

# Influence of varying magnetic field on ion acoustic solitary waves in dissipative plasma

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## Original Research

## Abstract:

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In this paper, we study obliquely propagating of small amplitude ion acoustic waves (IAWs) in non-relativistic cold plasma in which the ions are viscous fluid, electrons distribution is Maxwellian and external magnetic field varies in space. Using the reductive perturbation method, a nonlinear equation which complies with Korteweg–de Vries–Burgers (KdVB) equation is derived for this model. It is shown that a new effective dissipative which depends on the ion kinematic viscosity and varying in the magnetic field is appeared in the plasma. We show that the complete set of equations, by considering the varying magnetic field and viscosity effect, create IA waves which radiate energy as oscillatory shock wave during their travelling in the medium.

**Keywords:** Magnetized plasmas; Viscous fluid; Modified KdV-Burgers equation; Solitary and shock waves

## 1. Introduction

The study of nonlinear physical structures is of particular importance and attractiveness among scientists. Nonlinear waves and plasma phenomena such as soliton and shock arising out of competition between nonlinearity, dispersion and dissipation properties and can be investigated in space environment and laboratory. One of the most popular plasma waves to study among the authors is the nonlinear ion-acoustic wave. The first experimental observation of ion-acoustic solitons was made by Ikezi et al. [1]. The Viking spacecraft [2] and Freja satellite [3] have reported the existence of soliton-shaped structures in the magnetosphere. Ion-acoustic shock structures were first observed experimentally by Anderson et al. [4]. Also, Taylor et al. [5] observed the ionic shock structures in Double Plasma device. There has been also reported several observations proving the existence of nonlinear profiles, specially shocks in astrophysical plasmas [6–8]. In fact, the dissipative effects due to the heating, particle acceleration and particle trapping are sources for creating shock wave formation. On the other hand, numerous observations of space plasmas and laboratory findings [9–12] indicate clearly the presence of external magnetic field on the plasma. Therefore, many authors have investigated different aspects of waves

to understand behavior of magnetized astrophysical plasmas and plasma devices [13–16]. While in all of these studies, the magnetic field is considered constant, there are only a very few studies with variable magnetic field [17–20]. Schaeffer et al. [17] studied on the laser-driven magnetic piston, where the generated background magnetic field due to the piston pressure changes between 200 and 1500 G and finally they could describe set of shock waves for this model. To complete the previously researches, we consider a magnetized dissipative plasma with thermal electrons and non-uniform magnetic field. The dissipation is taken into account the kinematic viscosity among the ion particles. The basic equations are presented for this model in section II. Numerical simulations results based on the continuity, motion and Poisson equations to understand ion viscosity and non-uniform magnetic effects are presented in section III. In the section IV, the reductive perturbation technique is used to obtain the differential equation describing the wave. Based on the descriptive equation, the change in the initial wave form while moving in the plasma is numerically analyzed in this section too. Some concluding remarks will be provided in Sec. V.

## 2. Basic equations of plasmas in space dependent magnetic field

Consider a collisionless dissipative magnetized plasma comprising of cold ions and thermal electrons. We consider the external magnetic field on the  $z$  axis direction and there is an ion viscosity effect in the plasma model. Such plasmas are described by the following equations

$$\begin{aligned} \frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{u}_i) &= 0 \\ \frac{\partial \mathbf{u}_i}{\partial t} + (\mathbf{u}_i \cdot \nabla) \mathbf{u}_i &= \frac{-e}{m_i} (\nabla \phi + \mathbf{u}_i \times \mathbf{B}) + \eta \nabla^2 \mathbf{u}_i \\ \nabla^2 \phi &= \frac{-e}{\epsilon} (n_i - n_e) \end{aligned} \quad (1)$$

where  $n_i$ ,  $\mathbf{u}_i$ ,  $\phi$  and  $\eta$  are the ion number density, the ion fluid velocity, the electrostatic potential and the kinematic viscosity, respectively. We assume that the wave is propagating in the  $x$ - $z$  plane. Thus, the normalized form of the main equations 1 will be as follows [15]

$$\begin{aligned} \frac{\partial n}{\partial t} + \frac{\partial (nu_x)}{\partial x} + \frac{\partial (nu_z)}{\partial z} &= 0 \\ \frac{\partial u_x}{\partial t} + (u_x \frac{\partial}{\partial x} + u_z \frac{\partial}{\partial z}) u_x &= -\frac{\partial \phi}{\partial x} + bu_y + \eta \frac{\partial^2 u_x}{\partial x^2} \\ \frac{\partial u_y}{\partial t} + (u_x \frac{\partial}{\partial x} + u_z \frac{\partial}{\partial z}) u_y &= -bu_x \\ \frac{\partial u_z}{\partial t} + (u_x \frac{\partial}{\partial x} + u_z \frac{\partial}{\partial z}) u_z &= -\frac{\partial \phi}{\partial z} + \eta \frac{\partial^2 u_z}{\partial z^2} \\ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} &= n_e - n_i \end{aligned} \quad (2)$$

in which  $n$ ,  $\mathbf{u}$  and  $\phi$  are previously defined parameters normalized by  $n_{i0}$  (equilibrium value of ion number density),  $C_i = \sqrt{k_B T_e / m_i}$  and  $k_B T_e / e$  respectively, while  $k_B$  is the Boltzmann's constant and stands as the mass of positively charged ions.  $\eta$  has been normalized by  $\lambda_D \omega_{pi}$ . The time  $t$  and the distance  $r$  are normalized by the ion plasma frequency  $\omega_{pi}^{-1} = \sqrt{m_i / n_{i0} e^2}$  and the Debye length  $\lambda_D = C_i / \omega_{pi}$  respectively. The key variable in our problem is the space dependent parameter  $b_r = \omega_B / \omega_{pi}$  where  $\omega_B = eB(r) / m_i$ . Note that the magnetic field in our model is a function of space ( $r$ ). The electron distribution is Maxwellian, so it is given as:

$$n_e = e^\phi \quad (3)$$

### 3. KdV-Burgers equation

In order to investigate the ion acoustic wave behavior in the considered plasma model, we construct a weakly nonlinear theory of the electrostatic waves with small but finite amplitude which leads to a scaling of the independent variables through the stretched coordinates  $\xi = \epsilon^{1/2}(l_x x + l_y y + l_z z - \lambda t)$ ,  $\tau = \epsilon^{3/2} t$ , and  $\eta = \epsilon^{1/2} \eta_0$ ; where  $\epsilon$  is a small dimensionless parameter measuring the weakness of the dispersion and nonlinearity. Parameters  $x$ ,  $y$  and  $z$  are the directional cosines of the wave vector  $k$  along the  $x$ ,  $y$  and  $z$  axes, respectively, so that  $l_x^2 + l_y^2 + l_z^2 = 1$ . The  $\lambda$  is unknown phase velocity which will be determined later. In

the above transformation  $\lambda$  is normalized by  $C_i$ . We also expand  $n$ ,  $u_{x,y,z}$  and  $\phi$  in a power series of  $\epsilon$  as follows;

$$\begin{aligned} n &= 1 + \epsilon n_1 + \epsilon^2 n_2 + \dots \\ u_x &= \epsilon^{3/2} u_{1x} + \epsilon^2 u_{2x} + \dots \\ u_y &= \epsilon^{3/2} u_{1y} + \epsilon^2 u_{2y} + \dots \\ u_z &= \epsilon u_{1z} + \epsilon^2 u_{2z} + \dots \\ \phi &= \epsilon \phi_1 + \epsilon^2 \phi_2 + \dots \end{aligned} \quad (4)$$

Now we use the stretched coordinates and expansions Eq.4 in Eq.3, and collect same terms in different powers of  $\epsilon$ . The final result is the KdV-Burgers (KdVB) equation yields;

$$\begin{aligned} \frac{\partial \phi_1}{\partial \tau} + l_z \phi_1 \frac{\partial \phi_1}{\partial \xi} + \frac{1}{2} l_z (1 + \frac{1-l_z^2}{b^2}) \frac{\partial^3 \phi_1}{\partial \xi^3} \\ + \frac{1}{2} \left\{ l_z (1-l_z^2) \frac{\partial}{\partial \xi} \left( \frac{1}{b^2} \right) - \eta_0 \right\} \frac{\partial^2 \phi_1}{\partial \xi^2} = 0 \end{aligned} \quad (5)$$

Now it can be claimed that we have obtained an equation that describes the evolution of small amplitude IA solitary waves in this environment. It may be noted that the coefficient of the dissipative term in Eq. 5 has a spatial function, through non-uniform magnetic field encoded in  $b(\xi)$ . It is clear that the important parameter "b" in the above equation is related to the variable magnetic field. In fact, Eq. 5 shows that in addition to the ion-viscosity, variation of magnetic field can be a source of dissipation for shock structure production in a plasma. Also one can find easily from the mentioned equation  $l_z$ , the cosine angle, plays a serious role in the characteristics of solitary waves and their propagation. When the magnetic field is almost in the wave propagation direction ( $l_z \approx 1$ ), dissipative term due to magnetic field variation goes toward zero and thus the dissipation effect of magnetic field reduces, while for greater angles it becomes important, i.e. solitons emerge noticeable amounts of energies.

As a reminder, the general form of the KdV-Burgers equation is:

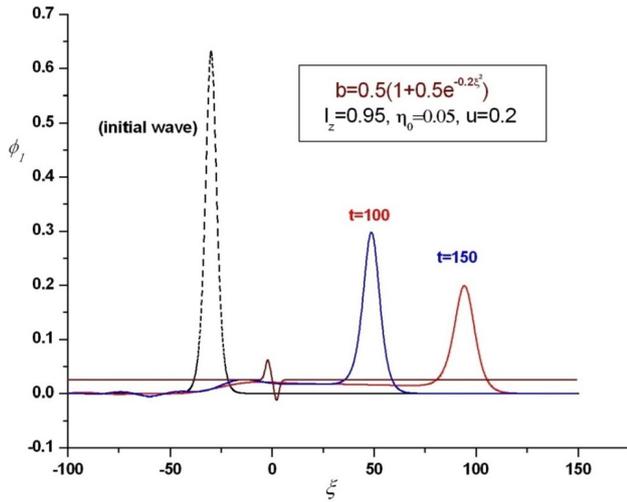
$$\frac{\partial \phi_1}{\partial \tau} + A \phi_1 \frac{\partial \phi_1}{\partial \xi} + B \frac{\partial^3 \phi_1}{\partial \xi^3} + C \frac{\partial^2 \phi_1}{\partial \xi^2} = 0 \quad (6)$$

In a modeled plasma with uniform magnetic field, evolution equation is a usual KdVB equation [16]. In the absence of dissipative term ( $C = 0$ ), nonlinear and dispersion effects may cancel each other so that the result becomes a stable solitary wave as [13, 14]:

$$\phi_1 = \phi_0 \operatorname{sech}^2 \left( \frac{x}{w} \right) \quad (7)$$

where  $x = \xi - u\tau$ ,  $\phi_0 = 3u/A$  is the soliton amplitude, and  $w = 2\sqrt{B/u}$  is its width and  $u$  is the soliton velocity. In our research, when the dissipative term exists ( $C \neq 0$ ), solitary solution changes into a shock profile and in this case soliton emerges amounts of energy radiation during its evolution in dissipative medium [21].

As it mentioned, Eq. (5) shows that the dissipative term appears in the presence of the ion kinematic viscosity and non-uniform magnetic field as a function of spatial position.

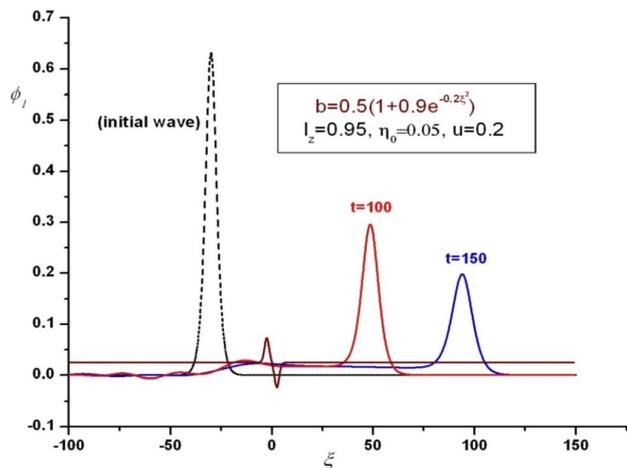


**Figure 1.** Time evolution of the normalized soliton while interacting with  $b = 0.5(1 + 0.5 \exp^{-0.2\xi^2})$  in the presence of  $\eta_{eff}$  perturbation when  $l_z = 0.95$ ,  $\eta_0 = 0.05$  and  $u = 0.2$ .

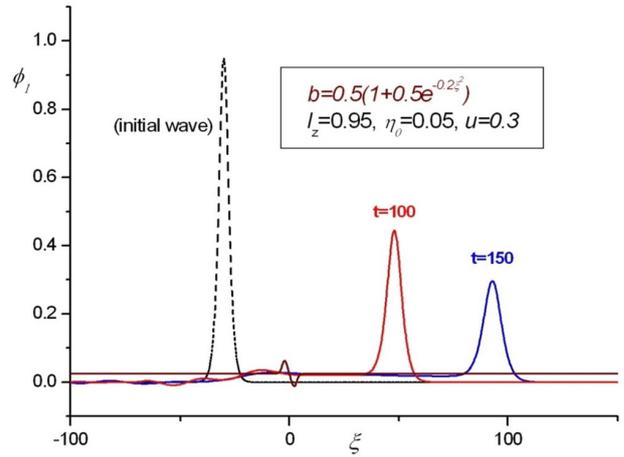
Therefore, we expect that stable solitons radiate energy as shock waves while moving in a variable magnetic field. It is an important result in most of plasmas like the earth atmosphere consisting of a viscous ion fluid. It is clear that the magnetic field around the Earth is not constant, thus we expect solitary waves change into shock waves while moving in the atmosphere. In order to study effects of varying magnetic field and ion-viscosity on the behavior of IA solitary waves, we consider a simple variation in the magnetic field as  $B = B_0(1 + B_p \exp^{-\alpha x^2})$  [22].

#### 4. Numerical results and discussion

In this section a comprehensive study on the features of localized waves is carried out. Normalized function of the magnetic field which appears in the equation of motion has the form of  $b = b_0(1 + b_m \exp^{-\alpha \xi^2})$  and in this case, dissipative term in Eq. 5 is presented as

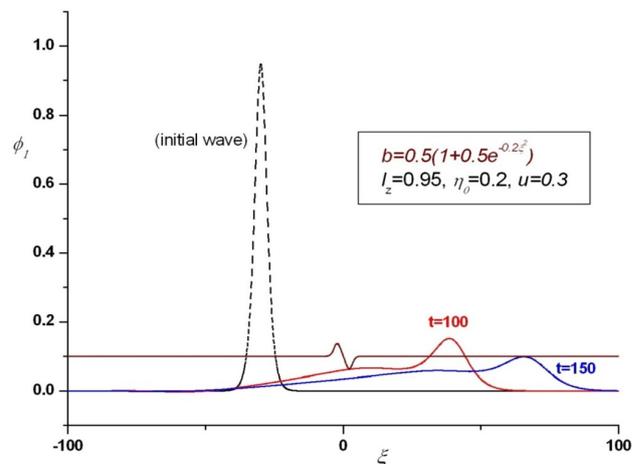


**Figure 2.** Time evolution of the normalized soliton while interacting with  $b = 0.5(1 + 0.9 \exp^{-0.2\xi^2})$  in the presence of  $\eta_{eff}$  perturbation when  $l_z = 0.95$ ,  $\eta_0 = 0.05$  and  $u = 0.2$ .

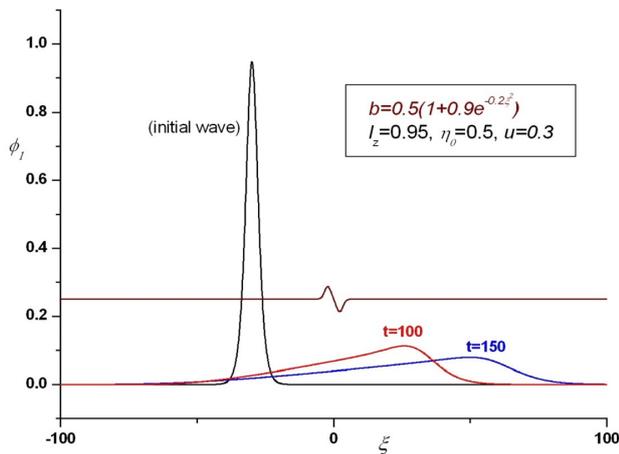


**Figure 3.** Time evolution of the normalized soliton while interacting with  $b = 0.5(1 + 0.5 \exp^{-0.2\xi^2})$  in the presence of  $\eta_{eff}$  perturbation when  $l_z = 0.95$ ,  $\eta_0 = 0.05$  and  $u = 0.3$ .

$\eta_{eff} = \frac{1}{2}[l_z - (1 - l_z^2) \frac{d}{d\xi}(\frac{1}{b^2}) - \eta_0]$ . At positions far from the varying magnetic field, the external magnetic field governing the plasma is uniform ( $b = b_0$ ). Therefore, the wave is initially in the solitary form which goes toward the varying magnetic field and finally spoils while passing the perturbed region. There has not known exact solution for the KdVB equation with space dependent coefficients. Thus we have to use numerical calculation for simulating the evolution of the localized solution in varying magnetic field. We solved equation (5) using the Runge-Kutta technique for time derivation and finite difference method for space derivations. The grid spacing has been taken  $\Delta\xi = 0.0001$  and  $0.005$  (as cross check for numerical stability of solution) and time grid spacing has been chosen as  $\Delta\tau = 0.0001$ . It is clear that the center of variation is  $\xi = 0$  and we also assume that directional cosine ( $l_z$ ) is 0.95. The effect of perturbation term ( $\eta_{eff} = \frac{1}{2}[l_z - (1 - l_z^2) \frac{d}{d\xi}(\frac{1}{b^2}) - \eta_0]$ ) on IA solitary waves is discussed in Figs. 1-5 and it has



**Figure 4.** Time evolution of the normalized soliton while interacting with  $b = 0.5(1 + 0.5 \exp^{-0.2\xi^2})$  in the presence of  $\eta_{eff}$  perturbation when  $l_z = 0.95$ ,  $\eta_0 = 0.2$  and  $u = 0.3$ .

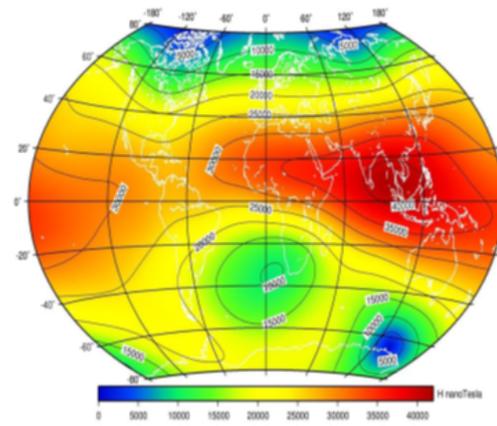


**Figure 5.** Time evolution of the normalized soliton while interacting with  $b = 0.5(1 + 0.5\exp^{-0.2\xi^2})$  in the presence of  $\eta_{eff}$  perturbation when  $l_z = 0.95$ ,  $\eta_0 = 0.5$  and  $u = 0.3$  values.

plotted by brown. These figures clearly show that the two effects of viscosity and non-uniform magnetic field affect the ion acoustic wave. At first, we study the effect of non-uniform magnetic field with different slopes in the weak viscosity ( $\eta_0 = 0.05$ ) in Fig. 1 and Fig.2. It is observed that initial soliton at  $\xi = -30$  with speed  $u = 0.2$  moves toward the  $b = 0.5(1 + 0.5\exp^{-0.2\xi^2})$  (Fig. 1) and  $b = 0.5(1 + 0.5\exp^{-0.2\xi^2})$  (Fig. 2) at  $\xi = 0$  when the ion fluid velocity ( $\eta_0$ ) is 0.05 and then it passes through the magnetic field. One can see that IA solitary waves can be propagated with harmonic style and shorter amplitude after interaction with magnetic field perturbation at two times ( $t = 100$  and  $t = 150$ ). These figures also demonstrate that a backward propagating shock structure at the location of varying magnetic field. Amount of the backward wave becomes greater for non-uniform magnetic field with larger magnitude. It is also observed that the propagation velocity of the perturbed wave does not depend on the magnetic field intensity.

To investigate the dissipative effect ( $\eta_0$ ) on the behavior of IA solitary waves in presence of varying magnetic field, numerically results are presented in figures 3-5 for a plasma with  $b = 0.5(1 + 0.5\exp^{-0.2\xi^2})$ ,  $l_z = 0.95$ ,  $u = 0.3$  and different values of  $\eta_0$  ( $\eta_0 = 0.05, 0.2$  and  $0.5$ ). It is obvious that the waveform changes after passing through the field perturbation and in the presence of plasma viscosity coefficient. From these figures, it can be easily concluded that the wave velocity and amplitude decrease and the soliton wave becomes as monotonic shock wave. An interesting and important point in this study is observed in these figures where backward oscillations produced by the non-uniform magnetic field are not emitted in high viscosity plasma.

Non-uniformity of magnetic field intensity in natural plasma and plasma devices is very probable. Figure 6 shows the changes in the intensity of the Earth's magnetic field [23]. It can be seen that the intensity of the horizontal component of the magnetic field reaches from 1000 nT at



**Figure 6.** Horizontal component of magnetic field intensity on the earth.

the  $-40^\circ : 0$  position to 30000 nT at the  $20^\circ : 0$  position. Then it decreases again to 10,000 nT at  $70^\circ : 0$ . Therefore, in this investigation, Gaussian behavior is chosen for non-uniform magnetic field variation.

In this study, we showed that the behavior of the harmonic wave (soliton) is unchanged in the presence of a uniform magnetic field, while if the intensity of the external magnetic field which governs the plasma environment is non-uniform, the wave is perturbed. The physical reason for this phenomenon is the non-uniform effect of the magnetic field on the vibrations of plasma particles in different parts of the plasma environment. More precisely, the magnetic force acting on the charged particles in different parts of the plasma is not the same, so the oscillating motion of the particles and subsequently the production of thin and dense regions will not be uniform, and in this case, ion wave propagation will be disturbed. In this condition, the harmonic wave will propagate as a shock wave under the influence of the non-uniform magnetic field. Therefore, it is possible to introduce the non-uniform magnetic field as a new factor in the production and propagation of shock wave.

## 5. Conclusions

We have investigated the effects of space dependent magnetic field and viscosity on the nonlinear propagation of small amplitude ion acoustic waves in electron-ion plasmas. Time evolution equation of small amplitude waves for plasmas is derived, which dissipative term emerges from space derivation of magnetic field and the kinematic viscosity among the ion particles.

Our derived equation shows that as an actual situation when magnetic field is non-uniform, solitary waves moving with some dispersions, related to the rate of changes in magnetic field. Our results also show that in such plasma, the solitary and shock waves of ion acoustic wave can exist and also the weak viscosity effect of the ion particles contributes to the stability of the IAW form in passing through the varying magnetic field. We show that some backward shock wave occurs in the wave propagation due to the spatial dependence of the magnetic field. Such problem has not been studied before. We have presented a compact form

for varying parameter in the plasmas which can be used for several plasmas with different distribution, constituent and.... Therefore, this method can be used as a starting point in upcoming investigation. The results obtained from this study are helpful to realistic understanding of ion acoustic wave behavior for viscosity plasma in a non-uniform magnetic field.

This investigation can be repeated about other sound waves such as dust acoustics and electron acoustic with non-thermal distribution of particles [24–29] and even its effect in the presence of dust particles with variable charge [30, 31].

## Appendix I:

Using the following plasma parameters expansion

$$\begin{aligned}n &= 1 + \varepsilon n_1 + \varepsilon^2 n_2 + \dots \\u_x &= \varepsilon^{3/2} u_{1x} + \varepsilon^2 u_{2x} + \dots \\u_y &= \varepsilon^{3/2} u_{1y} + \varepsilon^2 u_{2y} + \dots \\u_z &= \varepsilon u_{1z} + \varepsilon^2 u_{2z} + \dots \\\phi &= \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \dots\end{aligned}$$

in the basic equations 1 and the use of the stretched coordinates  $\xi = \varepsilon^{1/2}(l_x x + l_y y + l_z z - \lambda t)$ ,  $\tau = \varepsilon^{3/2} t$ , and  $\eta = \varepsilon^{1/2} \eta_0$ , the first order perturbed parameter are;

$$\phi_1 = u_{1z} = n_1, \quad \lambda = l_z \quad (\text{A1})$$

$$u_{1x} = \frac{l_y}{b} \frac{\partial \phi_1}{\partial \xi}, \quad u_{2x} = \frac{\lambda}{b} \frac{\partial u_{1y}}{\partial \xi} \quad (\text{A2})$$

$$u_{1y} = \frac{l_x}{b} \frac{\partial \phi_1}{\partial \xi}, \quad u_{2y} = \frac{\lambda}{b} \frac{\partial u_{1x}}{\partial \xi} \quad (\text{A3})$$

And the second order perturbed parameters are;

$$\begin{aligned}\frac{\partial n_1}{\partial \tau} - \lambda \frac{\partial n_2}{\partial \xi} + l_x \frac{\partial u_{2x}}{\partial \xi} + l_y \frac{\partial u_{2y}}{\partial \xi} + l_z \frac{\partial}{\partial \xi} (n_1 u_{1z} + u_{2z}) \\ - \eta_0 \frac{\partial^2 u_{2x}}{\partial \xi^2} = 0\end{aligned} \quad (\text{B1})$$

$$\frac{\partial u_{1z}}{\partial \tau} - \lambda \frac{\partial u_{2z}}{\partial \xi} + l_z u_z \frac{\partial u_{1z}}{\partial \xi} + l_z \frac{\partial \phi_2}{\partial \xi} = 0 \quad (\text{B2})$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} = \phi_2 + \frac{1}{2} \phi_1^2 - n_2 \quad (\text{B3})$$

At this step, by substituting Eqs. A in Eqs. B and removing the parameters the second order perturbed parameters (index 2), equation 5 can be obtained.

### Ethical approval:

This manuscript does not report on or involve the use of any animal or human data or tissue. So the ethical approval does not applicable.

### Authors Contributions:

All authors contributed equally to prepare, analyze data, and writing the paper.

### Availability of data and materials:

There is no data available to present this work.

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The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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