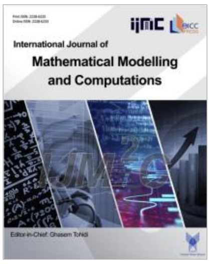

Int. J. Math. Model. Comput. 2026; 16(1): 57-63

International Journal of Mathematical Modelling & Computations (IJM^{2}C)

https://doi.org/10.57647/ijm2c.2026.160104

Analytical Solutions for Economic Dispatch via Wolfe's Duality and Exponential Convex Model

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Original Research

Received:
30 November 2025

Revised:
22 September 2025

Accepted:
24 September 2025

Publish online:
27 September 2025

Published in Issue:
31 March 2026

Abstract

Optimization plays a fundamental role in applied sciences. With the advancement of computational tools, numerical (non-analytical) methods for optimization have progressed considerably. However, such methods often face challenges related to computational accuracy, convergence time, and reliance on sophisticated hardware. These issues are particularly relevant in electrical energy production optimization. In power system economic dispatch (ED), the objective is to minimize total generation cost by coordinating multiple electrical generators—a problem that is inherently mathematical. Ensuring fast convergence and reduced computational time is essential. Traditionally, the production cost is modeled as a quadratic function of generator output, assuming a linearly increasing cost rate. However, for some generators, as efficiency declines, the cost rate increases non-linearly. Consequently, Quadratic Models yield sub-optimal solutions. In this study, we propose a new cost model based on an exponential function to better capture the nonlinear cost behavior. For the first time, an analytical solution using Wolfe's duality is derived for ED with exponential cost functions. This approach enhances accuracy and reduces computational overhead. Simulation results show that our method matches the accuracy of the exponential Lambda Iteration method while reducing execution time by 26.6%. It is especially suitable for real-time applications such as Optimal Power Flow (OPF) in power distribution systems.

Keywords: Analytical Solution, Wolfe's Duality, Convex Analysis, Power Structures

Cite this article: Javidani S, HamedooniAsli A, Analytical Solutions for Economic Dispatch via Wolfe's Duality and Exponential Convex Model, *Int. J. Math. Model. Comput.* 2026;16(1):57-63. https://doi.org/10.57647/ijm2c.2026.160104

1. Introduction

Optimization, a mathematical framework for identifying the best solution under given constraints, plays a vital role across various domains, including engineering, economics, and logistics [1]. In electrical engineering, optimization is essential for making systems more efficient, keeping costs down, and maintaining overall stability [2]. The Economic Dispatch (ED) problem is a fundamental optimization task in power system operations. Its primary goal is to determine the optimal power output of generating units such that the total generation cost is minimized while satisfying system constraints such as power balance and generator operating limits [3]. Classical ED formulations often assume a simple convex quadratic cost model for thermal units, allowing for efficient solution using gradient-based methods or Lagrangian multipliers [12,14]. Convexity is a key concept that enhances the effectiveness and reliability of optimization [5,6], especially in the stability of solving quadratic functions [13]. Sometimes, quadratic assumptions break down in real-world power systems, causing inefficient power generation and inaccurate cost calculations [15]. This complexity underscores the need for more precise mathematical models and efficient solution techniques. This paper presents a mathematically rigorous, analytical solution to the ED problem using an exponential cost model. We establish the convexity conditions of the exponential function and leverage Wolfe's duality theorem to derive closed-form algebraic solutions [7]. Unlike heuristic methods, the proposed approach is rooted in convex analysis and duality theory, ensuring provable global optimal and computational efficiency. It is important to note that duality offers a clear and powerful framework for modeling and solving complex optimization problems in many fields. Concepts such as strong duality, duality gaps, Slater conditions, and complementary slackness often play critical roles in deriving solutions [9,10,11]. In addition, Duality can be used when the function is non-convex or large-scale optimization problems, or even with modification [16,17,18]. In this paper, Wolfe's duality theory is applied for the first time to derive a unique and direct analytical solution to the ED problem. Furthermore, the integration of an exponential cost model enhances the accuracy of power system optimization. The proposed method guarantees computational efficiency, achieving the minimal calculation time while providing a unique analytical solution. The structure of the paper is organized as follows. Section 2 presents a historical overview and theoretical background relevant to the ED problem. Section 3 describes the proposed modeling approach using exponential cost functions. Section 4 introduces the convexity analysis and provides the analytical solution using Wolfe's duality theory. Section 5 presents simulation results and evaluates the performance of the proposed method. Section 6 discusses the practical implications and insights drawn from the results. Finally, the conclusion highlights the key contributions of the work and outlines possible directions for future research.

2. History and Background

Over the past decades, several solution approaches have been proposed for the ED problem. Early methods relied on convex optimization principles, such as the iteration technique and quadratic programming, which are efficient when the cost function is convex. To address this, a wide range of meta heuristic algorithms—such as genetic algorithms (GA), particle swarm optimization (PSO), and differential evolution (DE)—have been employed for solving ED problems with complex cost functions [4]. These methods are flexible and do not require derivative information, but they often suffer from slow convergence, randomness, and a lack of global optimality guarantees. Alternatively, convex optimization theory offers a framework with strong mathematical guarantees. Convex problem formulations ensure global optimal, and duality theory enables analytical treatment of constraints [20]. Wolfe's duality theorem, in particular, provides a powerful mechanism for solving convex problems by transforming them into tractable dual forms [7]. Recent studies have demonstrated the effectiveness of convexity-based approaches and dual formulations in solving various power system optimization problems, including unit commitment and optimal power flow. However, exponential cost models have not been utilized in this context. The present research builds on these mathematical foundations to, for the first time, address the ED problem analytically, providing a closed-form solution that eliminates dependence on numerical iterations or heuristic methods while achieving reduced production costs.

3. Modeling and Proposed Method

In a power system, multiple generators are integrated into a common electrical network, each producing power according to dispatch commands issued by the central operator. Initially, the cost of power generation increases gradually with output; however, as production nears the generator's capacity limits, the rate of cost escalation becomes significantly steeper. To investigate the cost behavior of each generating unit analytically, it is necessary to develop a precise mathematical model of the generator's characteristics. This entails formulating a cost function that accurately reflects the operational dynamics while omitting negligible factors. The modeling process involves identifying the appropriate mathematical relationship between key variables, as elaborated below. This formulation enables a clearer understanding of how generator costs evolve with output and provides a foundational basis for developing accurate and efficient optimization strategies in the power system operations.

3.1. Formulation and Model of the Production Unit Cost Function

Approximating generation cost with a quadratic function is often the first step in modeling generator cost behavior. In such a model, the cost rate is increasing but linear with respect to generation. However, the cost for some generating units increases nonlinearly, and the Quadratic Model lacks the precision to represent this escalation accurately [24]. In such cases, the exponential cost model offers a more realistic approximation due to its ability to describe the nonlinear growth in fuel consumption and maintenance costs. The results of this study confirm that exponential cost models outperform Quadratic Models—particularly for older units that exhibit exponential-like behavior as efficiency decreases. Accordingly, the cost function of each generator is modeled as:

$$f_i = \alpha_i e^{\beta_i p_i} + \gamma_i \tag{1}$$

Where, f_i is the production cost of unit *i*, p_i is the power output of unit *i*, α_i, β_i and γ_i are coefficients obtained through exponential curve fitting based on actual cost data. This formulation enables a more accurate representation of the true cost structure in modern and aging power systems. This model provide a more accurate and adaptable representation of real-world cost behaviors in thermal power units.

3.2. Objective Function Formulation for Economic Dispatch

Each generating unit is characterized by a unique cost function, shaped by its technical specifications and operational history. When multiple generators are employed to collectively satisfy the system's power demand, an optimal power distribution strategy is essential to minimize total generation cost. This task is formally known as Economic Dispatch (ED). Mathematically, the ED problem is formulated as a constrained optimization problem. The objective is to minimize the total generation cost—defined as the sum of the individual cost functions of all units—while satisfying power balance constraints:

$$min \sum_{i=0}^{n} f_i \tag{2}$$

subject to:

$$P_d \leq \sum_{i=1}^{n} p_i \tag{3}$$

Where, P_d denotes the total system demand.
This formulation establishes a foundation for applying convex optimization and duality principles, which are explored in the subsequent section.

4. Convexity and Analytical Solution Based on Wolfe Dual Function

Under convexity conditions, it is possible to determine an analytical solution for the optimization problem. Direct methods for solving the primal problem often require considerable computational time, which motivates the search for alternative approaches [25].
One such indirect method is the duality approach, in which the constraints of the objective function are expressed in a way that allows reformulation of the problem as a dual problem. Notably, duality methods can sometimes transform non-convex problems into convex ones [23]. Since the exponential function is convex, its convexity properties can be exploited. In many cases, instead of solving the primal problem directly, it can be more convenient to solve the problem via its dual formulation. This study uses the Wolfe dual formulation.
To derive the dual problem, the Lagrangian for the primal cost function is defined as:

$$L(\lambda,\ p_i) = \sum_{i=0}^{n} f(p_i) \ + \lambda(P_d - \sum_{i=1}^{n} p_i\ ) \tag{4}$$

$$L(\lambda,\ p_i) = \sum_{i=0}^{n} (\alpha_i e^{\beta_i p_i} + \gamma_i) \ + \lambda(P_d - \sum_{i=1}^{n} p_i\ ) \tag{5}$$

The first term represents the total cost (objective function), and the second incorporates the power balance constraint through the Lagrange multiplier λ, also referred to as the dual variable. To determine the dual function, we minimize the Lagrangian with respect to p_i:

$$g(\lambda) = \inf_{p_i} L(\lambda,\ p_i) \tag{6}$$

$$g(\lambda) = \inf_{p_i} \sum_{i=0}^{n} f_i(p_i) \ + \lambda(P_d - \sum_{i=1}^{n} p_i\ ) \tag{7}$$

$$g(\lambda) = \inf_{p_i} \sum_{i=0}^{n} (\alpha_i e^{\beta_i p_i} + \gamma_i) \ + \lambda(P_d - \sum_{i=1}^{n} p_i\ ) \tag{8}$$

This $g(\lambda)$, known as the dual function, is always concave, even when the original function f is non-convex. Given the differentiability of the exponential function, we take the derivative of $L(\lambda,\ p_i)$ with respect to p_i and set it to zero:

$$\frac{\partial L}{\partial p_i} = \frac{\partial}{\partial p_i}\left[\sum_{i=0}^{n} (\alpha_i e^{\beta_i p_i} + \gamma_i) \ + \lambda(P_d - \sum_{i=1}^{n} p_i\ )\right] = 0 \tag{9}$$

$$\frac{\partial L}{\partial p_i} = \quad \alpha_i e^{\beta_i p_i} + \gamma_i - \lambda = 0 \tag{10}$$

This yields a system of n equations with $(n+1)$ unknowns. Solving these provides p_i values, which are substituted back into the Lagrangian to obtain $g(\lambda)$:

$$g = g(\lambda) \tag{11}$$

$$g(\lambda) = \sum_{i=0}^{n} (\frac{\lambda}{\beta_i} + \gamma_i) \ + \lambda(P_d - \sum_{i=1}^{n} \frac{1}{\beta_i} ln(\frac{\lambda}{\alpha_i \beta_i})) \tag{12}$$

The dual problem is defined as finding the maximum of $g(\lambda)$ subject to:

$$\begin{cases} sup & g(\lambda) \\ s.t.: & \lambda \geq 0 \end{cases} \tag{13}$$

If $g(\lambda)$ is differentiable, the problem can be written as:

$$\begin{cases} max & g(\lambda) \\ s.t.: & \lambda \geq 0 \end{cases} \quad (14)$$

The effectiveness of solving the primal via its dual is captured by the concept of the duality gap, defined as:

$$d = f^* - g^* \quad (15)$$

Here, f^* and g^* denote the optimal values of the primal and dual problems, respectively. If the primal function is convex and strong duality holds, then $f^* = g^*$ at the optimal point [26,27]. Thus:

$$\begin{cases} min \sum_{i=0}^{n} (\alpha_i e^{\beta_i p_i} + \gamma_i) \\ s.t.: \quad P_d \leq \sum_{i=1}^{n} p_i \end{cases} \quad (16)$$

and the corresponding dual problem:

$$g^* = \sum_{i=0}^{n} \max_{\lambda} \left(\sum_{i=0}^{n} (\frac{\lambda}{\beta_i} + \gamma_i) \quad + \lambda(P_d - \sum_{i=1}^{n} \frac{1}{\beta_i} ln(\frac{\lambda}{\alpha_i \beta_i})) \right) \quad (17)$$

Thus, solving the dual problem becomes equivalent to solving the primal one. Given the convexity of the primal and the linear nature of its constraint, the Slater condition guarantees strong duality—ensuring the existence and uniqueness of the optimal λ^*.

While solving $g(\lambda)$ analytically can be challenging, it remains more tractable than directly solving the primal problem. To further streamline the process, the Wolfe dual formulation can be applied using the KKT(Karush-Kuhn-Tucker) conditions [28, 29], especially when the primal objective function is convex and the constraints are linear.

4.1. Examination of KKT Conditions in Wolfe Dual

The Wolfe dual method, under the Karush-Kuhn-Tucker (KKT) conditions, is characterized by four fundamental criteria:

1. the Stationarity Condition, involving the gradient respect to $\alpha_i e^{\beta_i p_i} + \gamma_i$;
2. the Primal Feasibility Condition, ensuring that the power demand constraint is satisfied;
3. the Dual Feasibility Condition, requiring a non-negative value for λ ; and
4. the Complementary Slackness Condition.

These conditions are expressed as follows:

1) $\frac{\partial L}{\partial p_i} = \frac{\partial f}{\partial p_i} - \lambda = 0$

2) $P_d \leq \sum_{i=1}^{n} p_i$

3) $\lambda \geq 0$

4) $\lambda(P_d - \sum_{i=1}^{n} p_i) = 0$

The fourth condition enforces strict equality:

$$P_d - \sum_{i=1}^{n} p_i = 0 \quad (18)$$

This shows that it is consistent with the third condition $\lambda > 0$, and Complementary Slackness holds. Moreover, given the exponential cost function model, the other KKT conditions are also satisfied. With the KKT conditions established, the proposed solution procedure can be derived as follows:

First, determine p_i :

$$p_i = \frac{1}{\beta_i} ln(\frac{\lambda}{\alpha_i \beta_i}) \quad (19)$$

and then substitute it into the constraint:

$$P_d = \sum_{i=1}^{n} \frac{1}{\beta_i} ln(\frac{\lambda}{\alpha_i \beta_i}) \quad (20)$$

Solving this equation leads to the optimal dual variable λ^*:

$$\lambda^* = \left(e^{P_d} \prod_{i=1}^{n} (\alpha_i \beta_i)^{\frac{1}{\beta_i}} \right)^{\frac{1}{\sum_{i=1}^{n} \frac{1}{\beta_i}}} \quad (21)$$

Thus, the closed-form expression for the optimal generator output p_i becomes:

$$p_i{}^* = \frac{1}{\beta_i} ln \left(e^{\frac{P_d}{\sum_{j=1}^{n} \frac{1}{\beta_j}}} \cdot \frac{1}{\alpha_i \beta_i} \prod_{j=1}^{n} (\alpha_j \beta_j)^{\frac{1}{\beta_j \sum_{k=1}^{n} \frac{1}{\beta_k}}} \right) \quad (22)$$

This algebraic expression provides an analytical solution for the optimal generator outputs. Simulation results demonstrate that the proposed method not only delivers precise results but also reduces computation time, while avoiding the numerical instability often encountered in iterative methods.

5. Simulation and Results

The test system uses generation cost data from three thermal units. Figure1 illustrates the raw data along with both quadratic and exponential curve fits, clearly showing the fitted coefficients for each model.

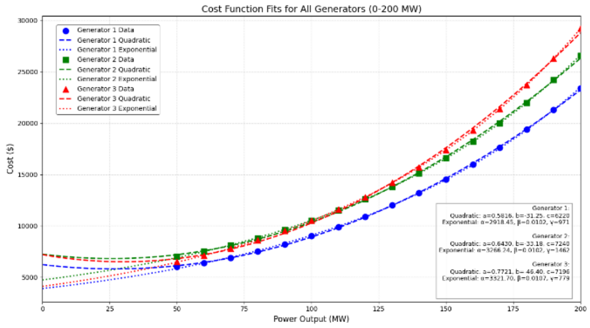

Figure 1. Quadratic and exponential curve fitting.

The simulation evaluates three distinct cost function models: (1) conventional quadratic cost functions solved via Lambda Iteration, (2) exponential cost functions using Lambda Iteration, and (3) the proposed closed-form analytical solution based on Wolfe's Duality with Exponential Modeling. Baseline solutions are first established using Lambda Iteration for both the quadratic and exponential cost models (Figures 2,3) to serve as benchmarks for assessing the performance of the proposed method.

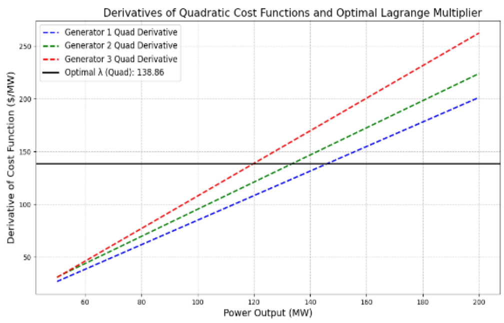

Figure 2. Cost function derivatives and optimal points in the Quadratic Model (using Lambda Iteration).

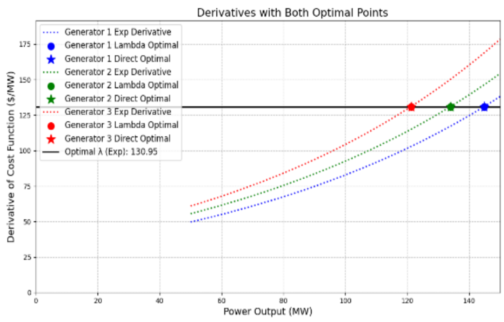

Figure 3. Cost function derivatives and optimal points in the Exponential Model (using Lambda Iteration).

Next, the proposed direct analytical approach based on Wolfe's Duality is implemented. All three methods are comprehensively evaluated using two key performance indicators: (1) computational efficiency, measured through precise execution time, and (2) solution quality, assessed via optimal generation dispatch and corresponding cost metrics. Comparative analysis highlights substantial differences in convergence behavior and computation time between iterative and closed-form techniques.

Table1. Execution Time Comparison.

Method	Execution Time (µs)	Speedup (WD vs. EI)
QI	123.8	-
EI	42.5	-
WD	31.2	26.6%

The results confirm that the Wolfe-duality-based analytical method significantly outperforms conventional approaches in execution time. Specifically, execution times were recorded as follows: (i) 123.8 µs for the quadratic cost model using Lambda Iteration, (ii) 42.5µs for the Exponential Model with Lambda Iteration, and (iii) 31.2 µs for the proposed closed-form method. This represents a 26.6% reduction in computation time compared with Lambda Iteration using the Exponential Model (see Table1). The improved efficiency of the direct method stems from its elimination of iterative processes while maintaining solution accuracy. This is evidenced by the nearly identical power outputs obtained via both Lambda Iteration and the proposed method, differing by less than 1×10^{-6} MW. The optimal dispatches were: Generator 1 at 144.829 MW, Generator 2 at 133.964 MW, and Generator 3 at 121.207 MW (Figure4).

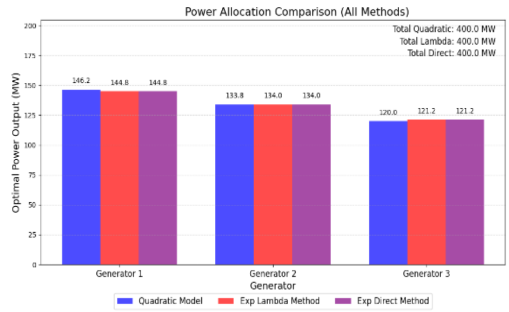

Figure 4. Power allocation across three methods.

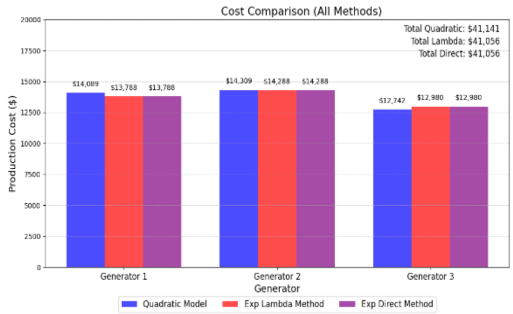

Figure 5. Production costs under the three approaches.

Moreover, the Exponential Model yielded a slightly lower total cost (41,055.90) compared to the Quadratic Model (41,140.78), despite producing similar dispatch results across all units (Figure5). The corresponding optimal Lagrange multipliers were 138.86 for the Quadratic Model and 130.95 for the Exponential Model, reflecting their differing cost curve characteristics.

10.57647/ijm2c.2026.160104

6. Discussion

The simulation results underscore the dual benefits of the proposed Wolfe-duality-based method: exceptional computational efficiency and high solution accuracy. The following key observations highlight the implications of the findings:

Computational Efficiency: The observed 26.6% reduction in execution time (from 42.5μs to 31.2μs) compared to the Lambda Iteration method highlights the structural advantage of the closed-form solution. This improvement is primarily attributed to the elimination of iterative computations while maintaining exact results, as confirmed by the machine-level agreement in generator outputs (differences 1×10^{-6} MW).

Economic Accuracy: The exponential cost model achieved a lower total cost (41,055.90$ versus 41,140.78$ for the Quadratic Model), demonstrating its superior fidelity in capturing the actual cost behavior of aging generators. Additionally, the Lagrange multipliers (130.95 for exponential vs. 138.86 for quadratic) further support the model's robustness and theoretical consistency.

Numerical Robustness: The identical optimal dispatch values for all three generating units between the Lambda and direct methods (G1:144.829~MW, G2:133.964~MW, G3:121.207~MW) confirm the numerical stability of the analytical solution, which is essential for real-time deployment in practical systems. Together, these findings validate that the integration of exponential cost modeling with Wolfe's Duality provides an optimal balance between computational performance and economic precision in the power system dispatch. The combination of mathematical rigor, exact solutions, and microsecond-scale execution time makes the proposed method a highly viable and efficient alternative for real-time applications such as Optimal Power Flow (OPF), where speed and accuracy are both critical.

7. Conclusion

This paper presented a novel closed-form analytical solution to the Economic Dispatch (ED) problem based on Wolfe's Duality (WD) using exponential generator cost models. Unlike traditional iterative techniques—such as Lambda Iteration—the proposed method eliminates the need for repetitive computations while retaining a high degree of solution accuracy. Simulation results from a benchmark three-generator test system validate the effectiveness of the proposed method. The WD approach matches the performance of the exponential Lambda Iteration (EI) method with machine-level precision, yet it achieves a 26.6% reduction in computation time. Additionally, it delivers a slightly lower total generation cost compared to both the EI and conventional Quadratic Models, confirming its economic optimality. The analytical tractability and numerical stability of the proposed WD formulation make it particularly well-suited for real-time power system applications, where both speed and accuracy are critical. By harnessing the strengths of convex optimization and duality theory, this method offers a scalable and efficient alternative for modern smart grid operations, addressing the growing need for responsive and automated decision-making in energy systems.

Authors Contribution
Shima Javidani: Conceptualization, Methodology, Formal Analysis, Writing – Original Draft Preparation, Experimentation, Visualization

Abbas Hamedooni Asli: Writing – Review & Editing, Validation, Experimentation, Simulation, Data Curation, Supervision

Availability of data and materials

The data that support the findings of this study are available from the corresponding author, upon reasonable request.

Conflict of interests
The author states that there is no conflict of interest

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