be made to Euclidean space without a time component by employing the Wick rotation in complex space. In this transformation, the imaginary unit "i" is converted to "$-i$" or "i/i" [12]. As a consequence, the Euclidean expression of the same concept will be as follows (Fig. 2):

$$\langle\varphi_2|e^{-H\tau}|\varphi_1)\rangle = \int_{\varphi(0)=\varphi_1}^{\varphi(\tau)=\varphi_2} D\varphi, e^{-S_E[\varphi]} \quad (5)$$

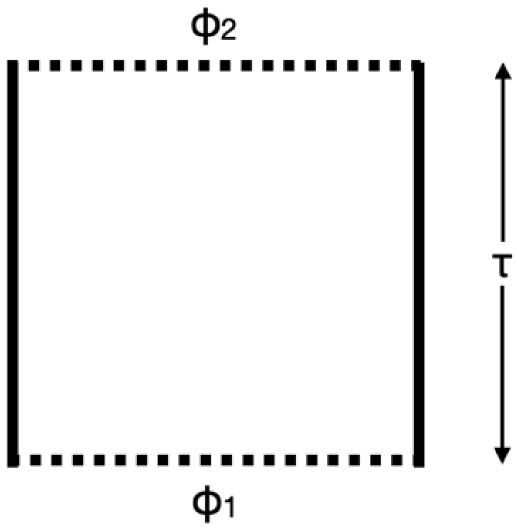

Figure 2. Using a path integral approach to transition from φ_1 to φ_2, which are no longer points but fields.

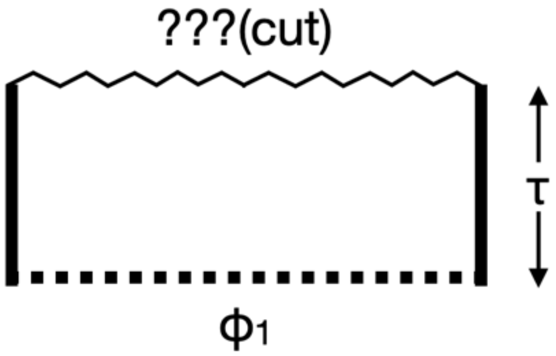

Figure 3. The picture depicts using path integral to construct a state function. In this process, we create a path integral for the transition between two fields. However, introducing a cut in their path will only be close to the initial field. From there, the paths are propagated towards the space under consideration. This approach provides a comprehensive understanding of the state function through path integrals and facilitates analysis in a specific field of interest.

In this formulation, τ is no longer a temporal component but rather represents a spatial separation between two points in Euclidean space.

2.2 Quantum states and density matrix

In this subsection, we explore quantum states and the density matrix. We provide detailed explanations of quantum states and their representations, enabling a deeper understanding of their properties and behavior.
Additionally, we explore the density matrix, a powerful tool for characterizing mixed states and capturing essential information about quantum systems.
Furthermore, we examine the significance of the trace of a density matrix. We offer insights into system purity, mixedness, and observables derived from the density matrix. We also explore analytical techniques for computing the density matrix in quantum mechanics. By doing so, we can analyze statistical properties and characterizations of quantum systems with precision and accuracy.
According to the provided definition of the path integral and the time evolution operator, if the initial point is specified, but we do not define a specific value for the final point, we form the state function associated with the initial point [13]:

$$e^{-H\tau}|\varphi_1\rangle = \int_{\varphi(0)=\varphi_1}^{?} D\varphi, e^{-S_E[\varphi]} = |\psi(\tau)\rangle \qquad (6)$$

To visualize in this context, imagine cutting or slicing through the picture depicted in the previous scenario, where we traveled from the initial point to the final point. By doing so, we gain insight into the evolution of the state function associated with the initial point over τ. In interpreting the path integral, we no longer confine paths to secondary points; instead, we unravel the propagation pattern of all possible pathways (Fig. 3).
In a similar manner and following the analogy observed in the thermofield double, a slice in the manifold can be interpreted in two ways:

1) In the form of a state function ($S^1 \times S^1$) encompassing both sides of the slice. In this approach, one of the two sides of the piece transforms into the ground state while the other continues its extension.
2) A localized operator on both sides of a slice that extends to a specific length. This operator establishes a connection between the values on the two sides of the piece [13].
In this formalism, we start with the fields at the initial point and construct the state function by considering all interactions in Euclidean space. Subsequently, utilizing this state function, we employ the Lorentz transformation by considering the fields and all interactions (in complex spacetime). By integrating these interactions, we find the time evolution of the initial state function.

$$D\varphi_1 \int_{\varphi_1} D\varphi, e^{iS} \int_{\varphi_1} D\varphi, e^{-S_E} \qquad (7)$$

In the definition of the state function, the farther the slice extends into the distant region, the broader the extent of Euclidean transformation. As the cut extends sufficiently far, it covers the entire Euclidean space, leading to the ground state. In other words, the time evolution reduces the excited state, ultimately reaching the ground state.

$$\tau \to \infty \Rightarrow |0\rangle$$

Utilizing the established definitions, it is evident that the density matrix, functioning as an operator, covers the interval between two slices. As these slices come closer to each other and merge into one, the resulting density matrix represents entangled states formed by the combination of the two states in bracket form. Thus, the density matrix encompasses all possible paths between the two slices in the manifold (Fig. 4).
Therefore, when the two created slices are combined, all possible paths find specific values. In other words, we sum over all states between the two regions, each associated

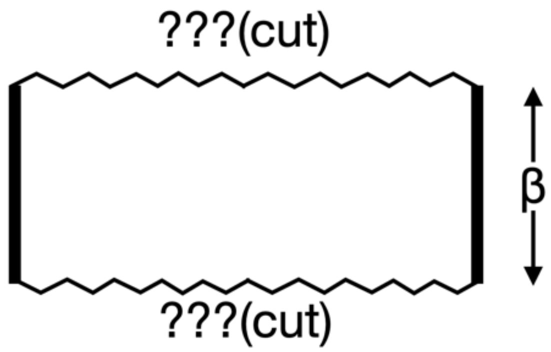

Figure 4. The representation of the density matrix of a thermal state is depicted, where the modes between the two slices are spaced apart by β. As the temperature increases, the value of β decreases. Consequently, when the two slices merge, we reach the maximum entanglement between the modes, the highest temperature, and a maximally mixed state. In this scenario, the density matrix is given by $\rho = e^{-\beta H}$.

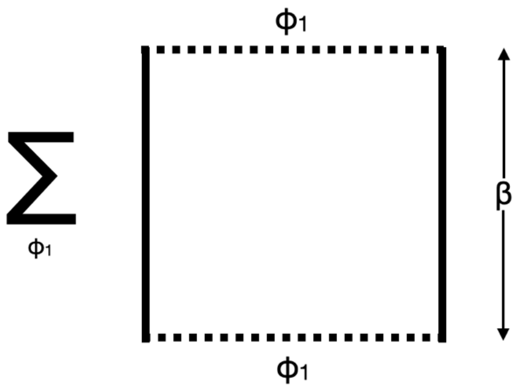

Figure 5. The summation of all existing fields within the distance of β is denoted as $Z(\beta)$.

with its unique energy level [14]. This definition is equivalent to the concept of the partition function. Consequently, when the trace operator is applied to the density matrix, it effectively merges the two created slices, resulting in the partition function corresponding to the distance between them. Therefore, the trace of a density matrix in a thermal state can be represented as follows (Fig. 5):

$$\langle \varphi_1|\rho|\varphi_1\rangle \Rightarrow Tr(\rho) = Z(\beta) \\ Tr(\rho) = \Sigma_{\varphi_1}\langle \varphi_1|e^{-\beta H}|\varphi_1\rangle \tag{8}$$

This concept can be represented cylindrically, with a cross-sectional surface area equal to β (the same surface where the two intersecting parts are glued together due to the overlap). As a result, $Z(\beta)$ is equal to Fig. 6 and Fig. 7.

As a result, part (d) in the figure indicates the effect of the reduced density matrix. This can be viewed in two ways. The reduced density matrix operator connects the cut's field values on both sides. Suppose the transformation due to the boost is considered along the theta direction. In that case, when we move from the upper part of the cut to the lower part, the transformation is equivalent to the effect of the reduced density matrix [15]. Therefore, the boost-induced transformation corresponds to the effect of the reduced density matrix. In fact,

$$\langle \varphi_2^A|\rho_A|\varphi_1^A\rangle = \langle \varphi_2^A|e^{-\int_0^{2\pi} Qd\theta}|\varphi_1^A\rangle = \langle \varphi_2^A|e^{-2\pi Q}|\varphi_1^A\rangle \tag{9}$$

Upon applying the Wick rotation, which transforms the Euclidean space into the complex Lorentzian space, the equivalent reduced density matrix is obtained in the Lorentzian spacetime (Fig. 8):

$$\rho_A = e^{-2\pi k_A} \tag{10}$$

Now, considering that the obtained value for the density matrix in the previous state is for a time slice of space, we can calculate the corresponding value in Lorentzian spacetime [13, 16] (Fig. 9).

Thus, the accelerating observer perceives a thermal temperature due to the boost present in Rindler space. This temperature increases as they approach the Rindler horizon. In essence, the number of modes that exhibit entanglement with one another increases significantly near the boundary created between the two regions on the left and right of space. This resulting temperature, known as Unruh radiation, directly correlates with the magnitude of the observer's acceleration (which, in turn, corresponds inversely to the distance from the Rindler horizon) [15, 17].

2.3 Unraveling spacetime: Rindler vs. Minkowski and entanglement

In the depicted scenario, we observe the temperature and spatial separation into two regions due to the defined type of acceleration and its relation to the inverse of the distance to the event horizon of the black hole. A similar phenomenon occurs when a black hole is present in Minkowski spacetime. A correspondence exists between the event horizon of the black hole and the Rindler horizon. In Rindler spacetime, acceleration in space creates entanglement between two regions, manifesting itself through the density matrix and the observed temperature at the horizon. However, in Minkowski spacetime, the presence of a black hole encompasses all the associated conditions.

Moving observers in Rindler spacetime observe something similar to a black hole and a thermal state (Hawking radiation). Simultaneously, an observer at rest in this spacetime (who requires energy to maintain their position) perceives other regions of the universe being accelerated by a force similar to a rocket, resulting in the emission of radiation towards the Rindler horizon.

Within quantum entanglement lies the enigmatic concept of the thermofield double, a bridge between different spacetime realms. Imagine two quantum systems, mirrored yet intertwined, dancing in joint Hilbert space [14, 15]. In Rindler space, where acceleration shapes the stage, this concept unveils a fascinating phenomenon: the entanglement between left and right Rindler wedges births thermal properties, revealing the Unruh effect—a vacuum perceived as a simmering thermal bath by accelerated observers.

The thermofield double threads extend beyond Rindler space, reaching black holes' cores in Minkowski space. Here, entanglement entropy mirrors black hole entropy. This connection sheds light on the interplay between entanglement and horizon thermodynamics, hinting at cosmic holographic resonance. The thermofield double becomes a key, unlocking spacetime's symphony and weaving together

Figure 6. Using cylindrical representations constrained by the distance between two fields in a planar representation. The cylinder's circumference in this representation is equal to the distance between the fields from a planar view so that it can be equivalent to the system partition function.

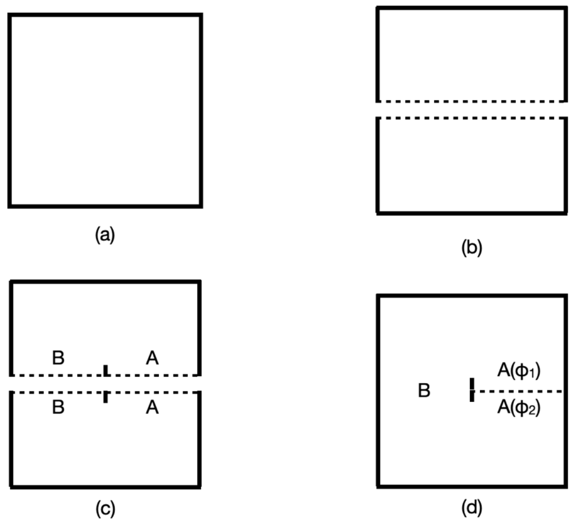

Figure 7. We create a similar manifold to calculate the reduced density matrix in Rindler spacetime. Initially, we consider a flat manifold (a) and then we create a cut within it (b). Next, we form two regions, left and right (c). The reduced density matrix involves the transfer operator between the upper and lower parts of the manifold. Now, with the definition of the reduced density matrix, we can connect half of the cut (d) using the trace operation. Consequently, the resulting reduced density matrix links the field values in the upper and lower sides of the right part of the manifold.

quantum threads and gravitational forces.
Within entanglement, particles engage in hidden interplay. When one particle is influenced, its partner responds instantaneously, defying distance and revealing entanglement's non-local nature. This phenomenon also offers insight into superluminal interaction—an illusion of faster-than-light communication that abides by cosmic limitations.
In this entangled interplay, chaos, and information express through entropy. As particles navigate their quantum realm, their shared enigmas materialize as entanglement entropy. This intricate quantification of their connection captures the delicate balance between order and disorder, echoing thermodynamics' second law principles. Through this entropic rhythm, we observe the quantum stage mirroring the cosmic theater—a profound interweaving of information, entanglement, and spacetime symphony.
It is also noteworthy to mention that in our exploration of the cosmos and its intricate tapestry, it is illuminating to revisit the visionary insights of historical figures like Pythagoras and Johannes Kepler. Pythagoras' notion of 'music of the spheres' and Kepler's 'musical geometry' represent pioneering attempts to understand the universe as a symphony of mathematical relationships. Pythagoras perceived celestial motions as harmonious musical intervals, revealing an early intuition of the cosmos as an entity governed by mathematical principles. Similarly, Kepler's exploration of planetary motions through harmonic ratios foresaw a universe where physical phenomena resonated through mathematical harmony. Their perspectives, while rooted in the science of their times, remarkably echo today's view of the universe as a complex yet harmonious construct where space, time, and matter dance to the rhythm of fundamental laws. Such historical insights not only enrich our scientific narrative but also underscore the timeless quest to decipher the cosmos through mathematics.

3. A Dance of dualities: AdS/CFT and spacetime emergence

This subsection delves into the intricate relationship between operators and states in the AdS/CFT correspondence. We analyze how operators are mapped between the gravitational theory in AdS and the corresponding CFT, unveiling the profound implications of this operator-state correspondence. By exploring this connection, we understand the physics underlying both sides of the correspondence. This illuminates the interplay between gravitational theory and the corresponding CFT operators and states [18].
The CFT is characterized by a particular symmetry property known as conformal symmetry. These properties give rise to various remarkable features which have proved crucial for analyzing physical phenomena [19].
CFT in space is defined by the following elements [18, 19]:
1. Firstly, scaling dimensions are the scaling exponents that arise from dilatational symmetry. These parameters indicate how system behavior changes with scale.
2. Operator Product Expansion (OPE) Coefficients: OPE coefficients constitute a pivotal aspect of CFT, characterizing operator product expansion. The OPE embodies a diverse set of powers representing the variations of fields and their effects in the CFT space.
3. Spin Constants: Spin constants emerge due to rotational symmetry in the system. They provide insight into the spatial properties of the fields under rotational transformations.
The holistic portrayal of CFT in space thus encompasses these fundamental features. Each contributes significantly to the understanding of intricate physical processes in conformal symmetry. This profound mathematical framework lays the groundwork for investigating diverse phenomena and advances modern theoretical physics.
A holistic depiction of CFT in space is essential to understand intricate physical processes in conformal symmetry. This mathematical framework allows diverse phenomena to be studied and advanced in theoretical physics.
In Quantum Field Theory (QFT), Operator Product Expan-

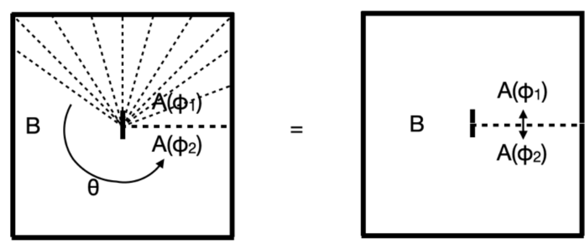

Figure 8. Illustration of two different methods of connecting fields on the two created slices in the manifold.

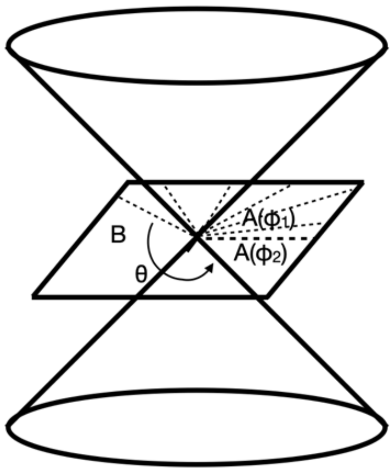

Figure 9. The density matrix created in Euclidean space is applicable in Lorentzian spacetime. As illustrated, a density matrix is formed in a slice of time.

sion (OPE) coefficients serve as numerical constants that emerge from expanding the product of two operators into the sum of a series of other operators. OPE provides a mathematical tool for analyzing correlation functions in quantum field theories. These coefficients represent the product of two operators at slight separations as the sum of operators at significant breaks [20].

Through OPE, valuable information about the theory under investigation becomes accessible. This includes insights into the interplay between operators and their contributions to correlation functions.

The significance of OPE lies in its ability to provide valuable quantitative details about the interaction strengths between various operators and the extent of involvement of different operators in correlation functions. It is an indispensable mathematical instrument for probing the intricate properties of quantum field theories. It enhances our understanding of the underlying physics.

3.1 Weyl mapping to cylinder and state-operator correspondence in CFT:

In the context of the Weyl mapping from the plane to the cylinder, the Dilaton operator transforms into the Hamiltonian of the system, leading to the creation of a gapped system with a non-zero ground state energy, which is dependent on the constant spacetime background ($c/12$). Consequently, through appropriate changes in variables, we can establish a connection between operators on the plane and operators on the cylinder [20]:

$$\mathscr{O}_{cyl}(t,n) = e^{\tau\Delta}\mathscr{O}_{pl}(x = e^{\tau}, n) \tag{11}$$

The Weyl transformation interconnected two distinct theories of the plane and the cylinder. With these definitions, we can express the path integral over the region on the plane and the cylinder where the operator is defined as follows Fig. 10.

In both theories, spacetime is flat. With this correspondence, we can obtain the path integral in Minkowski spacetime using the cylinder operator or consider the cylinder spacetime operators as Minkowski vacuum states on the plane.

We can define a state using an operator in all quantum field theories. To achieve this, we consider the operator at the center and take a path integral over a neighborhood around it. The outcome is a quantum state that can be utilized in any quantum field theory [20, 21] (Fig. 11).

Conformal Field Theory (CFT) allows us to use an operator instead of a state. In this definition, if we have a state, we can employ the dilatation transformation to shrink the state to a tiny point. After performing this transformation, we can use it as an operator in the path integral (Fig. 12).

3.2 Action in the presence of gravity and effects of stress-tensor

This subsection investigates the modifications to the action in the Ads/CFT context when gravity is present. We explore how the activity is altered to account for the gravitational effects within this framework. Additionally, we examine the impact of the stress tensor on the operators, shedding light on the implications of this interaction. By unraveling these aspects, we gain insights into the interplay between gravity, the modified action, and the behavior of the operators in the Ads/CFT correspondence.

3.2.1 Polyakov action

The Polyakov action represents the dynamics of an n-dimensional membrane in a $(n+1)$-dimensional spacetime. Thus, the curvature of the universe's spacetime can be considered based on this model. One can compute the partition function of this spacetime by utilizing the Polyakov action to describe a spacetime with curvature. Due to its nature being related to curved spaces, the Polyakov action can

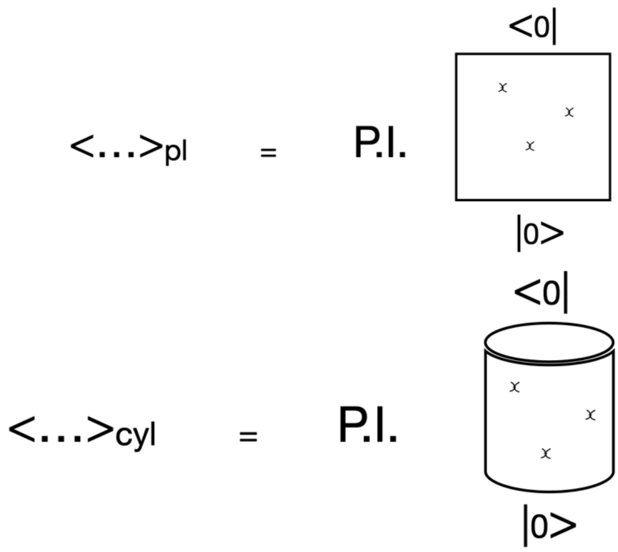

Figure 10. By utilizing the path integral approach, it is possible to investigate the effects of operators in two different theories that are related to each other via the Weyl transformation.

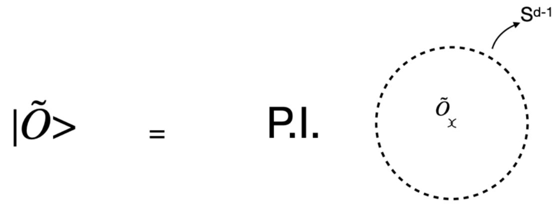

Figure 11. This visual depiction captures defining a quantum state using a central operator and surrounding neighborhood. By performing a path integral, the operator's attributes merge to create an individual quantum state.

generate correlation functions of the stress-energy tensor [22, 23]. Moreover, it enables the connection between the partition function of a flat space and the partition function of a curved space:

$$\begin{cases} z[g] = z[\delta]e^{-S_p(g)} \\ S_p(g) = \frac{T}{2}\int d^2\sigma\sqrt{-h}h^{ab}g_{\mu\nu}(X)\partial_a X^\mu(\sigma)\partial_b X^\nu(\sigma) \end{cases} \tag{12}$$

The Polyakov action incorporates all Conformal Field Theory (CFT) dependencies in the form of metrics, enabling it to provide correlation functions on the desired gravitational background. The action describes the dynamics of a string as it moves through spacetime and how it interacts with spacetime geometry. These dependencies are all related to the central charge of the space in question. Consequently, the central charge can be considered the stress-energy tensor coefficient. In essence, 'c' can serve as a measure of particles' degrees of freedom. The Polyakov action acts as a generator of the stress-energy tensor correlation functions [23].

3.2.2 Stress-tensor effects on operators of spacetime

The stress-energy tensor interaction with the vicinity of a primary operator (fields that transform nicely under the symmetries of the CFT space) causes the emergence of at least two types of singularities: one singularity occurs due to dilatation transformations, which are proportional to the inverse square of the difference between the flat and cylindrical space in the cylindrical space operators. Another singularity arises from translations, which are proportional to the inverse of the difference between flat and cylindrical space in the derivative of the cylindrical space operators [24]:

$$T(z)\tilde{O}(w,\bar{w}) \approx \frac{h}{(z-w)^2}\tilde{O}(w,\bar{w}) + \frac{\partial\tilde{O}(w,\bar{w})}{z-w} + \ldots \tag{13}$$

Therefore, when investigating the impact of the stress-energy tensor on cylindrical space operators, we will encounter singularities for each operator. It is worth noting that, due to the scale-invariant nature of conformal field theory (CFT), only singularities appear [24]:

$$< T(z)\tilde{O}(w_1)\tilde{O}(w_1) >= \frac{h(w_1-w_2)^2}{(z-w_1)^2(z-w_2)^2} \tag{14}$$

3.3 The role of action and stress-energy tensor

Understanding how the stress-energy tensor influences the action offers us a glimpse into the heart of spacetime emergence. We uncover the hidden connections between gravity and quantum reality by grasping these intricacies. These insights serve as guiding stars, illuminating the pathways through which gravity weaves itself into the structure of spacetime. As we unravel these threads, we inch closer to comprehending the complex interplay that shapes the cosmos, inching closer to deciphering the enigmatic symphony that binds the universe together.

As we ponder the modifications in action and the dance of operators, a compelling question arises: could these elements hold the key to the enigmatic Einstein-Rosen bridge? While the precise role may elude our grasp, the mere contemplation of this connection enriches our exploration. Acknowledging this potential link enhances the grand theme that binds our narrative—an intricate web of quantum threads and gravitational forces woven into the very essence of spacetime's fabric.

As observed, due to a boost in Rindler space, this spacetime divides into two distinct regions: the left and right halves. Consequently, some spacetime information becomes inaccessible, causing entropy [15]. The density matrix corresponding to this partition of Rindler space is expressed as follows:

$$\rho_A = \frac{1}{Z}e^{-\beta H} \tag{15}$$

Where $\beta = 1/T$ and H is the Hamiltonian of the system. The operational form of the density matrix is expressed as follows:

$$\rho_A = \frac{1}{Z}\sum_i e^{-\beta E_i}|0\rangle_A|0\rangle_{A^c} \tag{16}$$

When focusing on half of the space, the quantum system's state may become entangled with its unobserved complement, leading to a mixed-state representation. This entanglement with external degrees of freedom results in a probabilistic description of the system's quantum properties. On the other hand, when considering the entire space and having no entanglement with external factors, the system

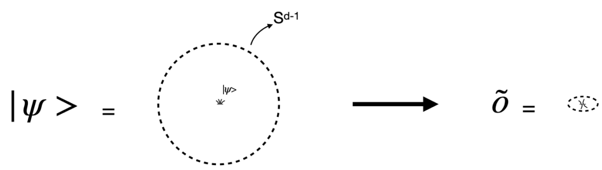

Figure 12. This visual representation elucidates the concept of operators in Conformal Field Theory (CFT). The process begins by considering a quantum state, which is then subjected to a dilatation transformation, symbolized by contraction to a minuscule point. This transformed state now functions as an operator in the path integral, emphasizing CFT's unique approach to bridging quantum states and operators.

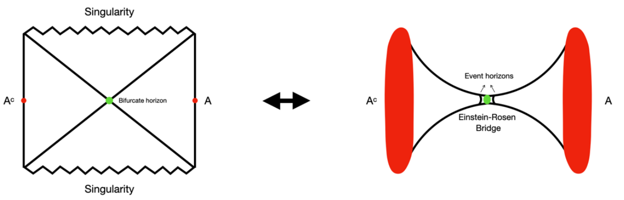

Figure 13. On the left side, we see the Penrose diagram of Rindler spacetime, while on the right side, a temporal slice of this spacetime is depicted. It should be noted that the event horizon and the Einstein-Rosen bridge in the image are represented in a manner that may not precisely capture their proper form. Whether the Einstein-Rosen bridge must permanently reside inside the event horizon or exhibit two distinct natures remains controversial.

can be accurately represented as a pure state containing detailed quantum information. This distinction between pure and mixed states is pivotal in understanding quantum correlations and entanglement phenomena in complex physical systems (Fig. 13).
The minimal surface that separates region A from its complement, denoted as Ac, is considered the event horizon (or Einstein-Rosen bridge). The area law, $S(A) = kA/(4l^2_{planck})$, governs the entropy of this spacetime, where $S(A)$ represents the entropy, k is a constant, A is the area of the horizon, and $l^{\square}_{planck}$ is the Planck length. It relies on the study's specific context of whether to use the event horizon or Einstein-Rosen bridge as the separating surface. We, therefore, refer to it as the bifurcate surface [7].
Based on the holographic entanglement entropy laws, when we examine the entropy of region A, we can look at its homologous region. As depicted in the figure, the bifurcated surface has homology with region A, indicated by the proportionality between the sizes of the areas ($m(A) \sim A$). This homologous region allows us to study region A's entropy through the holographic entanglement entropy framework.

4. Quantum information and black hole dynamics

This section explores the fascinating interplay between quantum information and black hole dynamics. Quantum information theory enables us to understand the behavior and properties of information in quantum systems. In contrast, black holes pose intriguing challenges and mysteries regarding the preservation and retrieval of information. By investigating the relationship between these two domains, we aim to uncover their profound connections and implications for our understanding of spacetime's fundamental nature. This section delves into crucial topics such as quantum entropy, fine-grained and coarse-grained entropies, the dynamics of black holes and Hawking radiation, and the intricate processes of information in the creation and evaporation of black holes. Furthermore, we explore the role of entangled modes in near and far regions, shedding light on their influence on information propagation and entanglement entropy within black hole systems. Through this exploration, we strive to deepen our understanding of the intricate relationship between quantum information and black holes' enigmatic nature.

4.1 Quantum entropy

In this subsection, we delve into quantum entropy as a fundamental measure of information in quantum systems. We explore the intricate landscape of quantum entropy, unveiling its role in quantifying hidden information within quantum states. Additionally, we discuss various information measures, including fine-grained and coarse-grained entropies, shedding light on their applications in characterizing quantum correlations and information storage.
We employ the von Neumann entropy to calculate the encoded entropy in a density matrix. This entropy is equal to zero for a pure-density matrix, and its upper bound is the entropy of a maximally mixed state. This entropy is given by the identity matrix in the inverse dimensions of the Hilbert space and Quantum Field Theory (QFT). This value diverges to infinity. To prevent divergence, a UV cutoff is utilized. The von Neumann entropy for the thermal state is as follows [3]:

$$\rho_{thermal} = \frac{1}{z} e^{-\beta H} \tag{17}$$

$$S\left(\frac{1}{z} e^{-\beta H}\right) = S_{thermal}(\beta) \tag{18}$$

There is a connection between this von Neumann entropy and the thermodynamic temperature present within spacetime. By contrast, the density matrix corresponds to the partition function, representing a statistical ensemble associated with the system's energy [3].
Quantum entropy is a fine-grained entropy; if we know a quantum system's density matrix, we can calculate the corresponding entropy. However, thermodynamic entropy is coarse-grained.

4.1.1 Relative entropy

By introducing the von Neumann entropy, the possibility of establishing a relationship between two distinct entropies emerges [25]. To achieve this, we define relative entropy,

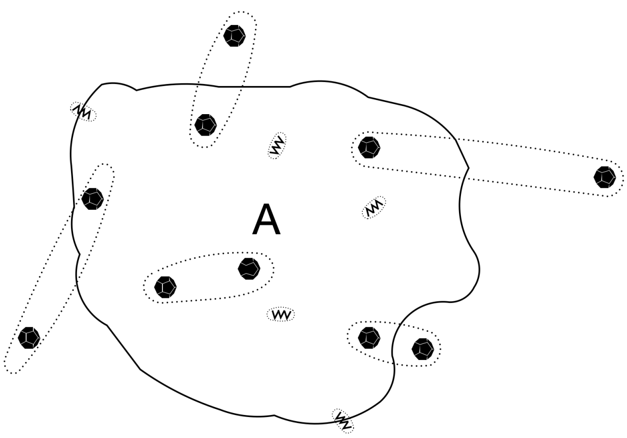

Figure 14. illustrates fine-grained entropy and coarse-grained entropy; fine-grained entropy cannot extend beyond a specific limit, but coarse-grained entropy can gradually move further apart from each other over time.

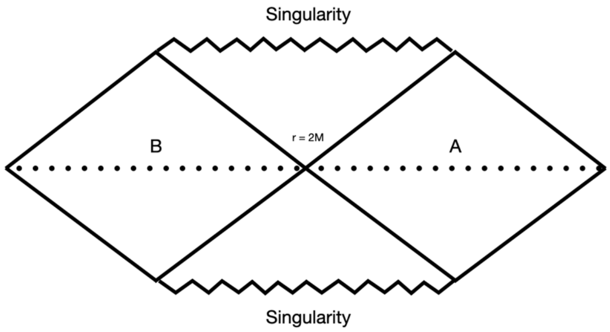

Figure 15. This visual representation illustrates the intriguing effect of a black hole with the thermofield double state. The emergence of a state like the thermofield dual state due to a black hole's presence disrupts the conventional causal connection between the right and left regions.

which is calculated as follows:

$$S(\rho \| \delta) = -tr\rho \log \sigma + tr\rho \log \rho \tag{19}$$

By comparing the disparity between two density matrices, we can determine how much their differences are and how many measurements are required to characterize those differences [25]. By comparing the density matrix with the Hilbert space dimensions, we can determine the amount of information in the system:

$$I_c = S(\rho \| \frac{1}{N} 1) = \log \dim \mathscr{H}_i - S_(\rho) \tag{20}$$

In the scenario where I_c equals $\log \dim \mathscr{H}_i$, the system's entropy is effectively zero, indicating the maximal possible information within it. Conversely, when I_c equals zero, it implies a maximally mixed system where no independent data can be extracted [3, 25].

4.1.2 Mutual information

With the help of the definition of relative entropy, we can redefine the meaning of mutual information:

$$I(A\square B) = S(A) + S(B) - S(AB) = S(\rho_{AB} \| \sigma_{AB}) = S(\rho_{AB} \| \rho(A) \otimes \rho(B)) \tag{21}$$

In this manner, the level of entanglement, both classical and quantum, between subsystems A and B is determined. The minimum amount of mutual information is zero. In most pure states, which include a complete chain of particles, if we focus on a tiny fraction compared to the rest of the system, this fraction is in the maximally mixed state [25].
Therefore, in the vacuum situation, the entropy of portion A of the whole system is equal to:

$$S(A) = \frac{c}{3} \log \left[\frac{l}{\varepsilon} \right] \tag{22}$$

Where l is the length of region A and ε is associated with the UV-Cut off. And when the system is in a thermal state:

$$S(A) = \frac{c}{3} \log \left[\frac{\beta}{\pi \varepsilon} \sin \frac{\pi l}{\beta} \right] \tag{23}$$

In this case, for l" β, the system does not exhibit any significant thermal effects and behaves entirely similar to a vacuum state; everything is considered empty at short distances. On the other hand, when l "β, the entropy grows linearly and proportionally with the system's temperature [25].
In this regard, the Von Neumann entropy in Quantum Field Theory (QFT) encompasses two types of entanglement, one associated with short-range entanglement related to the vacuum state and the other related to entanglement of fundamental particles on a larger scale. The fine-grained component occurs at short distances, and its contribution to the total entropy of region A is only present when one end of this short-range entanglement lies outside of area A and the other end lies inside it. The coarse-grained component involves fundamental particles, and entangled particle pairs can exist at significant distances from each other [26]. In essence, one part is associated with particle entanglement. The other characterizes statistical connections between different points in spacetime, which can carry spacetime's intrinsic meaning. Therefore, spacetime is an emerging

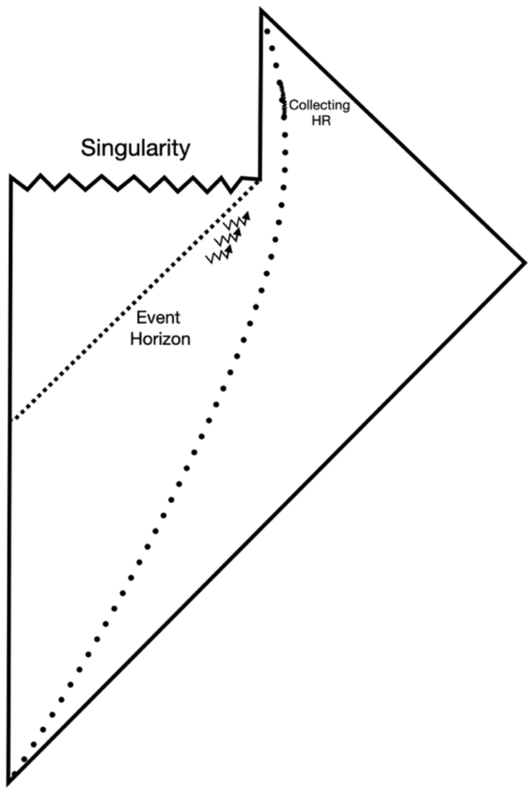

Figure 16. The intricate black hole creation and evaporation process through a Penrose diagram is presented. As the black hole evaporates, its event horizon radius diminishes, leading to the liberation of previously entangled particle pairs beyond the event horizon. With these particles existing outside the event horizon, lost information becomes accessible again, gradually reducing entropy. The diagram visualizes the distinct phases and information dynamics underlying the intricate relationship between black hole evaporation and Hawking radiation.

property of particle entanglement, and entangled particles can be used to probe spacetime at its elementary level. This is an essential tool for scientists seeking to understand the nature of spacetime better. This could provide insights into fundamental physical laws, such as quantum gravity, which could revolutionize our understanding of the universe. It could also lead to the development of new technologies, such as faster-than-light communications (Fig. 14).
Thus, the Von Neumann entropy of region A effectively represents all entanglements that have a portion inside area A and another portion outside of region A. It quantifies all the information reduced from the entire system due to removing region A from the system.

4.2 Black hole dynamics and information distribution process

This subsection delves into black hole dynamics and their profound implications for information distribution. We explore the intricate processes involved in the creation and evaporation of black holes. This sheds light on the complex interplay between information preservation and retrieval. By unraveling the mysteries of information flow in black hole systems, we gain insights into the fundamental nature of these astrophysical objects and their impact on the distribution of information within the vast reaches of spacetime.
The presence of a black hole produces a state similar to the thermofield double state [27]. Consequently, in Schwarzschild's spacetime, we have a world where the causal connection between right and left has been lost (Fig. 15).
In Hartle-Hawking coordinates, the spacetime is a cigar-shaped structure, entirely analogous to the Minkowski vacuum in far regions. However, traversing from area A to B and vice versa is impossible, as any path connecting these two regions terminates at the black hole. Consequently, in the vicinity of $r = 2m$, spacetime becomes curved. By employing the concepts of path integral, one can compute the density matrix associated with this spacetime. The result indicates that the Hartle-Hawking state is the thermal state with a temperature of $T_H = 1/(8\pi M)$. If we take the trace of this state and focus solely on its half, we obtain a thermal state as expected:

$$|HH\rangle = |TFD\square\beta = 8\pi M\rangle \tag{24}$$

$$\rho_A^{HH} = tr_B|HH\rangle\langle HH| \tag{25}$$

$$\rho_A^{HH} = e^{-8\pi M,H} \tag{26}$$

This density matrix corresponds to a thermal state with the Hawking radiation temperature.
As a result, the Euclidean path integral in a Schwarzschild spacetime yields a Hartle-Hawking state, which independently becomes a thermal state on either side. This thermal effect corresponds precisely to Hawking radiation. The Hartle-Hawking state describes a black hole in thermal equilibrium with its radiation. This state is analogous to placing a black hole inside a box that emits and absorbs radiation [15, 27].
The intriguing point is that, according to the concepts presented, space appears empty at the event horizon for an observer in free fall toward the black hole. This observer encounters no firewalls. Similarly, in Hartle-Hawking spacetime, an observer in free fall towards a black hole perceives space as empty in the same way that it is for an observer in Minkowski spacetime.

4.2.1 Mode entanglement: near and far regions

In regions near the event horizon, the correlation of modes manifests as an exponential summation of all energy contributions associated with each mode. As we move away from the event horizon while remaining gravitationally close to the black hole, the angular momentum becomes more dependent on the radial distance. This results in an increased magnitude. Additionally, at the event horizon, the rotation of the exterior and interior regions becomes identical, leading to the maximal possible degree of mode correlation [28]:

$$\sum e^{-\pi\omega}|n\rangle|n\rangle : \begin{cases} nearhorizon \approx |n\rangle|n\rangle \\ fargravitationalregion \approx \frac{1}{f(r)}|n\rangle|n\rangle. \end{cases} \tag{27}$$

Where $f(r)$ is a function of the distance from the event horizon and takes values greater than 1, the effect of entanglement diminishes and vanishes at far reaches, accompanying the transition to the redshift regime. Most entangled pairs are concentrated nearby, while the entanglement effect gradually fades away with increasing separation from the event horizon.

4.2.2 Black hole and hawking radiation time evolution

Considering quantum entanglement effects, a black hole gradually evaporates through interconnected pairs. In spacetime containing only one black hole, and where radiation does not interact significantly with the environment, the emitted radiation can be collected at a distant location. As long as the black hole has not entirely evaporated, some information remains in the entangled pairs which cannot be accessed. As a result of this phenomenon, the universe's entropy outside of the black hole also increases.
As the black hole evaporates, the radius of the event horizon diminishes. This causes some previously entangled pairs inside the event horizon to escape beyond it. With both particles existing simultaneously outside the event horizon, once lost, information becomes accessible again. This leads to a gradual reduction in entropy due to the black hole's presence.
Ultimately, when the black hole fully evaporates, spacetime returns to an empty state, accompanied by Hawking radiation. This description shows the entanglement between the radiation and the black hole increases until Page time. After that, it gradually diminishes until complete evaporation has taken place.
Due to the presence of Hawking radiation and the evaporation of black holes, the Penrose diagram corresponding to a black hole undergoes a distinct phase. In this phase, all emitted radiation is accessible and available. As a result, the entropy evolution of the universe due to a black hole can be divided into two distinct periods: the first corresponds to the time before radiation occurs to the point when the energy of the emitted radiation is less than the remaining

energy inside the black hole. During this period, the total entropy of the external system gradually increases with the increasing amount of Hawking radiation [29].
At this time, all the Hawking radiation is associated with half of the entangled pairs; hence, no additional information is added to the system. After passing the "Page time," the other half of the entangled pairs leave the black hole horizon in the second period. Consequently, as the entangled pairs are simultaneous on one side of the event horizon, the system's information gradually increases, and the entropy induced by the black hole decreases until the black hole completely evaporates. At that point, the system will have no additional entropy due to the black hole (Fig. 16).
One can conceptualize a black hole and Hawking radiation as a quantum mechanical model of qubits. This model represents the black hole as a collection of qubits. During specific time intervals, a certain number of qubits are separated from the black hole with a defined probability distribution. These separated qubits then form Hawking radiation.
To elaborate further, let's consider the black hole as an ensemble of qubits with distinct properties. Due to quantum effects near the black hole horizon, entangled pairs of qubits are continuously generated during the evaporation process. One qubit of the entangled pair falls into the black hole, while the other escapes and becomes part of the Hawking radiation. The evaporation process can be visualized as a continuous production of these entangled qubit pairs.
At each time step, several qubits are released as Hawking radiation. The black hole's quantum properties determine the probability distribution of this emission. As more and more qubits escape, Hawking radiation accumulates, carrying away information from the black hole's quantum state. During this period, as the black hole loses its energy through radiation, its mass decreases.
The entire system, comprising the black hole and the escaping qubits as Hawking radiation, undergoes complex quantum entanglement. This entanglement is responsible for the intricate correlations between the black hole quantum states and the emitted radiation. As evaporation continues, the entanglement pattern between the qubits evolves, gradually decreasing the black hole's entropy.

4.3 Information paradox and spacetime emergence

There is a paradox of information when entangled particles are emitted from the event horizon, with half of the particles remaining inside the horizon. On the one hand, the event horizon should not exhibit any particular region in space, and we should not witness any firewall at the horizon; hence, it is expected to be smooth. Consequently, the particles on both sides of this region must be fully correlated. On the other hand, when a considerable amount of emission is exerted, particles located at significant distances must remain entangled with their partners inside the event horizon. According to monogamy principles, they cannot be simultaneously correlated with particles in their surroundings (in 4-dimensional spacetime).
To address this issue, several approaches have been proposed. After the complete evaporation of the black hole, a region with high entropy (essentially equal to the black hole's entropy) remains wholly disconnected from spacetime. The main problem with this approach lies in its contradiction with thermodynamic principles, Bekenstein-Hawking entropy, and the area law.
As a result of this choice, there exists a region in space that, although its size is very close to zero, can have entropy comparable to the black hole's most significant amount of entropy. This is because there is no longer an upper bound on entropy in a limited area.
As a second approach, the black hole concept could be redefined entirely and considered a phenomenon that behaves like a black hole at a distance. However, this manipulation of black holes raises significant issues, as it eliminates any thermofield double to resolve the problem. Despite addressing this particular issue, it leads to numerous other problems, making it an unsuitable approach to understanding black holes correctly.
The approach adopted in this paper is based on considering a more general definition of locality, which behaves differently from the classical notion of locality. It should be noted that locality is a geometric concept with no specific purpose in quantum gravity. The main challenge we face is the close connection between Hawking radiation at a distance from the black hole and its entangled pairs inside the black hole. The central question addressed in this article is whether it is possible, with the help of higher dimensions, to define novel geometry related to this connection. This is where locality holds for particles far from the black hole and its nearby entangled pairs.
Here we consider two aspects: 1. The necessity of correlation between particles for smoothness of a region, and 2. The vacuum environment itself is subject to a form of von Neumann entanglement.
Therefore, we can infer that the spacetime continuum is an emerging subject due to entanglement. This suggests that the entanglement between particles gives rise to the smoothness and structure of the spacetime continuum. Furthermore, the entanglement between particles in a vacuum environment creates a form of von Neumann entanglement, which can explain the emergence of the spacetime continuum. This implies that the behavior of particles at the quantum level can be used to describe the universe's structure. As such, this entanglement can be used to understand the spacetime continuum better.
Consequently, a specific form of space is required when particles far from the event horizon are entangled with particles inside them. This additional space extends beyond the dimensional limitations of spacetime confined by the event horizon of the black hole. That's why the membrane that acts as a barrier for everything in spacetime no longer interferes with this connection. The spacetime model sees this as tunneling, which, as we know, should extend to higher dimensions.
One of the most effective approaches to analyzing this perspective more comprehensively while considering dimensional connections is Ads/CFT. This method allows for locality redefinition in QG.

5. Conclusion: unveiling the quantum symphony of spacetime

To understand the fundamental nature of our universe, we journeyed through the intricate tapestry of quantum entanglement, spacetime emergence, and the enigmatic interplay between black holes and quantum information. Our exploration delved deep into theoretical physics, unveiling a symphony of interconnected threads weaving spacetime fabric together.

At the heart of our voyage lies the mesmerizing phenomenon of entanglement. This is a dance of particles that defy distance and time. From the delicate choreography of particles responding instantaneously to their partners' actions to the entwined nature of quantum states transcending boundaries, entanglement emerges as a fundamental force, resonating with cosmic harmonies.

This symphony unfolds in spacetime, where reality is intertwined with entanglement. The emergence of spacetime from the intricate entanglement reveals a deeper connection between geometry and quantum information. Through the lens of AdS/CFT correspondence, we glimpse the unity between gravitational theories and quantum fields, offering a glimpse into the holographic nature of our universe.

Black holes take center stage in a cosmic theater. As we explored their dynamics and interactions with quantum information, we uncovered paradoxes that challenge our understanding of locality, entropy, and information fate. The process of black hole evaporation, driven by the complex interplay of entangled qubits, presents an image of conserving and recovering information that challenges conventional classical understanding.

Throughout this exploration, we came across the profound notion of the thermofield double, which links disparate spacetime domains. The entanglement binding the left and right Rindler wedges and its correlation with the entanglement entropy of black holes points to a fundamental interplay between information and gravitational dynamics on a cosmic scale.

As we stand at the crossroads of discovery, tantalizing future paths beckon us forward. These uncharted territories promise to expand our understanding even further.

1. Quantum Entanglement in Curved Spacetimes and the Holographic Perspective: The exploration of quantum entanglement within curved spacetimes opens doors to unraveling holographic mysteries. The replica trick and island effects offer potential avenues to delve deeper into the holographic nature of information and its intricate connection with spacetime curvature.

2. Tensor Networks: A Path to Unified Description: Tensor networks provide a promising framework for uniting holography, higher dimensions, and spacetime emergence. We may unlock new ways to describe the intricate relationships between entanglement, geometry, and spacetime emergence by harnessing the power of tensor networks.

As we conclude this exploration, the harmonious interplay between quantum information and spacetime dynamics beckons us to further investigate the mysteries beyond. The enigma of information paradoxes and the quest to reconcile quantum mechanics with gravity invite novel avenues of research. This promises to illuminate the deeper layers of our universe's symphony.

In the grand orchestration of reality, we find entanglement, spacetime, and quantum information harmonizing in ways that transcend our conventional understanding—a symphony of spacetime composed of vibrating quantum entanglement threads.

As we gaze toward the uncharted horizons of theoretical physics, we stand at the frontier of discovery. We are well-positioned to unravel the symphonic secrets of the universe's most intricate melodies.

We conclude our journey through quantum entanglement, spacetime emergence, and the dance of black holes and quantum information. This article offers a glimpse into the intricate interplay that shapes the cosmic symphony. It invites readers to contemplate the harmonies that resonate throughout reality. As we explore and unravel the mysteries, we are reminded that the universe's song is still being written—one note at a time; the future beckons with promises of more profound insights and new horizons as we strive to decode the cosmos' symphony.

Ethical approval

This manuscript does not report on or involve the use of any animal or human data or tissue. So the ethical approval is not applicable.

Authors Contributions

All the authors have participated sufficiently in the intellectual content, conception and design of this work or the analysis and interpretation of the data (when applicable), as well as the writing of the manuscript.

Availability of data and materials

Data presented in the manuscript are available via request.

Conflict of Interests

The author declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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