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Research Article

Designing a Smart Flipped Collective Learning Model for Reducing Conceptual Errors Grounded in APOS Theory: A Case Study on Linear Equations with Female Students in District 2 of Tehran

Somayeh Babaeisadr, Esmail Yousefi*^{ID}, Abolfazl Tehranian, Ahmad Shahvarani Semnani, Somaye Khalashi Ghezelahmad

Department of Mathematics, SR.C., Islamic Azad University, Tehran, Iran

*Corresponding author: esmaeil.yousefi@iau.ac.ir

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This research aims to design a novel hybrid learning model, known as the Smart Flipped Collective Learning Model. This model integrates artificial intelligence within a flipped learning environment with in-person collaborative learning grounded in APOS (Action-Process-Object-Schema) theory. The study focused on reducing students' conceptual errors in the topic of linear equations. In this novel approach, educational content related to linear equations was provided to students using artificial intelligence, and class time was dedicated to collaborative and interactive activities to deepen learning. The research examined how students' learning processes evolved, their understanding levels improved, and how conceptual errors were mitigated through the integration of in-person collaborative learning with flipped instruction, grounded in the stages of APOS theory. The research method was quasi-experimental with a pre-test and post-test design and a control group. The statistical population included all ninth-grade female students in District 2 of Tehran. Random cluster sampling was performed, and 120 students were selected, with 60 in the experimental group and 60 in the control group. The results showed that the smart flipped collective model is a novel integrative framework with statistically significant and cognitively profound effects on teaching mathematical concepts. The smart flipped collective model led to a considerable improvement in mathematical performance and a reduction in conceptual errors. Traditional instruction largely remained at the Action and Process stages, while the smart flipped collective model significantly facilitated the transition to the Object and Schema stages. The effect size at the Schema stage was reported to be over 0.23, and at the Object stage, it was over 0.25, which are considered very strong effects in educational studies. Overall, the proposed model explained approximately 45% of the variance in complex APOS cognitive components.

Keywords: In-Person Collaborative Learning; Flipped Learning; Artificial Intelligence; Smart Flipped Collective Learning Model; Conceptual Errors, APOS Theory; Linear Equations

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1. Introduction

Hybrid learning models are dynamic, and their optimization depends on many factors. Students, especially in secondary school, often struggle to develop a deep understanding of mathematical concepts, solve complex problems, and apply mathematical knowledge to real-world situations. This research seeks to provide a novel educational solution to address these challenges. To maximize mathematical performance, a model must be created where students are supported and prepared by flipped content enhanced with artificial intelligence. They then actively engage in deepening their understanding through collaborative problem-solving in a structured in-person environment. At the same time, the teacher acts as a facilitator, and artificial intelligence provides continuous insights and adaptation. The key is strategic integration and application [19, 21].

Traditional teaching approaches, which were often teacher-centered, have not been very successful in fostering critical thinking, problem-solving, or a deep understanding of core concepts [13]. Flipped learning and collaborative learning are two innovative educational approaches designed to promote effective student interaction and foster deep learning [1, 32]. Learning fundamental mathematical concepts, especially in topics like linear equations, is of great importance. It forms the foundation for understanding more complex concepts in higher education. However, students frequently encounter conceptual errors that create a serious barrier to deep learning. These misconceptions often stem from incomplete or incorrect cognitive structures [3, 6, 25]. The role of technology in mathematics learning cannot be overlooked [4]. As a powerful technology, artificial intelligence (AI) has provided unprecedented capabilities for personalizing, optimizing, and enhancing learning processes [19, 20]. AI systems can analyse student performance data, provide immediate and adaptive feedback, identify patterns of conceptual errors, and tailor educational content to the needs of each individual.

To identify and precisely investigate conceptual errors, various mathematical learning theories have been considered. Among these, APOS theory (Action, Process, Object, Schema) stands out as a powerful framework for understanding how learners construct mathematical knowledge [12]. By segmenting the process of understanding mathematical concepts into the stages of Action, Process, Object, and Schema, this theory provides an analytical tool for accurately identifying students' weaknesses and misconceptions, mainly concerning key concepts like the slope and equation of a line [12].

The history of each of these approaches—collaborative learning, flipped instruction, artificial intelligence, APOS theory, and conceptual errors—is crucial and essential for a valid, robust, and practical study.

Artificial intelligence can play a significant role in flipped learning and collaborative learning. In their

article “The role of technology in mathematics learning,” Aminifar et al. [4] revealed the positive and beneficial effects of technology, stating that its use can be effective in resolving students' misconceptions. This technology includes educational programs, software, and, most importantly, artificial intelligence. A review by Kovari [19] examined the current status of the integration and impact of collaborative learning with artificial intelligence in the higher education sector. This review concluded that AI-supported predictive analytics and multimodal approaches increase student engagement and motivation, while personalized learning systems and recommender algorithms ensure the effectiveness of collaborative learning environments. The research by Lo and Hew [21] found that using artificial intelligence in flipped learning can lead to benefits such as increased student interaction with educational content, improved class readiness, and data-driven teaching and learning. Naderi's [24] article results revealed that leveraging AI tools in the flipped learning method increases student motivation and participation, leading to an improved level of learning and academic achievement. Research by Chen et al. [9] highlights the increasing influence of technology on learning and teaching methods. Despite the diverse research mentioned, there is a perceived lack of studies that investigate the extent of AI's impact on the combination of collaborative and flipped learning.

Flipped learning was first observed in the research of Mazur [22]. In the early 21st century, two American teachers, Jonathan Bergmann and Aaron Sams, recorded instructional videos for students who were absent from class, thus establishing the flipped classroom model [8]. A study by Egara and Mosimege [11] found that the flipped classroom approach resulted in greater academic achievement and increased interest in mathematics among students compared to the traditional approach. The results also revealed that flipped instruction had a similar impact on the achievement and interest scores of both male and female students. A paper by Zainuddin and Halili [34] suggests that the flipped classroom has had a positive impact on student learning activities, including academic achievement, motivation, engagement, and interaction. The results of a study by Momeni Rad et al. [23] showed that the benefits of the flipped teaching method include increased self-confidence in learners, a sense of the teaching method being up-to-date, the active presence of the instructor, control over educational content, and time management. Research findings by Roozbahani [27] indicated a difference between flipped instruction and metacognitive strategies on the self-actualization of students at Sheikh Morteza Ansari Elementary School in Tehran, and this difference demonstrated good stability

over time. The findings of a study by Yousefi et al. [32] stated that the flipped teaching method has a significant impact on improving students' math performance, as well as on their motivation for learning mathematics and their collaboration. The results of research by Shahi et al. [28] demonstrated that the flipped learning method has a significant impact on all components of academic optimism.

Collaborative learning dates back many years. Ancient Greek philosophers emphasized the importance of dialogue and discussion for learning [15]. David developed collaborative learning as the formal, structured method that exists today, along with Roger Johnson and Spencer Kagan [17]. The results of a study by Siller and Ahmad [29] showed that collaborative learning appears promising for increasing mathematical achievement and fostering positive attitudes among elementary school students. In their research, Ahmad and Dogar [2] concluded that the experimental group performed better on the math achievement test than the control group, and their beliefs about mathematics improved.

Furthermore, collaborative learning is a more effective approach than traditional learning methods for enhancing the conceptual understanding abilities of fifth-grade students. The results of Jafari Dastjerd's [16] research, which included eighth-grade female students in Rey, indicated a significant effect of collaborative learning on the math academic performance of female students. A study by Karamian et al. [18] focused on developing and evaluating a novel active and collaborative learning framework. This framework was built upon five core principles: simplified instruction, the use of technology, personalized learning, varied teaching strategies, and the strengthening of mathematical self-confidence. The findings indicated that implementing this model resulted in a statistically significant improvement in students' math performance and a notable decrease in their anxiety levels. A particularly pronounced effect was observed among female students.

Despite the evidence regarding the separate effectiveness of collaborative learning and flipped learning, research that specifically investigates the integration of flipped instruction and in-person cooperative learning is more limited.

The roots of APOS theory can be traced back to Piaget's theories. Haghjoo and Reyhani [13] stated that APOS theory can be a helpful tool for improving the teaching and learning of mathematics if it is correctly understood and applied in classrooms. The theory emphasizes that if a concept is not understood stage by stage, an individual cannot advance to the next stage. The findings of Haghjoo and Reyhani's research [12] revealed that most

students understood the concept of slope according to APOS theory, and the understanding of most students on the topic of slope had progressed to the Object stage. Tsafe's [30] article investigated how APOS theory has positively transformed the teaching and learning of mathematics and has a close relationship with other cognitive abilities of the learner, such as intelligence and creativity. Studies have addressed the application of APOS theory in teaching mathematical concepts, but examining the integration of this theory with novel educational approaches such as flipped learning and in-person collaborative learning, based on artificial intelligence, at the ninth-grade level requires further research.

Conceptual errors have always been intertwined with mathematics and play a crucial role in learners' academic progress. An article by Zaeembashi et al. [33] made it clear that students have conceptual errors in several areas, such as the definition of an equation, the concept of the equals sign, the use of variables, solving equations, and converting word problems into equations. Their research emphasizes the importance of conceptual instruction and focusing on understanding the meanings of symbols and their relationship to mathematical concepts. Azadi et al. [6] research showed that a conceptual map helps in more precise identification of misconceptions, including in distinguishing variable types and choosing the appropriate graph. In their study, Aminifar et al. [3] found that students encounter numerous conceptual errors when working with variables, and teachers' awareness of these errors is crucial for effective teaching and for efforts to correct and develop students' understanding of the concept of a variable. A study by Delastri and Lolang [10] showed that conceptual errors in algebra result from misunderstanding specific concepts, creating equality between several concepts without considering the conditions, and ambiguity in interpreting mathematical symbols. However, the investigation of conceptual errors in the context of integrating collaborative learning and flipped instruction, grounded in APOS theory, requires further research and has not yet been adequately addressed.

2. Designing a Smart Flipped Collective Learning Model

The smart flipped collective learning model is a comprehensive and novel educational paradigm that seamlessly blends collaborative learning, flipped instruction, and artificial intelligence to optimize the learning process in mathematics. This framework invites learners to engage in dynamic interaction and the co-construction of knowledge. At the same time, the flipped method and the capabilities of artificial intelligence

enable this exchange and its analysis. The following definitions are first provided:

1. **Artificial Intelligence (AI):** In education, AI refers to the application of intelligent systems to improve and reshape teaching and learning processes. It includes using algorithms and data to personalize, self-regulate, and enhance educational experiences [14].
2. **Flipped Learning:** This is a novel educational approach where traditional classroom and homework tasks are reversed. Students study initial educational content at home using resources like AI, and class time is dedicated to practical activities, discussions, and problem-solving [8].
3. **In-Person Collaborative Learning:** This is an educational approach where students work together physically in small groups in a shared location (typically a classroom) to achieve common learning goals [17].
4. **APOS Theory:** This is a constructivist theory in mathematics education. According to this theory, learning mathematics involves four stages [5]:

Stage 1. Action: At this stage, the learner executes a new concept through specific step-by-step instructions or calculations. Understanding at this stage is typically superficial.

Stage 2. Process: The student can mentally visualize the operations without needing to perform them physically.

Stage 3. Object: At this stage, the learner considers the concept as an independent object and can reason about it.

Stage 4. Schema: At this stage, an individual can understand the relationships between different concepts and integrate them into new situations. APOS theory gives special attention to the internalization of actions into mental processes, the encapsulation of processes into manipulable objects, and the coordination and reversal of these structures to build a deeper understanding of mathematical concepts.

2.1. The Structure of the Smart Flipped Collective Learning Model

The SFCL(Smart Flipped Collective Learning) model is organized into four main stages that guide the learning process cyclically and dynamically:

Stage 1. Pre-Training: In this stage, students learn foundational concepts through various digital resources (including multimedia content and preliminary assessments) before physically attending class. This, in accordance with the principles of the flipped classroom, allows for self-directed learning.

Stage 2. Interactive Activities: In the classroom, students work in small groups to solve problems and complete collaborative tasks. These activities are designed to create cognitive conflict and conceptual reconstruction. The teacher's role here is to facilitate and guide constructive discussions.

Stage 3. Intelligent Data Analysis: An artificial intelligence-based system collects and analyzes data related to learner performance and interactions in real-time. This data goes beyond purely quantitative results and includes behavioural and cognitive patterns.

Stage 4. Adaptive Feedback: Based on the results of the data analysis, the system provides personalized and immediate feedback to students and instructors. This feedback includes suggestions for supplementary resources, identification of strengths and weaknesses, and guidance for optimizing the learning path.

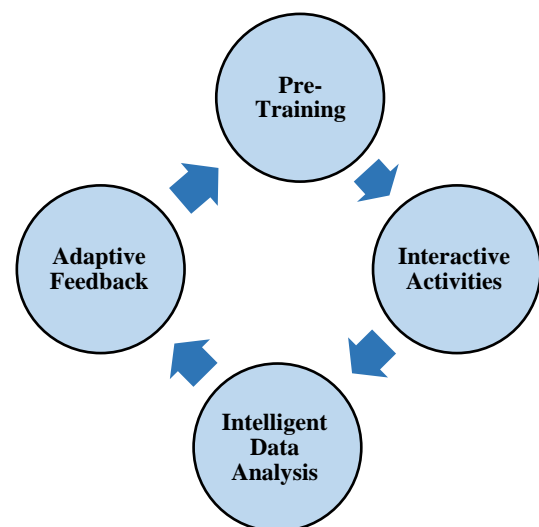


Figure 1. Diagram of the General Structure of the Smart Flipped Collective Learning Model

2.2. Key Players and Technology Infrastructure

This model operates based on synchronized cycles involving the student, the instructor, and technology. Students play an active role in their self-directed and collaborative learning process. The instructor acts as a facilitator and data analyst, while the AI system serves as an intelligent support tool, handling the analysis and feedback processes. The necessary tools include intelligent Learning Management Systems (LMS(Learning Management System)), multimedia platforms, and AI-based data analysis software.

2.3. The Definition of Learning in the SFCL Model

In this model, learning is defined as a dynamic socio-cognitive process. In this process, the learner internalizes complex concepts and creates new meaning through active interaction with adaptive digital content,

structured collaborative participation with peers, and intelligent feedback from AI-based systems. This facilitates the transition from superficial learning to deep understanding and leads to the activation of metacognitive control.

2.4. Learning Theories Related to the Smart Flipped Collective Learning Model

Some relevant learning theories are presented below, along with their connection to the smart flipped collective learning model.

2.4.1. Vygotsky Sociocultural Theory

This theory emphasizes the role of social interactions and cultural environments in the formation and development of higher mental functions. The fundamental concept in this theory is the Zone of Proximal Development (ZPD(Zone of Proximal Development)), defined as the gap between an individual’s current developmental level (what they can do independently) and their potential developmental level (what they can achieve with the help and guidance of a more knowledgeable individual). Table 1 outlines the stages of Vygotsky theory and their connection to the proposed model [31].

Table 1. Vygotsky theory

Vygotsky Theory Stages	Flipped Learning	Collaborative Learning	Smart Flipped Collective Learning Model	AI in Education
Social Interaction	Discussion after Observation	Learning in the ZPD	In-person/Online Conversation	Virtual Language Tutors
Zone of Proximal Development (ZPD)	Advanced Classroom Activities	Peer Learning	Blended Progress Assessment	Personalized Content
Psychological Tools (Language, etc.)	Visual Content Display	Peer Learning Conversations	Use of Multimedia	Personalized Language Analysis

Table 2. Piaget theory

Cognitive Development Stages	Flipped Learning	Collaborative Learning	Smart Flipped Collective Learning Model	AI in Education
Sensorimotor	Initial Visual Learning	Simple Group Games with Objects	In-person Blended Activities	Initial Diagnosis & Practice
Preoperational	Narrative-Visual Learning	Role-playing & Collaborative Activities	Blending Storytelling & Interaction	Behavioural Analysis for Learning
Concrete Operational	Concrete Representation of Concepts	Solving Concrete Problems	Digital and Physical Interaction	Error Analysis & Content Adjustment
Formal Operational	Abstract Problems in Flipped Classroom	Group Reasoning on Theoretical Concepts	Blending & Analysis of Theoretical Content	Abstract Thinking & Immediate Feedback

2.4.2. *Piaget Constructivism Theory*

This perspective is based on the principle that knowledge is actively constructed by the learner themselves, rather than being passively received. Piaget’s theory encompasses four main stages of cognitive development: sensorimotor (birth to 2 years), preoperational (2 to 7 years), concrete operational (7 to 11 years), and finally, formal operational (11 years and above). [Table 2](#) outlines the stages of Piaget theory and their connection to the proposed model [\[26\]](#).

2.4.3. *Zimmerman Self-Regulated Learning Theory*

This perspective emphasizes an individual’s ability to control their own learning process through a four-stage cycle: first, planning for goals; next, executing and monitoring performance during the activity; then, self-reflection; and finally, reviewing and adjusting strategies based on the results to improve learning. This cycle helps students become independent and effective learners. [Table 3](#) outlines the stages of Self-Regulated Learning theory and their connection to the proposed model [\[35\]](#).

Table 3. Self-regulated learning theory

Self-Regulation Stages	Flipped Learning	Collaborative Learning	Smart Flipped Collective Learning Model	AI in Education
Planning	Goals & Strategies at Home	Shared Planning & Goals	Online & In-person Planning	Planning & Guidance Tools
Execution	Progress Monitoring at Home	Mutual Peer Feedback	Blended Practical Practice	Adaptive Performance Feedback
Self-Reflection	Performance Assessment at Home	Analytical Group Feedback	Online Assessment & In-person Reflection	Reflective Performance Analysis
Review & Adjustment	Adjusting Strategies with Results	Group Discussion for Improvement	Adjusting Blended Learning Path	Improvement Suggestions & Guidance

Table 4. Bandura theory

Bandura Theory Stages	Flipped Learning	Collaborative Learning	Smart Flipped Collective Learning Model	AI in Education
Attention	Observing Models at Home	Observing Peers’ Behavioural Models	Online Content, Presence of Models	Smart Attention-Grabbing
Retention	Repetition for Memory Consolidation	Explaining to Enhance Retention	Conceptual Consolidation with Interaction	Spaced Repetition for Recall
Reproduction	Practicing Observed Skills at Home	Collaborative Practice with Peers	Online & Practical Practice	Simulated Performance Feedback
Motivation	Reinforcing Self-Efficacy with Videos	Group Successes and Mutual Motivation	Flexibility and Diverse Resources for Motivation	Personalized Motivational Feedback

Table 5. APOS theory

APOS Theory Stages	Flipped Learning	Collaborative Learning	Smart Flipped Collective Learning Model	AI in Education
Action	Initial Practice at Home	Discussion about Actions	Blended Practical Practice	Immediate Feedback on Practice
Process	Understanding Online Processes	Group Discussion of Processes	In-person Discussion of Online Processes	Monitoring the Understanding of Processes
Object	Encapsulation of Process into an Object	Building a Shared Object	Blended Work on the Object	Analysis of Object Understanding
Schema	Organizing Schemas at Home	Building Group Schemas	Online Knowledge Structuring, In-person Schema Application	Diagnosing Knowledge Gaps

2.4.4. Bandura Social Cognitive Theory

It is a comprehensive framework that emphasizes the vital role of learning through observation, cognition, and social interactions in shaping and regulating human behaviour. This theory includes four essential components: attention, retention, reproduction, and motivation. Table 4 outlines the stages of Bandura theory and their connection to the proposed model [7].

2.4.5. APOS Theory

The APOS theory (Actions, Processes, Objects, Schemas), which describes the construction of complex mathematical concepts in the minds of learners, consists of four stages: action, process, object, and schema. Table 5 outlines the stages of APOS theory and their connection to the proposed model [5].

3. Methodology

First, we collected and reviewed all articles and references related to smart flipped collective learning with the aid of artificial intelligence in reducing conceptual errors, grounded in APOS theory, in the context of linear equations. We then summarized the overall framework of the topic along with the presented methods. In the next phase, given the availability of a suitable study environment, we observed and investigated the combination of collaborative learning and flipped instruction with the help of artificial intelligence in reducing conceptual errors among ninth-grade female students in selected schools in District 2 of Tehran. We also conducted a review of learning theories, which included studying theories relevant to the subject.

The research method is quasi-experimental, implemented using a pre-test and post-test design with both control and experimental groups. This design allowed us to compare the impact of the independent variable (Smart Flipped Collective Learning Model) on the dependent variable (reduction of conceptual errors in the topic of linear equations) between the two groups. A pre-test was administered to ensure the homogeneity of the groups in terms of initial academic level and to assess students' prior knowledge. A post-test was administered to evaluate the extent of learning and changes that occurred after the educational intervention.

The statistical population for this study included all ninth-grade female students in District 2 of Tehran. A random cluster sampling method was employed. Several schools were randomly selected from District 2, and then specific classes from each of these schools were invited to participate in the research. The sample consisted of 120 ninth-grade students, who were randomly divided into two groups: an experimental group (60 students) that received instruction using the smart flipped collective teaching method supported by artificial intelligence, and a control group (60 students) that was taught using traditional methods. Inclusion criteria for selecting students, ensuring the internal validity of the research, included the absence of specific learning disabilities, access to the internet and digital tools for the experimental group, and homogeneity of the groups in terms of initial academic level, based on their previous mathematics scores.

A Mathematics Performance Test (pre-test and post-test), with questions designed grounded in the stages of

APOS theory, was used for data collection. In addition, separate questionnaires for teachers and students, along with teachers' academic notes, were utilized to gather qualitative information and more comprehensive insights. It was emphasized that participation was entirely voluntary, and unwillingness to complete the questionnaires would not result in any coercion. This approach fostered a two-way discourse and helped acquire more comprehensive data. To ensure the face validity of the instruments, they were reviewed and revised by eight mathematics education specialists and two university professors. For content validity, the Content Validity Index (CVI) was calculated, yielding a value of 0.81, which indicates high content validity. The reliability of the instruments was also confirmed by calculating Cronbach's Alpha coefficient, with a value of 0.88 demonstrating the instruments' high reliability.

Before the start of the educational intervention, we held an orientation session for the students and administered a mathematics pre-test to both groups. The teaching intervention was conducted over six weeks in the form of six 90-minute sessions. During this period, the experimental group initially studied topics related to linear equations at home with the aid of AI-powered, personalized content. They were allowed to use various free AI tools (Gemini, ChatGPT, DeepSeek) and were also permitted to purchase and use advanced AI or substitute educational software for their learning. The students were asked to use these software programs or AI tools twice a day for 30 minutes each time for their instruction. Then, in class, they engaged in interactive activities like group work on complex problems, analysis of common mistakes (with AI feedback and teacher guidance), and mathematical discussions and reasoning. Here, AI helped by providing adaptive exercises and immediate feedback. To address differences in learning levels within this group, we assigned varied tasks appropriate to different proficiency levels. In contrast, the control group received traditional instruction: the teacher initially explained concepts, then solved sample problems, and assigned homework for students to complete at home without direct guidance. After the educational intervention concluded, both groups took a mathematics post-test.

The types of input data included students' responses to exercises, the time spent on solving each question, and their patterns of conceptual errors. This data was continuously analysed to provide immediate and personalized feedback.

The design and development of this model required a significant initial investment, which included the costs of AI software development, digital content creation, and teacher training. The teaching time was also limited

due to the teacher's concern about completing all the relevant topics. However, its long-term benefits included increased learning efficiency, reduced teaching time for foundational concepts, and an enhanced, deeper understanding. Although the initial costs may be high, the reduced need for remedial classes and improved student academic performance over time can justify these expenses.

During the implementation of this research, challenges arose, including some students' resistance to the new learning method (which was resolved through orientation and motivational sessions), internet access issues for the experimental group (which were overcome by providing the necessary internet and tools), and varying learning levels within the experimental group (which were managed by providing tailored assignments for different levels). These measures helped to increase the internal validity and practical feasibility of the research.

4. Data Analysis and Research Findings

To investigate the effectiveness of the smart flipped collective model in reducing students' conceptual errors in the topic of linear equations, data from 120 ninth-grade students (60 experimental and 60 control) were analyzed using a pre-test-post-test design with a control group.

For the analysis of the data obtained from the students' mathematics performance pre-test and post-test, descriptive statistics, including mean and standard deviation, were first used. Then, to examine the significant difference between the experimental and control groups, an independent samples t-test and an Analysis of Covariance (ANCOVA) test were utilized. The significance level (α) was set at 0.05. The analysis was conducted using SPSS version 26 software.

The main analytical tools included:

- Parametric ANCOVA and MANCOVA tests
- Component analysis of APOS stages
- Effect size (Partial Eta Squared)
- Interpretive analysis of conceptual responses

4.1. Descriptive statistics

As seen in [Table 6](#), in the pre-test phase, the mean scores of the two groups were very close with a negligible difference (0.16), which indicates the initial homogeneity of the groups before the intervention. The standard deviations were also similar at this stage. In the post-test phase, the mean score of the experimental group was significantly higher than that of the control group, showing a remarkable difference of 4.36 points. This increase in the experimental group was

accompanied by a decrease in the standard deviation (1.82) compared to the pre-test, suggesting greater score homogeneity after the implementation of the learning model. The improvement in scores from pre-test to post-test for the experimental group (6.75 points) was much greater than that of the control group (2.55 points). This strongly indicates a positive and significant effect of the learning model intervention on the experimental group's scores. This descriptive data shows a strong trend for the experimental group following the intervention.

Table 6. Descriptive statistics of the mean pre-test and post-test scores of the students in the two groups

Group	Test	Mean	Standard Deviation
Experimental (Collective)	Pre-Test	11.27	2.43
Control	Pre-Test	11.11	2.50
Experimental (Collective)	Post-Test	18.02	1.82
Control	Post-Test	13.66	2.28

4.2. Independent samples t-test for pre-test comparison

Initially, to ensure the homogeneity of the two groups in terms of initial level, an independent samples t-test was performed on the pre-test scores. The results of the independent samples t-test for the pre-test showed that there was no statistically significant difference between the mean pre-test scores of the experimental and control groups ($t(118)=0.248$, $p = 0.805$), and the initial homogeneity was confirmed.

4.3. Univariate Analysis of Covariance (ANCOVA)

To examine the difference in post-test performance while controlling for the effect of the pre-test, a univariate Analysis of Covariance (ANCOVA) was used. The results are presented in Table 7.

Table 7. ANCOVA test on overall math performance scores (with pre-test as covariate)

Source of Variation	SS	Df	MS	F	P	η^2
Pre-test	121.5	1	121.5	13.94	0.000	0.107
Experimental group	244.6	1	244.6	28.07	0.000	0.193
Error	1019.1	117	8.71			

As is evident from Table 7, a significant difference ($p<0.001$) with a large effect size ($\eta^2=0.193$) is apparent. The results showed that the pre-test, with $F(1,117)=13.94$ and $p=0.000$, was significantly related to post-test performance. This confirmed that controlling for initial scores in the analysis was necessary. With the pre-test scores controlled, there is a significant

difference between the performance of the two groups on the post-test. The experimental group performed better than the control group. To calculate the Effect Size, the Partial Eta Squared coefficient was used. For $\eta^2=0.193$, the effect size is large. This means that approximately 19.3% of the variance in post-test scores can be attributed to the type of instruction (a combination of flipped learning, collaborative learning, and artificial intelligence).

4.4. Component analysis on the stages of APOS theory

The post-test instrument is designed to measure the four stages of APOS.

Table 8. Comparison of the mean of APOS theory stages in the post-test between groups (independent samples t-test)

APOS Stages	Experimental Group	Control Group	t	P-value	η^2
Action	4.62 ± 0.51	4.23 ± 0.68	3.41	0.001	0.09
Process	4.51 ± 0.64	3.94 ± 0.77	4.12	0.000	0.12
Object	4.39 ± 0.73	3.21 ± 0.89	7.31	0.000	0.25
Schema	4.17 ± 0.82	2.98 ± 0.91	6.89	0.000	0.23

From Table 8, we can see that for all four stages, the p-values are less than 0.001, indicating a highly significant statistical difference between the two groups in each of these stages. The effect sizes for the Action (0.09) and Process (0.12) stages are at a medium level, which signifies a noticeable impact of the learning model intervention on these foundational levels. More importantly, the Object (0.25) and Schema (0.23) stages have a large effect size, which demonstrates the success of the learning model intervention, especially in promoting a deeper understanding and more complex organization of mathematical concepts. The independent t-statistics for the object (7.31) and Schema (6.89) stages are also remarkably larger than the first two components, reflecting a stronger effect at these higher levels. The smart flipped collective model had a positive and significant impact on all aspects of mathematical understanding, grounded in APOS theory, with its effect being more pronounced at higher cognitive levels.

4.5. Multivariate Analysis of Covariance (MANCOVA)

To simultaneously examine the effect of the educational group on all APOS stages, we used a Multivariate Analysis of Covariance (MANCOVA). The results of this test are shown in Table 9.

Table 9. Multivariate Analysis of Covariance (MANCOVA) on the four stages of APOS theory

Statistical Index	Value	F	Hypothesis df	Error df	P-value
Wilks' Lambda	0.563	12.97	4	113	0.000
Pillai's Trace	0.437	12.97	4	113	0.000
Hotelling's Trace	0.776	12.97	4	113	0.000
Hotelling's Trace	0.776	12.97	4	113	0.000

From [Table 9](#), we find that all the multivariate statistical indices are decisively significant in this analysis. The F-value for all of them is 12.97, and p is equal to 0.000 ($p < 0.001$). This very high statistical significance indicates a significant difference between the experimental and control groups in terms of the overall combination of the four APOS stages, after controlling for pre-test scores. These results confirmed that the learning model intervention had a comprehensive and overarching impact on all dimensions of mathematical understanding (according to APOS theory). The smart flipped collective model led to significant and simultaneous changes in the different levels of students' mathematical comprehension.

4.6. Analysis of students' mental structures (interpretive-qualitative) grounded in APOS theory

In [Table 10](#), we observe that the Action (0.09) and Process (0.12) stages have a medium effect size, which indicates a significant improvement at the more foundational levels of mathematical understanding. In contrast, the object (0.25) and Schema (0.23) stages, as well as the overall performance score (0.193), all show a large Effect Size. These findings clearly state that the learning model intervention not only had a strong overall effect on mathematical performance but was also particularly effective in promoting deeper and more complex levels of mathematical thinking (Object and Schema concepts).

Table 10. Summary of the model's effect size at different APOS stages based on Partial η^2

Stage	Partial η^2	Effect Size stage (according to Cohen's criteria)
Action	0.09	Medium
Process	0.12	Medium
Object	0.25	Large
Schema	0.23	Large
Overall Performance Score	0.193	Large

A qualitative analysis of the descriptive responses and group activities showed that:

Action Stage: The conversion of algebraic expressions into simple equations was performed accurately.

Process Stage: Students were capable of mentally reasoning about changes in slope and their effect on the graph.

Object Stage: The linear equation was understood as a geometric and algebraic object, not just a symbolic relationship.

Schema Stage: The responses demonstrated the ability to link the concepts of function, slope, intercepts, and graphs.

The smart flipped collective model has paved the way for the reconstruction of stronger mathematical schemas by integrating digital content, active learning, group interaction, and adaptive feedback. In the experimental group, a significant percentage of students advanced from the Action and Process stages to the object and even the Schema stages. A meaningful reduction in conceptual errors was observed in cases such as understanding slope, equality, and writing the equation of a line. In the control group, the majority of students remained at the lower levels and continued to understand concepts in a step-by-step and procedural manner. The number of conceptual errors (e.g., a linear equation with zero slope or mistakes in coordinate axes) was higher.

In addition, based on the descriptive responses of students in the experimental group, it was observed that:

- Abstract concepts like slope, understood as a rate of change, were better comprehended, and the understanding of the relationships among slope, points, and the general form of the equation was enhanced.
- Errors such as coordinate transposition, misunderstanding of negative slopes, and mistakes in constructing the equation were addressed, and many responses had a logical and generalizable structure.
- In schema-stage questions, students were able to compare and interpret equations for several different cases, which was rare in the control group. In fact, high-level questions, such as analysing the relationship between coefficients and changes in graphs, were answered with complete and reasoning-based interpretations.

The conceptual model of the smart flipped collective framework grounded in APOS theory is shown in [Figure 2](#).

Traditional instruction largely remained at the Action and Process stages, while the smart collective model significantly facilitated the transition to the Object and

Schema stages. The effect size at the Schema stage was reported to be over 0.23, and at the Object stage, it was over 0.25, which is considered a very strong effect in

educational studies. In total, the proposed model explains about 45% of the variance in the complex cognitive components of APOS.

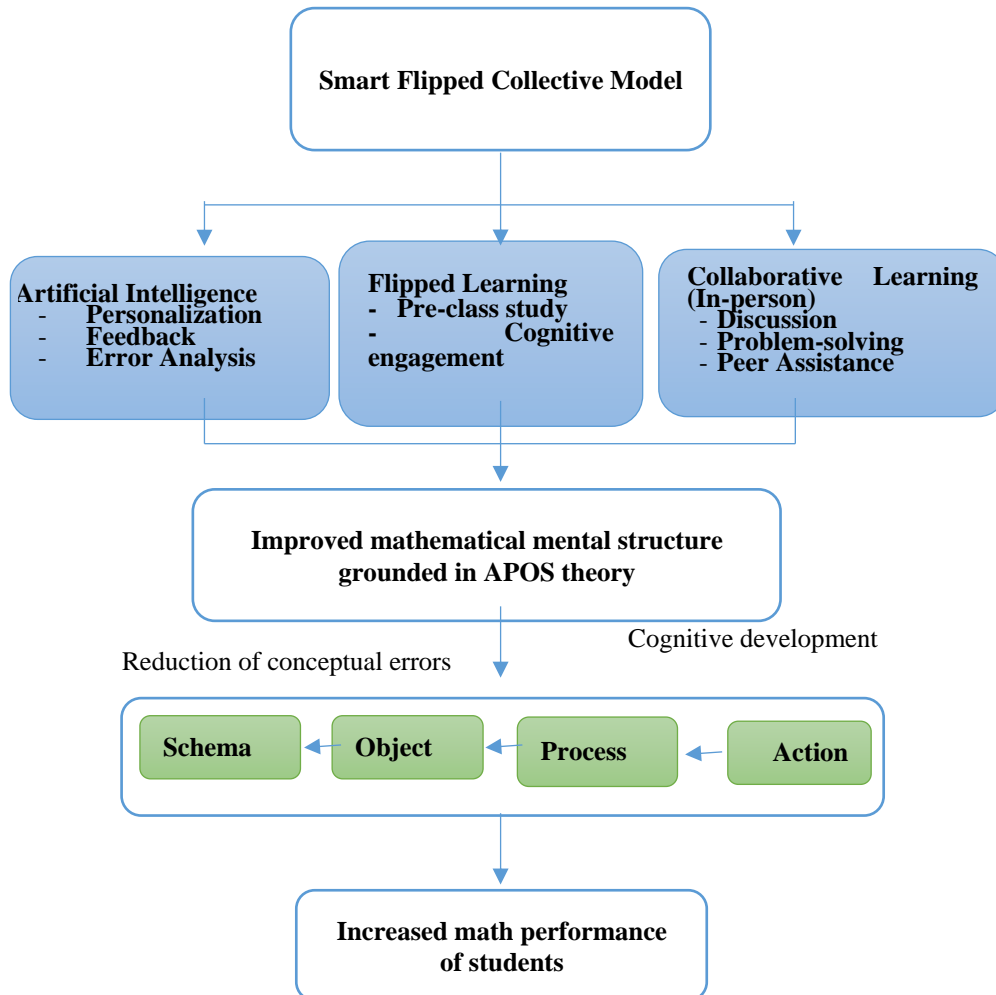


Figure 2. The conceptual model of the smart flipped collective framework is grounded in APOS theory

5. Conclusions

The Smart Flipped Collective Learning Model is a novel and effective integrated framework that combines flipped learning, collaborative learning, and artificial intelligence into a coherent and powerful model. This research demonstrated that this model has a statistically significant impact on improving mathematical performance and reducing conceptual errors in linear equations. The results, in particular, highlight its profound effect on elevating students' cognitive levels from the initial stages to the higher Object and Schema stages in the APOS theory. Contrary to many existing studies that focus on a single educational approach (such as flipped learning or collaborative learning), the main innovation of this paper lies in the strategic combination of these approaches with artificial intelligence. While previous research has confirmed the effectiveness of

each of these methods individually, the present study filled a gap in the scientific literature by providing an integrated model, and its results are consistent with previous research. For example, this study aligns with the research of Siller and Amad [29] and Jafari Dastjerd [16], who have considered collaborative learning to be highly effective in improving students' mathematical performance. Similarly, the studies by Naderi [24] and Zainuddin and Halili [34], which focused solely on flipped learning, have emphasized the efficiency of flipped education and thus share similar results with the present research. However, our proposed model went further by adding the components of in-person collaborative learning and artificial intelligence, addressing social interaction and personalized feedback. In doing so, it extended learning beyond the virtual space. This feature distinguishes the Smart Flipped Collective Learning Model from other existing models

and makes it a powerful educational tool. Our findings, with a large effect size at the Object and Schema stages, demonstrated the high potential of this model for generalization to other mathematical topics and serve as a testament to the effectiveness of the intelligent combination of technology, modern education, and cognitive theories in improving mathematics education.

6. Limitations of the Study

The limitations of this study include several aspects. The design of the hybrid learning model required significant time and financial resources for producing educational content and developing artificial intelligence tools. Flipped learning can be challenging for students who do not have sufficient access to technology. Collaborative learning requires careful classroom management and facilitation of group interactions, which can be difficult in high-density student environments. Furthermore, this study was conducted with ninth-grade female students in District 2 of Tehran. This limitation may affect the generalizability of the findings to other populations, including male students, different age groups, and various geographical locations. The level of acceptance of this learning model by students and teachers can affect the results. The generalizability of this model to other mathematical topics requires further study.

7. Research and Practical Suggestions

The effectiveness of the designed model can be investigated in other mathematical topics, such as geometry and functions, to determine its generalizability. Additionally, it is essential to evaluate the model's impact at different educational levels and to conduct long-term studies to assess the sustainability of the learning outcomes. Schools can facilitate the learning of foundational mathematical concepts by utilizing artificial intelligence tools and digital content tailored for flipped learning. By participating in specialized training courses, teachers can acquire the necessary skills to manage in-person collaborative learning and use new technologies. Furthermore, developing technological infrastructure in schools is essential for implementing this model and improving students' access to learning resources.

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Authors Contribution

All the authors have participated sufficiently in the intellectual content, conception and design of this work or the analysis and interpretation of the data (when applicable), as well as the writing of the manuscript.

Availability of data and materials

The data that support the findings of this study are available from the corresponding author, upon reasonable request.

Conflict of interests

The author states that there is no conflict of interest.

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