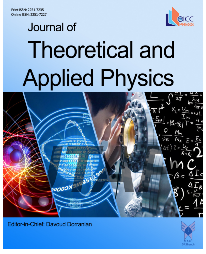

Volume 18, Issue 2, 182419 (1-7)

Journal of Theoretical and Applied Physics (JTAP)

https://dx.doi.org/10.57647/j.jtap.2024.1802.19

The wave-like concept hidden behind the definition of Planck mass

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Original Research

Received:
14 June 2023
Revised:
10 September 2023
Accepted:
24 September 2023

Abstract:

In this paper, the role of physical constants in the definition of Planck mass is reviewed. The hidden concepts of Planck base units are then examined in three quantities: Planck length, Planck time or Planck frequency, and Planck mass. In an extension, a new combination of physical constants reveals the hidden nature behind the definition of Planck mass while establishing a relationship between the two fields of general relativity and quantum physics. Here it is shown that the Planck mass can be considered a wave packet. It is also shown how it is possible to define the origin of primordial black holes. Finally, the frequency characteristics of dark matter and energy are studied based on mass wave nature.

Keywords: Physical constants; Wave nature of matter; Mass definition; Primordial black holes

1. Introduction

A physical constant, also denominated as a universal constant, is a quantity that is part of a combination of fundamental constants of physics. The majority accept that each member of this assortment is unique; some investigations consider that these constants undergo minute changes over time. Each physical constant can admit two fundamental roles: to retain the laws of physics accurately within their definitions; the second is to preserve effective relationships among the group members to maintain fundamental physical theories. Any physical constant expresses a particular numerical value but cannot be measured directly in any independent physical experiment. Some of the well-known physical constants are $c \simeq 2.99 \times 10^{8}$ ms^{-1}, the speed of light in a vacuum or, in principle, the highest possible velocity that can assume for an object, $G \simeq 6.67 \times 10^{-11}$ kg^{-1}m^{3}s^{-2}, the Newtonian gravitational coupling constant, $h \simeq 6.62 \times 10^{-34}$ kgm^{2}s^{-1}, the quantum action or $\hbar = 1.05 \times 10^{-34}$ Js, the quantum momentum action, ε_0 electrical conductivity constant in a vacuum, k_B Boltzmann constant, α fine-structure constant, and e fundamental electric charge [1]. Physical constants can take different dimensional forms. For example, the speed of light-the maximum possible velocity for a massless particle in physics-has a dimension of the length over time, while the fine-structure constant, a characteristic of electromagnetic interaction power, is dimensionless.
Today, the role of these constants in physics has matured so crucially that on November 16, 2018, the International Bureau of Weights and Measures decided to redefine the base units of the International System of Units (SI), i.e., meters, kilograms, seconds, amps, Kelvin, moles, and candles in physical constants [1]. They have emphasized that the new definitions should depend on the most accurately measured physical constants' values. So it can be expected that as the accuracy of experiments in measuring physical constants increases, the base units of physics will change slightly from time to time. However, this is not the only purpose that fundamental constants play in physics. At the turn of the twentieth century, Max Planck introduced different connections between physical constants that relied more on their phenomenological aspects. He presented a collection of physical units known today as Planck units by combining several physical constants. The most critical Planck base units are Planck length, l_p, Planck time, t_p, and

Planck mass, m_p, with values listed below [1]

$$l_p=\sqrt{\frac{\hbar G}{c^3}}=1.62\times10^{-35}\ \text{m} \tag{1a}$$

$$t_p=\sqrt{\frac{\hbar G}{c^5}}=5.39\times10^{-44}\ \text{s} \tag{1b}$$

$$m_p=\sqrt{\frac{\hbar c}{G}}=2.18\times10^{-8}\ \text{kg} \tag{1c}$$

It is worth noting that the Planck units were not the only human effort to introduce a unit system. About thirty years earlier, George Johnstone Stoney proposed another group of physical constants known as Stoney units composed of c, G, e, and k_e, where the last one is the Coulomb constant. Other collections also exist, including atomic units composed of e, $\hbar$, k_e, and m_e, where the last parameter is the rest mass of an electron, and natural units composed of c, $\hbar$, m_e, and ε_0. An essential feature of all three sets mentioned is the existence of electron charge or electron mass in the definition of base units of length, time, and mass. Thus, they can be called a set of units depending on the physical properties of the electron. Another set of units that uses proton mass instead of electron mass in its definitions is called quantum chromodynamic units. Note that the fundamental difference between Planck units and the other sets of units mentioned above is that the Planck units define the length, the time, and the mass independent of the electric charge or mass of fundamental particles. In determining three Planck units, only three fundamental constants (c, $\hbar$, and G) combine differently.

Perhaps it can be said that this independence in the definition of Planck units can create a kind of boundary between different physics fields in the form of an expression such as "The Cube of Theoretical Physics" [2]. Let us consider a cube formed by three perpendicular axes named c (practically $1/c$), $\hbar$, and G, respectively, in the epistemological space of physics. In this case, each of the eight vertices of this cube will represent one of the theoretical physics fields as follows: (0, 0, 0), classical physics or non-relativistic mechanics, (c, 0, 0), special relativity, (0, $\hbar$, 0), non-relativistic quantum mechanics, (0, 0, G), classical or Newtonian gravity, (c, $\hbar$, 0), relativistic quantum mechanics and theory of quantum fields in flat space-time, (c, 0, G), general relativity, (0, $\hbar$, G), nonrelativistic quantum gravity, (c, $\hbar$, G) quantum field theory in curved space-time or quantum gravity. Among these vertices, or rather the areas mentioned above, except the last two, which are still open questions in physics, the rest that has received extensive scientific and humane investments in recent history and especially in the twentieth century for a better understanding of them.

Other applications of Planck base units include the extraction of different physical quantities known as Planck derived-quantities, such as Planck surface, Planck volume, Planck density, Planck energy, and Planck force. Most of these subunits result from several mathematical operations and placing them in the physics equations.

However, the physical concepts and interpretations underlying Planck base units are more significant. The existence of particular structures or theories depends on Planck's base quantities to the extent of the physical world they describe. Examples of this in which the Planck length is considered a measure of space elements include the concept of quantum foam [3] and the size of quantum black holes [4], thus providing the basis of quantum gravity [5]. We can also refer to the length of strings in string theory [6] by Planck length scales. However, there is no general agreement on attributing the shortest possible distance in the physical world to Planck length.

Similarly, people may consider Planck time the shortest period in the physical universe, so even in the standard cosmological model, there is no analysis of the universe's physics before this time. The concept of Planck frequency, which is the inverse of Planck time, can also consider as a maximum in the wave representation of physical phenomena. The only surviving member of this set is Planck mass, which does not yield to definition as the most minor or even most considerable quantitative limit known in physics, unlike its other two counterparts. Examples include the mass of protons, which is 10^{19} times smaller than Planck mass, and the mass of astrophysical black holes, much larger than Planck mass. As a result, there is no expectation that Planck mass plays a role similar to that of its other two counterparts in quantum gravity or the physics of the universe's beginning. Nevertheless, that is not all we may know about Planck mass.

2. The wave nature of Planck mass

What is concluded from Planck units today is more than at the time of these units' presentation. The excellent perception of fundamental particle physics and cosmology has significantly impacted this progress. From the beginning of the twentieth century to the present, the historical course shows that introducing the Planck constant can be considered the key to the story of the particle view of light, energy quantization, and particle-wave duality theory. However, the fact cannot be ignored that no unique and independent definition of mass is available in either particle or wave view. Perhaps, as Planck combined the three physical constants c, G, and $\hbar$ (or h) to represent base units, a new combination that conceptually illustrates a new manifestation of nature could be conceived. This quantity is $(\hbar/cG)^{1/2}$, whose numerical value is

$$\varpi=\sqrt{\frac{\hbar}{cG}}\simeq7.26\times10^{-17}\ kgm^{-1}s. \tag{2}$$

We call ϖ a linear mass density per unit frequency, and its dimension is $\mathrm{ML^{-1}T}$. From this point of view, its numerical value seems to be, in fact, the same as the ratio of Planck mass to the speed of light, with the difference that here the concept of Planck mass does not interfere with the definition of the new combination. It is also evident that linear density comes from a mass distribution over one dimension. In this case, multiplying ϖ by a specific longitudinal interval obtains a mass quantity per unit frequency, while multiplying ϖ by a particular frequency results in a linear density. We get a mass quantity if we multiply ϖ by a specific frequency and length. With the help of limiting values such as Plank length and Planck frequency, some conceptual aspects of

ϖ can be extracted within the definition of known masses in physics. For example, multiplying Plank length by ϖ obtains

$$m_f = \varpi l_p = \sqrt{\frac{\hbar}{cG}}\sqrt{\frac{\hbar G}{c^3}} = \frac{\hbar}{c^2} \tag{3}$$

which is the mass per unit frequency of the photon. Because by multiplying the above equation's two sides by c^2, the photon's energy per unit frequency obtains as the relation $m_f c^2 = \hbar$. Now we only need to multiply a specific frequency by both sides of it so that the equivalence between mass and energy for a moving photon obtains as $mc^2 = hf = \hbar\omega$ (where $\omega = 2\pi f$), which is, in fact, the famous formula of special relativity. This result may consider a photon as an oscillatory Planck length part of space by a frequency f. On the other hand, multiplying Planck frequency (which is the inverse of Planck time) by ϖ obtains

$$m_l = \varpi f_p = \sqrt{\frac{\hbar}{cG}}\sqrt{\frac{c^5}{\hbar G}} = \frac{c^2}{G} \tag{4}$$

which is the mass per unit length or linear density of the mass. This quantity may not be familiar to us in the form shown in the above equation. Still, it is sufficient to multiply both sides of the equation by l to give the equation as $m = (c^2/G)l$, and with a slight displacement, we get the same famous form of $l = Gm/c^2$ which represents the length of the Schwarzschild radius. This result may consider a black hole as an oscillatory part of space by a Planck frequency. Now, if we multiply Eq. (3) by Planck frequency or Eq. (4) by Planck length, we will arrive at an expected result, which is Planck mass. Hence Planck mass can be defined as the embedded or replaced mass in a part of space with the Planck length that fluctuates with the Planck frequency. This result suggests new interpretations of Planck mass as a minimum and maximum value simultaneously. If we consider the Planck frequency the highest frequency limit in nature, then the Planck mass can be defined as the mass equivalent to the most energetic photon that may occur in nature. If we consider Planck length the shortest possible length in nature, then Planck mass can be defined as the lowest mass of a black hole. So the combination $(\hbar/cG)^{1/2}$ can be considered a mass field.

If we look at this assumption from the perspective of the theoretical physics cube, we will see that in $(\hbar/cG)^{1/2}$ quantity, all three fundamental constants of physics are present. This quantity belongs to the vertex (c, $\hbar$, G) or the field of quantum gravity or quantum fields theory in curved spacetime. Simultaneously, according to Eq. (4), by multiplying a particular length, we obtain a quantity that belongs to the vertex (c, $\hbar$, 0) or the field of relativistic quantum mechanics and the theory of quantum fields in flat spacetime. On the other hand, according to Eq. (3), multiplying a specific frequency results in a quantity that belongs to the vertex (c, 0, G) or the field of general relativity. By multiplying a particular length or frequency by $(\hbar/cG)^{1/2}$, we reach an image of it in a conceptual space with a lower phenomenological dimension.

On the other hand, it can be pointed out that the quantity $(\hbar/cG)^{1/2}$ makes the definition of mass depend on two different physical parameters, namely, frequency, which has a wave nature, and length, which has a geometric nature $m \equiv m(l, f)$. Therefore, the two interpretations of Planck mass can be considered a double view rooted in the dual wave-particle theory. Hence, the combination $(\hbar/cG)^{1/2}$ can be considered a mass field, which works well in wave and particle theories.

In this case, using the above concept, we reach two conclusions. First, based on the fact that only waves of a specific frequency can be excited at a given length, it can be expected that the resulting mass would not contain continuous values, or in other words, it will be a quantum quantity. Second, it is possible to specify a range of lengths and frequencies for which the resulting mass would be comparable to that of fundamental particles such as electrons. Of course, it is noteworthy that the obtained mass values are of a qualitative comparative nature regarding the fundamental particles and are independent of their spatial distribution.

3. Photon-black hole incorporation

One might ask, given the definition of Planck mass in the previous section, is it possible to have a spacetime scale in which a black hole with Planck mass is equivalent to a photon with Planck mass? To find the answer to this question, we first consider mathematically the general equation that expresses the differential form of mass in terms of two main variables, frequency, and length, as follows

$$dm = \varpi d(fl) = \sqrt{\frac{\hbar}{cG}}(fdl + ldf) \tag{5}$$

In this case, if we define the mass per unit length at a constant frequency and the mass per unit frequency at a constant length differentially, we will have

$$m_l = (\frac{\partial m}{\partial l})_{f=const.} = \sqrt{\frac{\hbar}{cG}}f, \tag{6}$$

and

$$m_f = (\frac{\partial m}{\partial f})_{l=const.} = \sqrt{\frac{\hbar}{cG}}l, \tag{7}$$

In exceptional cases for $f = f_p$ in the first equation or $l = l_p$ in the second equation, we will have

$$m_l = (\frac{\partial m}{\partial l})_{f=f_p} = \frac{c^2}{G} \tag{8}$$

and

$$m_f = (\frac{\partial m}{\partial f})_{l=l_p} = \frac{\hbar}{c^2} \tag{9}$$

which are the mass of unit length scale of black hole and the mass of unit frequency of a photon, respectively. Now, if we equate the mass of each of the states of Eq. (8) and Eq. (9), $m_l l = m_f f$, then we have

$$\frac{c^2}{G}l = \frac{\hbar}{c^2}f \tag{10}$$

which results, we equate the mass of a photon to the mass of a black hole. With a slight displacement, we will have the coefficients in the above equation

$$\frac{l}{f} = \frac{\hbar G}{c^4} \tag{11}$$

which introduces a quantity as a constraint with the *LT* dimension. On the one hand, this combination represents an estimate of a quantum area of spacetime and, on the other hand, means the dependence of length and frequency on those dimensions

$$\aleph = \frac{\hbar G}{c^4} \approx 8.71 \times 10^{-79} \ \text{ms}, \tag{12}$$

where $\aleph$ is the length per unit frequency. One might say that this value has been obtained by multiplying the Plank length by the Plank time, $\aleph = l_p t_p$. The two are somewhat equal, but Eq. (12) is a constraint on the equality of two masses that expresses Planck mass's concept in two different approaches. One is the mass of a black hole, and the other is the mass of a photon. The equality of the two in the presence of the above constraint means that in spacetime dimensions around $\aleph$ and less, the photon's mass and the mass of the black hole can be equal. As a result, they may be able to become indistinguishable or even have a common nature under 8.71×10^{-79} ms.

Primordial black holes could be one of the closest phenomena to the discussed photon-black hole incorporation could be primordial black holes. Hypothetical black holes appeared shortly after the Big Bang and contributed to the condensation of baryonic matter around them [7]. So far, no valid reason has been provided to describe what these black holes are, and their physical origin is still unknown. Here we can point out that the primordial black holes may be massive wave packets limited to a physical boundary. This perception of primordial black holes can be the beginning of an exciting path for their study from the point of view presented in this article.

4. Frequency dependent energy

The combination of physical constants that led to $\aleph$ reminds us of the Planck constant's role in defining a quantum oscillator's energy. The Planck constant equals the product of energy over time, which ties the photon energy concept to frequency. Similarly, the quantity $\aleph$ defines as the product of length over time. Therefore, it is not far-fetched to consider length as a frequency-dependent quantity with this definition's help. Of course, this interpretation is not valid on all spacetime scales and can only be confirmed on scales below $\aleph$, as the Planck constant holds only to determine the energy of a photon, not the energy of any other classical oscillators. Now, with this result, we can go one step further and use the result of Eq. (12) and the definition of Eq. (2), the effect of length can be removed from the definition of mass as follows

$$m_{f^2} = \sqrt{\frac{\hbar^3 G}{c^9}} \approx 6.33 \times 10^{-95} \ \text{kgs}^2 \tag{13}$$

This result is a definition for mass per unit frequency squared in scales smaller than $\aleph$. In this case, with the help of the mass and energy equivalence relationship, energy per unit frequency squared will be obtained from the following equation

$$E_{f^2} = \sqrt{\frac{\hbar^3 G}{c^5}} \approx 5.68 \times 10^{-77} \ \text{Js}^2 \tag{14}$$

Therefore, at scales more minor than $\aleph$, where the length can also be considered a function of frequency, there will be only one degree of freedom of frequency for the bulk of spacetime. In this case, the energy equation changes from a linear $E = hf = \hbar\omega$ to a quadratic form of frequency as

$$E = \sqrt{\frac{\hbar^3 G}{c^5}} f^2 \tag{15}$$

As the spacetime scales increase to values greater than $\aleph$, the Planck length for the photon and the Planck frequency for the black hole will fix, and the two will be separated. In this case, the photon energy relation reduces to the well-known form of $E = hf = \hbar\omega$, and the black hole mass from the form of Eq. (13) changes to $m = (c^2/G)l$. This is the state that quantum field existence separates from gravitational field existence, the state that particle physicists name as symmetry breaking.

5. Energy-frequency relation

In this section, we calculate the energy-frequency relationship for various items, such as the universe's dark energy and dark matter.

5.1 Dark energy

In physical cosmology, dark energy is another form of unknown universe content that affects the largest scales [8]. The idea of dark energy, which is known to be directly responsible for the accelerated expansion of the universe, arose to justify the observed brightness of supernovae in disagreement with the theory that the universe expands at a constant rate [9–11]. As the essential dark energy model, the cosmological constant represents a constant energy density, which supplies the ultimate universe homogeneously [8]. The cosmological constant accompanying Cold Dark Matter (ΛCDM) indicates that dark energy contributes %69 of the total energy in the current period of the observable universe. Considering that dark matter allocates %26 of the universe's mass and energy in this model, the contribution of ordinary (baryonic) matter (excluding neutrinos and photons) will be only %5 [12]. The equivalent density of dark energy is about $\rho_{DE} \simeq 5.96 \times 10^{-27}$ kgm^{-3} $\simeq 5.36 \times 10^{-10}$ Jm^{-3}.

Following the definition of Eq. (2) for a linear mass density per frequency, here we define a new parameter for a volume mass density per frequency cubed, respectively

$$m_{l^3,f^3} = \sqrt{\frac{\hbar}{c^5 G}} \simeq 8.08 \times 10^{-34} \ \text{kgm}^{-3}\text{s}^3 \tag{16}$$

The energy density per cubic frequency obtains by multiplying the two sides of the above equation by c^2

$$E_{l^3,f^3} = \sqrt{\frac{\hbar}{cG}} \simeq 7.26 \times 10^{-17} \ \text{Jm}^{-3}\text{s}^3 \tag{17}$$

Now, we are in a situation where we can ask an important question. If the density of the universe's energy content is to be described by a wave concept, what should be the physical characteristic of that wave? Preliminary estimates show that a frequency value of about 1.94×10^2 Hz, which belongs to the domain of Super Low-Frequency waves $(30-300$ Hz), in each of three physical dimensions, can produce a minimal dark energy density. This amount of radio waves seems to guarantee the necessary energy for the universe's accelerated expansion.

5.2 Dark matter

Dark matter is a type of matter that has been hypothesized in astronomy and cosmology to explain phenomena that appear to be due to a certain amount of mass greater than the mass observed in the universe [13]. It is established that dark matter does not have any electromagnetic interaction with ordinary matter; instead, its existence and properties can be deduced indirectly through gravitational effects on visible matter, radiation, and the universe's large-scale structure. In galactic scales, the assumption of dark matter resolves the difference between gravitational mass and the observed mass of the luminous material inside them (stars, gas, dust). Jan Oort first proposed this hypothesis in 1932 to explain the stellar velocities of stars in the Milky Way galaxy [14], and then by Fritz Zwicky in 1933 to explain the evidence for "lost mass" in the motion of galaxies in galaxy clusters [15] and [16]. Many of these motions appear relatively uniform, so according to the Virial theorem, the total kinetic energy must equal half the energy of galaxies' gravitational potential. Although empirically, the observed kinetic energy is much higher; to be more precise, if we assume that the existing gravitational mass is due only to the visible matter in the galaxies, the stars that are far from the center of the galaxy have velocities much higher than what the Virial theorem predicts. The diagrams of the galaxy's rotation curves, which show the rotational speed based on distance, cannot be explained using visible matter alone [17]. The simplest way to explain this is to assume that visible matter is only a tiny part of the cluster. Evidence shows that galaxies are made up mainly of an almost spherical halo of dark matter with more focus at the center, with the visible matter at its center like a disk [18] and [19]. Although the scientific community generally accepts the existence of dark matter, alternative theories of gravity, including Modified Newtonian Dynamics (MOND) [20–22], Modified Gravity (MOG) [23], and several actions-based extended theories of gravity, indicated that they attempt to explain these unusual observations without the need to introduce additional mass, which we have refrained from describing them in this brief. To have a proper estimate of the value of frequency from the length-frequency relation for dark matter, one can consider a homogeneous spherically distribution of dark matter as a three-dimensional volume introduced by the length scale $l_{DM} \simeq 10^{20}$ m as a typical radius of spiral galaxies, known as the halo of the galaxy with a mass of about 10^{42} kg ($\approx 10\times$ visible matter in our galaxy). Therefore the average value of dark matter density around a typical galaxy is about $\rho_{DM} \simeq 2.39 \times 10^{-18}$ kgm^{-3}. Then the specific frequency of dark matter for a typical spiral galactic radius of about $l_{DM} \simeq 10^{20}$ m attains an approximate value of $f_{DM} \simeq 1.44 \times 10^5$ Hz. This frequency belongs to the Low Frequency (LF) range of electromagnetic radiation $(30-300$ kHz).

6. Conclusion

Mass is an essential concept in classical mechanics that is the product of a particle-like view of an object. However, we know that a particle can also exhibit wave behavior according to the theory of wave-particle duality. Now it does not seem far-fetched to ask, can any wave property be considered related to the particle's mass? Arguably, this is a question that has not yet been extensively explored. One of the possible options for examining mass from a wave-like hypothesis is the same view we have for energy and momentum. A possible suggestion for explaining this hypothesis is that the elementary particles of the waves originate from a vacuum. Therefore, the difference between them is the same as the difference in the excitation modes of the waves. In this case, mass is no longer an intrinsic property of the particle but a measure of the particle's energy. Inevitably, the relations between energy and mass will be subject to the wave properties of the particles. On the other hand, it will be determined why some particles are wave-like, and others are particle-like. In addition to interpreting the wave nature of mass from a mathematical point of view, we also introduce a parameter from a physical point of view with the help of fundamental physical constants that can explain the mass of a wave arising from a vacuum and can be adapted to different dimensions and scales. It also has some achievements in quantum mechanics and general relativity. A straightforward suggestion in this regard is to examine the results of this fundamental definition in the estimation of dark matter and dark energy. With the help of estimates made for the frequency of dark energy and dark matter alternative waves, it can be concluded that instead of finding the origin of these dark contents, one can look for the origin of cosmic radio waves whose mass and equivalent energy are equal to the mass of dark matter and the amount of dark energy in the universe.

Ethical approval
This manuscript does not report on or involve the use of any animal or human data or tissue. So the ethical approval is not applicable.

Authors Contributions
All the authors have participated sufficiently in the intellectual content, conception and design of this work or the analysis and interpretation of the data (when applicable), as well as the writing of the manuscript.

Availability of data and materials
Data presented in the manuscript are available via request.

Conflict of Interests
The author declare that they have no known competing financial interests or personal relationships

that could have appeared to influence the work reported in this paper.

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