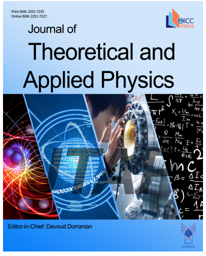

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The effect of nonextensive electrons on the head-on collision of dust-ion acoustic solitons

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Original Research

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Abstract:
The manuscript is about the head-on collision of two dust-ion acoustic solitary waves related to dusty plasma including inertial ions, negatively charged stationary dust particles and nonextensive electrons. For this purpose, using the extended Poincaré–Lighthill–Kuo approach, two Kortwege–de Vries equations with solitonic solutions are derived and also the analytical phase shifts after the head-on collision of two DIAS waves are obtained. The effects of two parameters of nonextensivity and density on the phase shifts are investigated. The results show that the nonextensive electrons has considerable effect on the properties of DIAS waves' collision. We believe that the results of this research can be useful in understanding plasma phenomena such as laboratory plasmas, space plasmas and their applications related to the DIAS waves' collision.

Keywords: Dust ion acoustic; Head-on collision; Phase shifts; Nonextensive

1. Introduction

Researches around the linear and nonlinear wave phenomena in multi-fluid plasmas has achieved an important popularity during the last decades [1–4]. This amount of attention originates from the frequent attendance of these plasma models in active galactic nuclei [5], pulsar magnetosphere [6, 7], solar atmosphere [8] and in the inner regions of the accretion disks surrounding the central black holes [9, 10]. Meanwhile, dust-ion acoustic solitary (DIAS) waves, which are in the category of low frequency waves, have a major role in nonlinear dust system which is ubiquitous in most space and astrophysical media, such as protostellar disks, asteroid zones, circumstellar and interstellar clouds, cometary tails, planetary atmospheres, nebula, interstellar media, Earth's ionosphere, and etc. [11–13]. In this regard, Shukla et al. [14] have theoretically proved that due to the equilibrium charge density conservation of three-component dusty plasma (which is consisting of electron, ion, and negatively charged dust) and the strong inequality $n_{e0} \ll n_{i0}$ (where n_{e0}, and n_{i0} are, respectively, electron, and ion number density at equilibrium) a dusty plasma supports low-frequency dust-ion acoustic (DIA) waves with phase speed much smaller than the electron thermal speed. Later, this assertion was also corroborated in several experimental studies [15, 16] and subsequently the propagation of DIA waves were investigated in different situations [17–19]. It is important to note that in comparison with ordinary linear systems, the propagation of soliton type DIA waves is accompanied by some special features because of their asymptotic form preservation when they undergo a collision [20]. Localized waves in non-linear environments are generally formed from medium dislocations and disturbances which can be happened throughout the system. For this reason, the chance of simultaneous formation of two or more alternating waves is significant. The collision of two or more solitons occurs because of a complicated collective force. Solitary waves collide and then scatter from each other with only small changes in their shapes. In this whole scenario, colliding solitons remain stable by preserving their sizes and shapes. On the other hand, the collision of single solitons formed in different parts of the environment is also a common phenomenon. After each collision, solitons can enhance or reduce their positive/negative phase shift which makes their size and shape unchanged after head-on collision. Therefore, investigation of such behavior is very important. The velocity of the solitary wave solutions in the KdV model, depends on the wave amplitude. Thus, created single solitons in such an environment have different velocities. This issue makes the collision of solitons inevitable. In other words, collision between solitary waves generally is unavoidable, for the multi-soliton states. Gardner et al. [21] showed that when two soliton waves come towards each other and collide, they interchange their energies and exchange their positions with each other and then separate off. Throughout the whole of this process, the solitary waves are remarkably stable entities preserving their identities throughout the interaction. And the only effect created in the solitons due to the collision is their phase shifts. The Poincare-Lighthill-Kuo (PLK) is an effective perturbation method in studying the head-on collision of waves as well as the resonance phenomenon between them [22]. Over the past decades, this technique has been repeatedly applied in the literature to characterize the head-on collision of solitary waves in various plasmas [23–27]. Another point for this investigation is that space plasma receptions testify that the electron populations in the plasma systems can presence in a non-equilibrium form [28–34]. In more general sense, there are many physical systems which cannot be explained correctly using the classical statistical description. Some of them can be described by proper framework of nonextensive statistics. Nonextensive statistical mechanics which is based on the deviations of Boltzmann–Gibbs–Shannon (B-G-S) entropy measure is a non-equilibrium distribution states. Renyi [35] was first proposed an appropriate nonextensive popularization of the B-G-S entropy for state of statistical equilibrium. This concept subsequently developed by Tsallis [36]. He suitably extended the entropies standard additivity to the nonextensive, and nonlinear case where entropy index q as a particular parameter characterizes the degree of nonextensivity. The parameter q that underpins the Tsallis's generalized entropy is related to the system's underlying dynamics and measures the rate of its nonextensivity. Indeed, from the aspect of statistical mechanics and thermodynamics, in the nonextensive systems the whole entropy is distinct from the sum of the respective parts entropies. In other words, for $q < 1$ (superextensivity) the total generalized entropy is more than the sum of the parts entropies, whiles the system generalized entropy is lower than the sum of the entropies of the parts for $q > 1$ (subextensivity). Till now, nonextensive statistics was well utilized to a wide types of astrophysical and cosmological scenarios like the solar neutrino problem [37], stellar polytropes [38], particular velocity distributions of galaxies [39], and also systems with long-range interactions [40, 41]. As an extension to previous investigations, our aim in current research is to evaluate the propagation and head-on collision of DIAS waves in a dusty plasma with nonextensive electrons. In recent years, interesting studies have been done on ion acoustic, dust acoustic and dust-ion acoustic waves as solitary and shock structures in non-extensive and superthermal plasmas [42–45], in which the waves propagation properties have been well studied.

Continuation of the topics in the manuscript is formed as follows. In Section 2, the basic equations which control the desired plasma model are presented. Then, the Korteweg–de Vries (KdV) equation, and the analytical phase shifts and trajectories after the head-on collision of two solitary waves are derived. In Section 3, a detailed discussion of the analytical numerical results obtained from this research are presented. And finally, it is given a summary of conclusions in Section 4.

2. Basic equations

We consider a dusty plasma system containing of cold ion-fluid, nonextensive distributed electrons and static negatively charged dust particles. The nonlinear dynamics of the DIA waves in this system is governed by

$$\frac{\partial n}{\partial t}+\frac{\partial (nu)}{\partial x}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=-\frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2}=\mu n_e-n+(1-\mu) \tag{3}$$

where n, u and ϕ are the number density of ions, the ion fluid speed and the electrostatic wave potential, respectively in which are normalized by the ion equilibrium value (n_0), $C_i=\sqrt{k_BT_e/m_i}$ and k_BT_e/e, respectively. It should be mentioned that m_i is the mass of positively ions, k_B is Boltzmann's constant and T_e is the electron temperature. The time t and the distance x are normalized by the ion plasma frequency $\omega_{pi}^{-1}=\sqrt{m_i/(4\pi n_{i0}e^2)}$ and the Debye redius $\lambda_{Dm}=\sqrt{(k_BT_e)/(4\pi n_{i0}e^2)}$, respectively. We have denoted $\mu=n_{e0}/n_{i0}$. The normalized distribution of nonextensive electrons is also given by [22]

$$n_e=[1+(q-1)\phi]^{\frac{q+1}{2(q-1)}} \tag{4}$$

In the above equation, the parameter q represents the intensity of nonextensivity. It is noteworthy that when $q < -1$, the nonextensive electron distribution (not mentioned here) is unnormalizable. In the extensive limiting mode ($q \to 1$), the electron distribution reverts to the famous Maxwell-Boltzmann velocity distribution.

Now to investigate the collision of two DIA soliton waves, we consider a couple of solitary waves, marked A and B, which are initially far apart and moving towards each other. So, after a while they interact, collide, and eventually depart. We also suppose that both solitary waves have small amplitudes $\sim \sigma$ (where σ is a formal, small perturbation parameter describing the strength of nonlinearity) and between DIA solitons there are weak interactions. In addition, the collision is quasielastic and therefore phase shift occurs in this interaction. Here, to study the influence of collision, we employ an extended Poincaré–Lighthill–Kuo (PLK) perturbation method. According to this method, the dependent variables are expanded as

$$\begin{aligned} n_i &= 1+\sigma n^1+\sigma^2 n^2+\sigma^3 n^3+\dots,\\ u_i &= u_0+\sigma u^1+\sigma^2 u^2+\sigma^3 u^3\dots,\\ \phi &= \sigma\phi^1+\sigma^2\phi^2+\sigma^3\phi^3+\dots. \end{aligned} \tag{5}$$

The stretched coordinates are presented in the following form [28]

$$\begin{aligned} \xi &= \sigma(r - c_1 t) + \sigma^2 P_0(\eta, \tau) + \sigma^3 P_1(\eta, \xi, \tau) + ... \\ \eta &= \sigma(r + c_2 t) + \sigma^2 Q_0(\xi, \tau) + \sigma^3 Q_1(\xi, \eta, \tau) + ... \\ \tau &= \sigma^3 t \end{aligned} \quad (6)$$

where ξ and η indicate the direction of two solitary waves' movement towards each other. c_1 and c_2 are the phase velocity of DIAS waves which are determined later. Substituting Equations (5)-(6) into Equations (1)-(3) and equating the quantities with equal power of σ, we attain a set of coupled equations for different orders of σ as

$$(u_0 - c_1)\frac{\partial n^1}{\partial \xi} + (u_0 + c_2)\frac{\partial n^1}{\partial \eta} + \frac{\partial u^1}{\partial \xi} + \frac{\partial u^1}{\partial \eta} = 0 \quad (7)$$

$$(u_0 - c_1)\frac{\partial u^1}{\partial \xi} + (u_0 + c_2)\frac{\partial u^1}{\partial \eta} + \frac{\partial \phi^1}{\partial \xi} + \frac{\partial \phi^1}{\partial \eta} = 0 \quad (8)$$

and

$$n^1 = \left[\frac{\mu(q+1)}{2}\right]\phi^1 \quad (9)$$

The solutions of Equations (7)-(9) read as

$$\phi^1 = \phi_1^1(\xi, \tau) + \phi_2^1(\eta, \tau) \quad (10)$$

$$n^1 = \left[\frac{\mu(q+1)}{2}\right][\phi_1^1(\xi, \tau) + \phi_2^1(\eta, \tau)] \quad (11)$$

$$u^1 = \frac{1}{c_1 - u_0}\phi_1^1(\xi, \tau) - \frac{1}{c_2 + u_0}\phi_2^1(\eta, \tau) \quad (12)$$

Using the solvability condition, the phase velocities $c_1 = [2/(\mu(q+1))]^{1/2} + u_0$ and $c_2 = [2/(\mu(q+1))]^{1/2} - u_0$ are derived. To find the unknown function ϕ_1^1 and ϕ_2^1 we should apply the next orders. Equations (10)-(12) show that we have two waves at the leading order. One of them, $\phi_1^2(\xi, \tau)$, is traveling to the right, and the other one, $\phi_2^2(\eta, \tau)$, is traveling to the left. From the next order we have

$$\phi^2 = \phi_1^2(\xi, \tau) + \phi_2^2(\eta, \tau) \quad (13)$$

$$n^2 = \left[\frac{\mu(q+1)}{2}\right][\phi_1^2(\xi, \tau) + \phi_2^2(\eta, \tau)] \quad (14)$$

$$u^2 = \frac{1}{c_1 - u_0}\phi_1^2(\xi, \tau) - \frac{1}{c_2 + u_0}\phi_2^2(\eta, \tau) \quad (15)$$

Higher-order approximations for perturbed dusty plasma variables can be inferred by separating the next higher-order terms in σ. Therefore, in the next perturbation order by imposing suitable condition on potential components and velocity component, we have

$$\begin{aligned} -2\left[\frac{1}{(q+1)/2}\right]u^3 = &\int\left(\frac{\partial \phi_1^1}{\partial \tau} + A\phi_1^1\frac{\partial \phi_1^1}{\partial \xi} + B\frac{\partial^3 \phi_1^1}{\partial \xi^3}\right)d\eta \\ &+ \int\left(\frac{\partial \phi_2^1}{\partial \tau} - A\phi_1^1\frac{\partial \phi_2^1}{\partial \eta} - B\frac{\partial^3 \phi_2^1}{\partial \eta^3}\right)d\xi \\ &+ \int\int(C\frac{\partial P_0}{\partial \eta} - D\phi_1^2)\frac{\partial^2 \phi_1^1}{\partial \xi^2}d\xi d\eta \\ &- \int\int(C\frac{\partial Q_0}{\partial \xi} - D\phi_1^1)\frac{\partial^2 \phi_2^1}{\partial \eta^2}d\xi d\eta \end{aligned} \quad (16)$$

where

$$\begin{aligned} A &= \frac{1}{2\lambda^{3/2}}\left[3\lambda^2 - \frac{\mu(q+1)(3-q)}{4}\right], \\ B &= \frac{1}{2\lambda^{3/2}}, \quad C = \frac{2}{\lambda^{1/2}}, \\ D &= \frac{1}{2\lambda^{3/2}}\left[\lambda^2 + \frac{\mu(q+1)(3-q)}{4}\right], \\ \lambda &= \frac{\mu(q+1)}{2} \end{aligned} \quad (17)$$

The above coefficients are placed in the structure that will be introduced below for the collision of two waves. These results are converted to [46] for thermal ion-electron plasmas ($q = 1$, $\mu = 1$). Eqs. (17) consistent with the results obtained in [47] for $\mu = 1$ for the collision of two ion acoustic solitons in nonextensive plasma. Also, these coefficients are in agreement with what has been obtained in the study of collision of two dust-ion acoustic waves with superthermal electrons [48].

Since, the integrated function is independent of $\eta(\xi)$, so the first and second terms in the right hand side of Equation (16) will be proportional to $\eta(\xi)$. Thus, the first two terms of Equation (16) are all secular terms, which must be eliminated in order to avoid spurious resonances as follows

$$\frac{\partial \phi_1^1}{\partial \tau} + A\phi_1^1\frac{\partial \phi_1^1}{\partial \xi} + B\frac{\partial^3 \phi_1^1}{\partial \xi^3} = 0 \quad (18)$$

$$\frac{\partial \phi_2^1}{\partial \tau} - A\phi_1^1\frac{\partial \phi_2^1}{\partial \eta} - B\frac{\partial^3 \phi_2^1}{\partial \eta^3} = 0 \quad (19)$$

Two last terms of Equation (16) are not secular terms in this order, but they can be secular in the next order [49, 50]

$$C\frac{\partial P_0}{\partial \eta} = D\phi_2^1 \quad (20)$$

$$C\frac{\partial Q_0}{\partial \xi} = D\phi_1^1 \quad (21)$$

The corresponding solutions of Equations (18) and (19), which are KdV equations in the reference frames of ξ and η for two side traveling wave, can be expressed as

$$\phi_1^1 = \phi_A \sec h^2\left[\left(\frac{A\phi_A}{12B}\right)^{\frac{1}{2}}(\xi - \frac{1}{3}A\phi_A\tau)\right] \quad (22)$$

$$\phi_2^1 = \phi_A \sec h^2\left[\left(\frac{A\phi_B}{12B}\right)^{\frac{1}{2}}(\eta + \frac{1}{3}A\phi_B\tau)\right] \quad (23)$$

where ϕ_A and ϕ_B are the initial amplitudes of the solitons. From Equations (20) and (21), the variations of leading phase due to the collision can be written as

$$P_0(\eta, \tau) = \frac{D}{C}\left(\frac{12B\phi_B}{A}\right)^{\frac{1}{2}}\left[\tanh\left(\frac{A\phi_B}{12B}\right)^{\frac{1}{2}}(\eta + \frac{1}{3}A\phi_B\tau) + 1\right] \quad (24)$$

$$Q_0(\xi, \tau) = \frac{D}{C}\left(\frac{12B\phi_A}{A}\right)^{\frac{1}{2}}\left[\tanh\left(\frac{A\phi_A}{12B}\right)^{\frac{1}{2}}(\xi - \frac{1}{3}A\phi_A\tau) - 1\right] \quad (25)$$

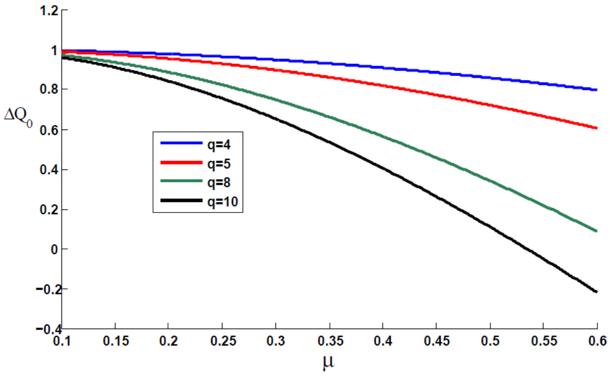

Figure 1. Phase shift variation of DIAS ΔQ_0 against μ $(0.1 < \mu < 0.6)$ for different values of $q = 4$, 5, 8 and 10 when $\phi_A = \phi_B = 1$.

Now, we can see that up to $O(\sigma^2)$, the trajectories of the two solitary waves for weak head-on collisions are

$$\xi = \sigma(x - c_1 t) + \sigma^2 \frac{D}{C}\left(\frac{12B\phi_B}{A}\right)^{\frac{1}{2}}\left[\tanh\left(\frac{A\phi_B}{12B}\right)^{\frac{1}{2}}(\eta + \frac{1}{3}A\phi_B\tau) + 1\right] + \ldots \tag{26}$$

$$\eta = \sigma(x + c_2 t) + \sigma^2 \frac{D}{C}\left(\frac{12B\phi_A}{A}\right)^{\frac{1}{2}}\left[\tanh\left(\frac{A\phi_A}{12B}\right)^{\frac{1}{2}}(\xi - \frac{1}{3}A\phi_A\tau) - 1\right] + \ldots \tag{27}$$

In order to obtain the phase shifts after a head-on collision between two DIA solitons, it is necessary that the two waves are far enough apart from each other at the beginning With this assumption soliton A is at $\xi = 0$, $\eta = -\infty$ and soliton B is at $\eta = 0$, $\xi = +\infty$, respectively. After the collision $(t = +\infty)$, soliton A is at $\xi = 0$, $\eta = +\infty$ and soliton B is at $\eta = 0$, $\xi = -\infty$. From Equations (26) and (27) we can obtain the corresponding phase shifts ΔQ_0 and ΔP_0 as follows [51, 52]

$$\Delta Q_0 = 2\sigma^2 \frac{D}{C}\left(\frac{12B\phi_A}{A}\right)^{\frac{1}{2}} \tag{28}$$

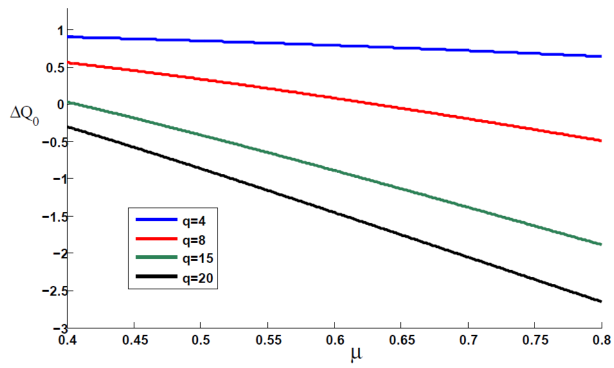

Figure 2. Phase shift variation of DIAS ΔQ_0 against μ $(0.4 < \mu < 0.8)$ for different values of $q = 4$, 8, 15 and 20 when $\phi_A = \phi_B = 1$.

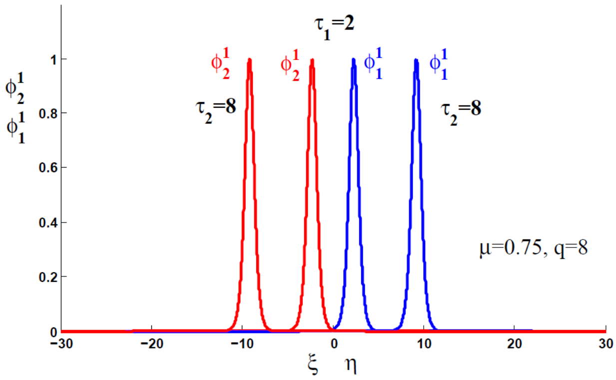

Figure 3. Plot of ϕ_1^1 and ϕ_2^1 against ξ and η, for two values of τ, where $\mu = 0.75$, $q = 8$ and $\phi_A = \phi_B = 1$.

$$\Delta P_0 = -2\sigma^2 \frac{D}{C}\left(\frac{12B\phi_B}{A}\right)^{\frac{1}{2}} \tag{29}$$

In the next section, we proceed with some numerical calculations to identify the behavior of DIAS waves during a head-on collision.

3. Results and discussion

In this section, we numerically show that how the nonextensive quantity q and density ratio μ affect the phase shift and amplitudes ϕ_A and ϕ_B. Since, the propagation path of the soliton A is to the right and for soliton B it is to the left, we find out from Equations (26) and (27) that each soliton has a negative phase shift in its traveling direction due to the collision. A negative phase shift means that the velocity of the wave increases after the collision. The magnitudes of the phase shifts are associated with some physical parameters such as μ, σ, q, ϕ_A and ϕ_B. We have presented this intricate dependence using some graphs. In our results all physical quantities suppose dimensionless, and we consider $\sigma = 1$ and $\phi_A = \phi_B = 1$.
Figures 1 and 2 represent the variation of the phase shift ΔQ_0 with μ for different values of q. Figure 1 shows the variation of the phase shift (ΔQ_0) with the concentration of the negatively charged dust particles $0.1 < \mu < 0.6$ for different values of q-nonextensive parameters. It is seen that ΔQ_0 decreases when μ increases and so in the presence of

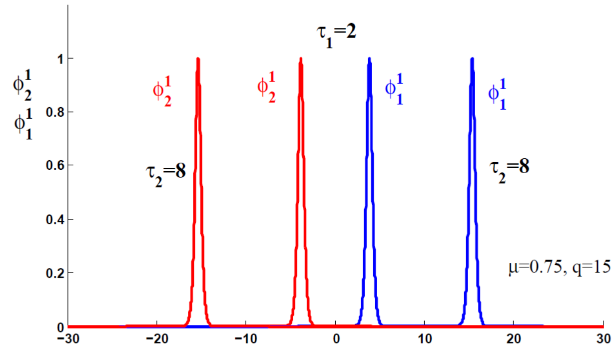

Figure 4. Plot of ϕ_1^1 and ϕ_2^1 against ξ and η, for two values of τ, where $\mu = 0.75$ and $\phi_A = \phi_B = 1$.

dust particles, the velocity of solitary waves decreases after the head collision. It is also observed that ΔQ_0 decreases as q-parameter increases. This means that the phase shift made in the dust ion acoustic solitons passing through the head-on collision is weaker in the non-extensive plasma. The results also is in agreement with the result obtained in [47] for ion-electron ($\mu \to 1$) nonextensive plasma. But the important and interesting point of this research is where some values of this plasma model allow no phase shift to occur in the colliding solitons (ΔQ_0). For example in Figure 1; when $q = 10$ and the relative density is equal to 0.534 , ΔQ_0 becomes zero. That is, in this situation, there is no phase shift between two incident and passing waves. Values of density in which the phase shift vanishes ($\Delta Q_0 = 0$) are called critical values ($\mu = \mu_c = 0.534$).

In Figure 2, the phase shift is plotted for, $0.4 < \mu < 0.8$ and different values of q-parameter. It can be seen that ΔQ_0 is negative for some values of nonextensive parameter, which means that the velocity of the passing waves increases after the head-on collision. Therefore, the higher the non-extensive property of the plasma, the lower the wave propagation velocity after the collision. There are critical values in this situation too. Figure 2 shows that for two values of q ($q = 8$ and $q = 15$), the critical values of relative density will be 0.633 and 0.408, respectively. Obviously, this phenomenon does not happen in the thermal e-i plasma ($q = 1$) [47]. Therefore, it can be said that in the presence of negative dust particles in the nonextensive plasma, there is a possibility that the wave velocity does not change before and after the head-on collision. In other words, the presence of negative dust particles and nonextensive electrons in the plasmas causes the dust ion acoustic solitons do not lose their energies when they head-on collision and so continue to move at the same velocity as before. A similar case of the disappearance of the phase shifts in quantum plasma has also been reported [49].

In Figures 3 and 4, the time evolution the waves in head on collision (ϕ_1^1 and ϕ_2^1) is depicted with two values of the non-extensive parameters. It is seen that the soliton ϕ_1^1 shifted toward right whereas solution ϕ_2^1 shifted toward left as time increases. It can be seen that the wave velocity is more in Figure 4 where the non-extensivity in the plasma is higher. This is consistent with our analysis because Figure 2 shows that as q increases for $\mu = 0.75$, the phase shift increases with negative values.

4. Conclusion

As we know, solitary waves undergo small changes in their amplitude and phase after head-on collision. The resulting changes depend on the important features of the system environment. As we will present in this work, by calculating/measuring the changes made in the scattered solitons after the collision, we can obtain critical information about some important characteristics of the plasmas. In this manuscript, we have theoretically investigated the head-on collision of dust-ion acoustic solitary (DIAS) waves in electron-ion-dust plasma when the electrons distribution is nonextensive. For this purpose, by employing the extended Poincaré–Lighthill–Kuo (PLK) perturbation method and numerical results, the leading-order analytical phase shifts of head-on collisions between two DIAS waves were derived. Analytical calculations showed that the phase shift of DIAS waves directly depends on the electron-ion density ratio and also the nonextensive parameter. It was shown that the phase shift values can be positive, zero or negative. Higher-order corrections, not considered here, may give some secondary structures in the collision event, especially, for the larger wave amplitude case. The present results of our investigation are believed to be useful in explaining the collective phenomena related to DIAS waves collisions that are of vital importance in tokamaks and in space plasmas.

Ethical approval
This manuscript does not report on or involve the use of any animal or human data or tissue. So the ethical approval is not applicable.

Availability of data and materials
Data presented in the manuscript are available via request.

Conflict of Interests
The author declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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