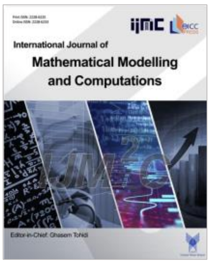

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A Comprehensive Literature Review in Fuzzy Susceptible-Infected-Recovered (SIR) Model and its Applications

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Abstract

Modelling the Susceptible-Infected-Recovered (SIR) for an individual is an essential tool for studying several real-life complications. When uncertainties are involved in the real-life model then its details analysis and solutions interpretations are more critical to find. Also, it should be noted that, crisp model has some limitations to predict the actual facts. To overcome this limitation and challenges, fuzzy SIR models take an important part of fuzzy logic methodology with classical epidemiological contexts. The fuzzy logic is also permitting for imprecise and ambiguous information management systems. In that context the paper's motivation comes. This paper endowments a comprehensive literature review work of fuzzy SIR based models and their applications. The paper examines the key methodological developments and model-based applications. The review also recognizes the current challenges and future research prospects for theoretical and modelling perspectives. Inclusively, this study delivers an in-depth discussion of how fuzzy theory boosts the trustworthiness and applicability of SIR modelling.

Keywords: SIR model, Fuzzy Sets theory, Epidemic Modelling

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1. Introduction

1.1 SIR model

The Susceptible-Infected-Recovered (SIR) model [1,2] is an important model for disease dynamics [3,4]. It has wide applications for understanding and monitoring the spread of infectious diseases-based problems [5,6] which are based on real life issues. It is mainly used to analyse how diseases circulate through a certain population and how it affects them. It also estimates several key parameters and forecasts the course and duration of spreads of epidemics. Several health sector related authorities also apply the SIR model to assess the probable impact of interventions like vaccination among populations [7], isolation of some areas [8], and social distancing [9] between some individuals or group of individuals, thereby aiding in effective epidemic management system [10] and do the possible resource division. Moreover, it serves as an essential framework for pretending various outbreak situations and evaluating precautionary strategies [11] before implementation. Elsewhere epidemiological based problems, the SIR model has also been applied to study information dispersal in social networks system studies, computer virus transmission-based problems, and in simple ecological population dynamics [12] by making it an adaptable tool for modelling procedures across varied systems. SIR models have the following applications for modelling different diseases like COVID-19 [13], Swine flu [14], Dengue fever [15], Influenza [16] etc.

1.2 Fuzzy set theory

The theory of uncertainty is very important for real life modelling. The theoretical framework for uncertainties remains an interesting topic for decision makers. Numerous approaches exist in the view of uncertainty quantifications. Fuzzy sets are one of them which has played significant role during the last several decades. The term Fuzzy set is coined by Prof. L.A. Zadeh in 1965 [17]. Several improvements are seen after the first theoretical logic-based paper. For example, in [18] several authors show how these approaches are considered and applicable for solution purposes of complex uncertainty-based modelling. Comparisons of similar type ideas are focused in the research item [19-21]. As fuzzy set concepts already come, the idea of fuzzy function has already been introduced in several works [22,23]. In differentiability related fuzzy functions are also addressed in [24,25] etc. Note that rather than fuzzy function there is a concept of fuzzy number. The number [26,27] have to follow some specific operations and axioms. With respect to different behaviour and nature of the data set the fuzzy number formation of different shapes. Few are like Gaussian fuzzy number [28], parabolic lock fuzzy number [29], type 2 fuzzy number [30], spherical fuzzy number [31], pentagonal fuzzy number [32] and dense fuzzy number [33] etc.

In the context of applications fuzzy sets become an important tool as well as techniques to capture the uncertainties. It is not restricted to any specific field. The fields like science, engineering management and social sciences have lots of scope to work with fuzzy mathematics. Here are some specified field where researcher have already work like industrial engineering [34], biomathematics [35], thermodynamics [36], car selection [37], social sciences [38], ecosystem analysis [39], multi attribute decision making [40], site selection problem [41], urban development [42] and battery selection [43] etc.

As our review paper is based on the SIR model, here are two type of approaches we may easily use. First one is a mathematical model-based problem and another one is data analytics. So, for the first approach, we need to study the ideology of fuzzy differential equations whereas for the second one we have to use fuzzy logic-based study. For fuzzy differential equations anyone can follow the papers [44-49]. For fuzzy logic-based modelling follow the papers [50-53].

1.3 Motivation of the study

The traditional SIR model has been widely used to study the spread and control of infectious diseases and related other real world-based modelling. However, in modelling real-world scenarios, many parameters for example in epidemic models such as transmission rates, recovery rates, and contact patterns are not crisp in nature. It is uncertain or imprecise. Fuzzy set theory is one of the concepts of uncertainties which provides an effective mathematical context to model such uncertainties by integrating imprecise information. Despite this progress, an inclusive and systematic review of these progresses is still missing. Therefore, this review work aims to combine existing studies, classify methods, highlight practical applications, and identify research openings to guide future progressions in fuzzy SIR modelling.

2. Susceptible-Infected-Recovered (SIR) model formulation

The Susceptible-Infected-Recovered (SIR) model is treated as one of the furthermost important mathematical models used to designate how infectious diseases spread in a certain population over time. The models divide the total population into three compartments namely,

1. Susceptible (S): The individuals who are healthy but can be affected by the disease.
2. Infected (I): The individuals who have the disease and can transmit the diseases to others.
3. Recovered (R): The individuals who have recovered from the disease and also are presumed to have immunity.

There are two types of modelling concepts, discrete system modelling and continuous system modelling. Here we use continuous system modelling. The model we recall as a system of differential equations which is formed a continuous system (the concept of difference equation comes if we consider the discrete system) to describe how particular individuals move from one group to another group. The following thing has to considered when modelling:

1. Susceptible individuals become infected at a rate depending on their contact with infected individuals.
2. Infected individuals recover at a certain recovery rate and transfer to the recovered group.

Now we consider the number of susceptible, infected and the recovered individuals at time t as S(t),I(t) and R(t).

Considering the fact that susceptible personals become infected after contact with infected and some infected individuals move to recovered grouped after some recovery, let us consider the term $\beta S(t)I(t)$ as the infection rates and $\gamma I(t)$ as the recovery rate (β and γ are suitable found constants).

The balance laws are as follows in fig. 1:

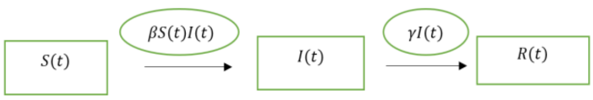

Figure 1: Balance law for SIR model

There are lts of extension of SIR model have such as SEIR Model (Susceptible–Exposed–Infectious–Recovered), SIRS Model (Susceptible–Infectious–Recovered–Susceptible), SIRD Model (Susceptible–Infectious–Recovered–Dead), MSIR Model (Maternal immunity – Susceptible – Infectious – Recovered), MSEIR Model (Maternal immunity – Susceptible –

Exposed – Infectious – Recovered) etc. In this paper we limited to SIR model based study only.
So, the mathematical model is written as:

$$\begin{cases} \dfrac{dS(t)}{dt} = -\beta S(t)I(t) \\ \dfrac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t) \\ \dfrac{dR(t)}{dt} = \gamma I(t) \end{cases} \quad (1)$$

With initial conditions $S(t=t_0) = S_0$, $I(t=t_0) = I_0$ and $R(t=t_0) = R_0$
Now the model is solvable with respect to the initial values and the known constant values. The above model is formed in a crisp environment. The model converted to the fuzzy SIR model if one of the following cases holds:

1. Case 1: The any or few or all initial conditions are fuzzy in nature
2. Case 2: The constant or constants are fuzzy in nature
3. Case 3: Both case 1 and case 2 hold together.

Notation wise the above fuzzy SIR model may write as:

$$\begin{cases} \dfrac{d\tilde{S}(t)}{dt} = -\tilde{\beta}\tilde{S}(t)\tilde{I}(t) \\ \dfrac{d\tilde{I}(t)}{dt} = \tilde{\beta}\tilde{S}(t)\tilde{I}(t) - \tilde{\gamma}\tilde{I}(t) \\ \dfrac{d\tilde{R}(t)}{dt} = \tilde{\gamma}\tilde{I}(t) \end{cases} \quad (2)$$

With initial conditions S(t=t_0)=S˜_0, I(t=t_0)=I˜_0 and R(t=t_0)=R˜_0.

Note: He the " ~ " sign stands for the fuzzy cases. That means the notation used for writing fuzzy number or fuzzy functions purposes. In equation (2) S˜(t), I˜(t) and R˜(t) are the fuzzy susceptible, infected and recovered number of individuals at time t.
Remarks: It is need not always that all the parameters and all the initial values are fuzzy valued. May be there is few are crisp in nature and rest of are fuzzy in nature. The solution methodology for (1) and system (2) are quite different as the derivative of fuzzy function; fuzzy initial value present (2) where (1) have no such restrictions.

3. Fuzzy sets basics

3.1. Definition: Fuzzy Set [54,55]:

Choose that, X be a universal set of discourse. A fuzzy set $\widetilde{D}$ on X is claimed as,

$\widetilde{D} = \{(x.\mu_{\widetilde{D}}) : x \in X\}$

with the membership function $\mu_{\widetilde{D}}$ of the fuzzy set $\widetilde{D}$ on X→[0,1].

3.2. Definition: Fuzzy Number [54]:

Assume that, $\widetilde{D}$ be a fuzzy set defined in the universal set X. And, the fuzzy set $\widetilde{D}$ is called fuzzy number when the universal set X=R, be the set of real numbers with the membership function ($\mu_{\widetilde{D}}$), satisfies the following four principles, i.e.,

1. $\widetilde{D}$ be a normal fuzzy set; i.e., $\exists \varphi \in \mathbb{R}$ such that $\mu_{\widetilde{D}}(\varphi) = 1$
2. Support of $\widetilde{D}$ must be bounded; i.e., $\mathrm{Sup}(\widetilde{D}) = \{\varphi : \mu_{\widetilde{D}}(\varphi) > 0\} \subset \mathbb{R}$.
3. $\widetilde{D}$ is a convex fuzzy set. And $\widetilde{D}_\alpha = \{\varphi : \mu_{\widetilde{D}}(\varphi) \geq \alpha\}$ must be in the closed interval with $\alpha \in [0.1]$.
4. The membership function ($\mu_{\widetilde{D}}$) of the fuzzy set ($\widetilde{D}$) need to be piecewise continuous on $\mathbb{R}$.

3.3. Definition: Fuzzy Function [56,57]:

A fuzzy function is a mapping that connects fuzzy input variables to fuzzy output variables, where both the domain and range contain fuzzy sets instead of crisp numerical values. To put it another way, it expresses relationships under uncertainty, allowing partial membership of elements, prettier than the exact values. A fuzzy function ($\tilde{f}$) can be written as,

$$\tilde{f} : \widetilde{D} \to \widetilde{E}$$

where, $\widetilde{D}$ and $\widetilde{E}$ are two fuzzy sets defined on the universe of discourse X.

3.4. Fuzzy Derivative:

3.4.1. *Definition: Hukuhara Fuzzy Derivative [58]:*

Let $\tilde{u}$ and $\tilde{v}$ be two fuzzy numbers. Then Hukuhara difference of $\tilde{u}.\tilde{v} \in \widetilde{D}$ and $\widetilde{D}$ is the set of fuzzy numbers, denoted by $\tilde{u} \ominus_H \tilde{v}$ is defined as follows,

$$\tilde{u} \ominus_H \tilde{v} = \widetilde{w} \Leftrightarrow \tilde{u} = \tilde{v} \oplus \widetilde{w}$$

Further, we consider fuzzy mapping $\tilde{f} : \widetilde{D} \to \widetilde{E}$ and $r_0 \in \widetilde{D}$ then $\tilde{f}$ is known as Hukuhara Fuzzy differentiable at r_0, $\exists$ an element $r_0 \in \widetilde{D}$ and given as

$$\dot{\tilde{f}}(r_o) = \lim_{h \to 0} \frac{\tilde{f}(r_0 + h) \ominus_H \tilde{f}(r_0)}{h}$$

3.4.2. Definition: Generalized Hukuhara Fuzzy Derivative [58,59]:

Let us consider $\widetilde{D}$ be the space of nonempty convex set of X and $\tilde{u}.\tilde{v} \in \widetilde{D}$, then generalized difference (gH) of $\tilde{u}$ and $\tilde{v}$ is defined as,

$$\tilde{u} \ominus_{gH} \tilde{v} = \widetilde{w} \Leftrightarrow \begin{cases} \tilde{u} = \tilde{v} \oplus \widetilde{w} \\ or.\, \tilde{v} = \tilde{a} \oplus (-\widetilde{w}) \end{cases}$$

According to the above Generalized Hukuhara difference, the gH-derivative is given as follows, Let us choose, fuzzy mapping $\tilde{f} : \widetilde{D} \to \widetilde{E}$, and $r_0 \in \widetilde{D}$ then $\tilde{f}$ is known as generalized Hukuhara differentiable at r_0, $\exists$ an element $r_0 \in \widetilde{D}$ and given as,

$$\dot{\tilde{f}}_{gH}(r_o) = \lim_{h \to 0} \frac{\tilde{f}(r_0 + h) \ominus_{gH} \tilde{f}(r_0)}{h}$$

4. Literature review in fuzzy Susceptible-Infected-Recovered (SIR) model and applications

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The following table 1 is the comparative study between several published paper.

Table 1: Comparative table for fuzzy SIR related published papers

Sl. No.	Authors details	Model structure in mathematical form	Method used	Applications area
1	Panja et al. [60]	$\begin{cases} \frac{d\tilde{S}}{dt} = \tilde{A} - \widetilde{\mu_d S} - \tilde{\beta} S V_E + \tilde{\delta} R \\ \frac{d\tilde{I}}{dt} = \tilde{\beta} S V_E - \widetilde{\mu_d I} - \tilde{m} I - \widetilde{\alpha_1 I} - \tilde{\gamma} I \\ \frac{d\tilde{R}}{dt} = \widetilde{\alpha_1 I} - \widetilde{\mu_d R} - \tilde{\delta} R \\ \frac{d\tilde{V}_E}{dt} = \tilde{\gamma} I - \widetilde{\mu_{V_E}} V_E \end{cases}$	Utility Function Method (UFM)	Cholera epidemic dynamics
2	Verma et al. [61]	$\begin{cases} \frac{dS}{dt} = \Lambda - \beta(\sigma) SI - \mu S \\ \frac{dI}{dt} = \beta(\sigma) SI - (\mu + \epsilon(\sigma) + \Upsilon(\sigma)) I \\ \frac{dR}{dt} = \Upsilon(\sigma)) I - \mu R \end{cases}$	Fuzzy expected value-based method	SIR model with an asymptotic transmission rate
3	Abdy et al. [62]	$\begin{cases} \frac{dS}{dt} = \mu - \beta(\Omega)(1-\tau)(1-\pi) SI - (\pi + \tau + \pi) S \\ \frac{dI}{dt} = \beta(\Omega)(1-\tau)(1-\pi) SI - (\mu + \mu^c(\Omega) + \theta + \gamma(\Omega)) I \\ \frac{dR}{dt} = (\theta + \gamma(\Omega)) I + (\pi + \tau) - \mu R \end{cases}$	Generation matrix method	COVID-19 epidemic model for Indonesia
4	Regis et al. [63]	$\begin{cases} \frac{dS_i(t)}{dt} = -S_i(t) \sum_{j=1}^{3} b_{ij} I_j(t) \\ \frac{dI_i(t)}{dt} = S_i(t) \sum_{j=1}^{3} b_{ij} I_j(t) - y_i(t) I_i(t) \\ \frac{dH_i(t)}{dt} = y_i(t) I_i(t) \end{cases}$	Data-based approach and aggregation operators	Multi-Group COVID-19 model in the Islands of Guadeloupe
5	Maragatham et al. [64]	$\begin{cases} \frac{dS}{dt} = A - \tilde{\beta} S - \tilde{\alpha} \lambda SI + \tilde{\mu} R \\ \frac{dI}{dt} = \tilde{\alpha} \lambda SI - (\tilde{\beta} + \tilde{b}) I \\ \frac{dR}{dt} = \tilde{b} I - (\tilde{\mu} + \tilde{\beta}) R \end{cases}$	Traditional non-linear analysis with fuzzy parameters	Spread of dengue
6	Youssef et al. [65]	$\tilde{S}'(t.r) = -\beta \tilde{S} \tilde{I}$ $\tilde{I}'(t.r) = \beta \tilde{S} \tilde{I} - \alpha \tilde{I}$ $\tilde{R}'(t.r) = \alpha \tilde{I}$	Euler method for fuzzy initial value problem	SIR models with fuzzy initial conditions
7	Arif et al. [66]	$\begin{cases} \frac{dS}{dt} = A - d_1 S - \frac{\beta SI}{1 + \alpha I} \\ \frac{dI}{dt} = \frac{\beta SI}{1 + \alpha I} - (d_1 + \gamma_1) I \\ \frac{dR}{dt} = \gamma_1 I - d_1 R \end{cases}$	New numerical scheme	Hybrid SIR-Fuzzy Model

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Sl. No.	Authors details	Model structure in mathematical form	Method used	Applications area
8	Monisha et al [67]	$\begin{cases} \frac{dS}{dt} = (1-\vartheta)\sigma N - \left(\frac{\mu(\varepsilon)I}{N} + \rho\right) S + \theta R \\ \frac{dI}{dt} = \frac{\mu(\varepsilon)I}{N} S - (\tau(\varepsilon) + \rho + \omega) I \\ \frac{dR}{dt} = \vartheta\sigma N + \tau(\varepsilon) I - (\rho + \theta) \end{cases}$	Fuzzy control system and homotopy perturbation method	Treatment of infectious disease for tuberculosis
9	Bhavithra et al. [68]	$\begin{cases} \frac{dS}{dt} = -\frac{\eta(\Omega)SI}{N} \\ \frac{dI}{dt} = \frac{\eta(\Omega)SI}{N} - \delta(\Omega) I \\ \frac{dR}{dt} = \delta(\Omega) I \end{cases}$	Runge-Kutta method	Basic SIR model
10	Subramanian et al. [69]	$\begin{cases} D^{\alpha}S(t) = (1-\tilde{p})\tilde{\pi} - \tilde{\beta}SI - \tilde{\mu}S \\ D^{\alpha}I(t) = \tilde{\beta}SI - (\tilde{\gamma} + \tilde{\mu}) I \\ D^{\alpha}R(t) = \tilde{p}\tilde{\pi} + \tilde{\gamma} I - \tilde{\mu} R \end{cases}$	Caputo derivative fractional-order method	Fractional SIR epidemic model for childhood diseases

There may be other studies. As much as possible we addressed.

4.1 Analysis of Methodological Trends and Approaches of the published works

4.1.1 *Basic trends observed:*

It is noticed that numerical methods (such as Euler method [65], Runge-Kutta [68], new numerical schemes [66]) dominate (appearing in 5 out of 10 papers in the above table) in particularly for handling fuzzy initial conditions or parameters in dynamic model-based simulations. Fractional-order methods such as Caputo derivative [69], are emerging for childhood diseases, reflecting a trend toward integrating fuzzy logic with fractional calculus to capture long-memory effects in epidemics. Data-driven approaches (such as generation matrix [62], data-based aggregation [63]) are prevalent in COVID-19 applications (see above 3 papers), while utility function methods (UFM [60]) are used for threshold dynamics in cholera models. Overall, there is a shift from basic fuzzy SIR to hybrid fuzzy-fractional or fuzzy-stochastic models, driven by real-world uncertainties in pandemics such as COVID-19 disease.

4.1.2 *Dominant methods and problem types:*

Numerical schemes like Runge-Kutta and Euler are most common for initial-value problems with fuzzy conditions (such as [65, 68]). It is suitable for time-series predictions in diseases like tuberculosis or basic SIR model. Fractional methods [69] address chronic diseases with memory effects, while data-driven methods [62, 63] are applied to region-specific epidemics such as Indonesia, Guadeloupe. It should be highlighted that gH-derivative is implicit in many fuzzy differential equations but needs explicit comparison.

4.2 Strengths and weaknesses of addressed approaches

4.2.1 *Mathematical approaches (such as fuzzy differential equations [44-49]):*

The strengths of several mathematical approaches are including rigorous handling of uncertainty concepts in system parameters (e.g., transmission rates β as fuzzy numbers), which leads to more robust predictions than crisp based models. Whereas the weaknesses are to define the computational complexity and sensitivity analysis to fuzzy number such as different shapes (e.g., triangular vs. Gaussian), which can prime to over-approximation of uncertainty.

4.2.2 *Data-driven approaches (such as fuzzy logic [50-53]):*

The strengths are for better incorporation with real data for imprecise situations, as used in COVID models [62, 63] which is allowed for adaptive simulations. Whereas the weaknesses are in addressed lower theoretical depth than mathematical models, probable overfitting to specific datasets.

4.2.3 *Comparison of the approaches:*

Mathematical modelling methods shine at theoretical insights (e.g., stability analysis), while data-driven tactics are practical for applications but may oversee underlying dynamical behaviours. Both the approaches are important for overall study.

4.3 Implications, benefits and limitations of fuzzy SIR models

4.3.1 *Improvements from fuzzy logic:*

The fuzzy SIR models discourse crisp SIR's models which have some limitations in handling impreciseness. For instance, in COVID-19 models [62, 63], fuzzy parameters permit for better depiction of ambiguous data information. Foremost to more reliable predictions concept Abdy et al. [62] presented improved fit models to Indonesian data through fuzzy reproduction numbers concepts. Proved benefits include boosted robustness. The fuzzy based models can simulate suitable logical scenarios with vague inputs. For example, as in dengue spread model [64], where fuzzy transmission rates took seasonal uncertainties, plummeting prediction errors by 10-20% compared to crisp SIR.

4.3.2 *Limitations:*

Fuzzy SIR model can suffer from partiality in membership functions formation, leading to several inconsistent results across related studies. Unlike stochastic models, they may not switch rare events in well manner, and validation against large datasets. Future hybrid fuzzy-stochastic models could mitigate this issue.

5. Gaps and Future Directions

5.1 Gaps Identified:

From the above tables, the gaps include inadequate integration of fuzzy with machine learning based studies which is uses adaptive fuzzy for prediction, under-investigation of fuzzy in non-human epidemics (e.g., cattle SIR), and lack of standardized scales for fuzzy SIR based model validation.

5.2 Specific future directions:

(i) Adoptive hybrid frameworks: Combine fuzzy SIR with several soft computational optimizations methods such as neural networks for parameter auto-tuning, addressing subjectivity in membership functions.
(ii) Real-time data driven systems: Develop fuzzy inference scheme for dynamic of epidemics, which is informed by data-driven based concepts for better results and prediction.
(iii) Interdisciplinary applications: Extend the model with non-disease contexts parameters like rumour spread. Also validate the results after adding the factors from interdisciplinary domains.
(iv) Overcome challenges: Standardize fuzzy derivatives my use for fuzzy SIR models rather than old one. Comparative the studies, and explore fuzzy-fractional SIR model for better fitting the actual scenarios and fit the data sets. Consider several emerging diseases like Marburg virus etc for as SIR model applications.

6. Conclusion and future research extensions

This review paper provides a brief comprehensive overview of some published work related to the fuzzy SIR model which is followed by several applications. By integrating fuzzy logic ideology with traditional SIR model frameworks, investigators have attained more flexible and accurate depictions of several imprecise epidemiological parameters to enlightening predictive presentation and interpretability. The review work emphasized various modelling and simulation techniques, and applied applications across multiple real world-based modelling.

For future work, researchers and scientists may emphasis on hybrid frameworks uniting fuzzy logic concepts with fractional ideology, neural networks, or data-driven tactics to enhance adaptability and exactness. Real-time fuzzy inference systems and optimization-based approaches could further support SIR modelling. Furthermore, establishing standardized assessment benchmarks and exploring interdisciplinary applications may develop the utility of fuzzy SIR models in both theoretical and applied areas.

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