Fuzzy Optim. Model. J. 2026; 7(1) : 32-42

Fuzzy Optimization and Modeling Journal (FOMJ)

https://doi.org/10.57647/fomj.2026.0701.02

Original Research

A General Approach to Construct Intuitionistic Fuzzy Regression Model Based on a New Least Absolute Discrepancy

Abdullah Al-Qudaimi*, Walid Yousef

Information Technology Department, Faculty of Computing & IT University of Science and Technology, Sana'a, Yemen

*Corresponding author: aalqudaimi@yahoo.com

Article History

Received:
20 December 2025
Revised:
23 February 2026
Accepted:
26 March 2026
Published in Issue:
30 March 2026

Abstract:
The data sufferance from uncertainty and vague events impose statistical regression modeling to be treated in fuzzy environment for more accuracy upon causal relationship between independent and dependent variables. For more informative fuzzy, among limited studies on intuitionistic fuzzy regression models (IFRMs), this study proposed the most general approach to construct a full IFRM (the model's parameters and output as well as input variable(s) are represented as symmetric and/or asymmetric positive and/or negative and/or mixed of neither negative nor positive triangular intuitionistic fuzzy numbers (TIFNs)). The estimated parameters of the proposed IFRM determined on solving linear programming problems based on intuitionistic fuzzy least absolute of discrepancies (IFLAD). The proposed approach respects homogeneity principle in modeling such that the constructed IFRM for fitting symmetric TIFNs is symmetric either (the intercept and slope(s) are symmetric in this case). Furthermore, two illustrative examples are used to show the soundness and robustness of the proposed model and compared with existing IFRM.

Keywords: Triangular intuitionistic fuzzy regression model (IFRM); Full IFRM; Triangular intuitionistic fuzzy numbers (TIFNs); Intuitionistic fuzzy least absolute of discrepancies (IFLAD); Homogeneity principle

Cite this article: Al-Qudaimi A, Yousef W. A General Approach to Construct Intuitionistic Fuzzy Regression Model Based on a New Least Absolute Discrepancy. Fuzzy Optim. Model. J. 2026;7(1):32-42. https://doi.org/fomj.2026.0701.02

1. Introduction

Statistical Regression studies the causal effectuation of one or group of independent variables on dependent variables. When the available data are represented with presence of uncertainty and/or the relation among the variables is expressed as indispensable impreciseness, regression can be dragged into fuzzy environment. Tremendous fuzzy regression methods have been emerged [1] since the first work of regression modeling with fuzzy data initiated by Tanaka et al. [2] in 1982, which explains the relationship between crisp numerical independent variables and fuzzy dependent variable. In the trend of hybridizing traditional regression with fuzzy set theory, fuzzy least square regression (FLSR) has been extended such as those in [3, 4, 5, 6, 7, 8, 9, 10, 11].

FLSR due to maintaining the positivity of errors, the errors are squared; Fuzzy least absolute regression (FLAR). FLAR same as its non-fuzzy least absolute regression which based on L_1-norm is recommended to handle the errors in studied datasets, for example Dielman [12] show the robustness of least absolute approach and its superiority to least square approach. With the growing of computing tools which facilitated the solving of mathematical programming problems, Fuzzy least absolute of discrepancies (FLAD) based methods have been developed by many researchers. Chang and Lee [13] proposed FLAD approach using ranking method that reduced to linear programming problem to estimate the parameters of their fuzzy regression model. Inuiguchit et al. [14] investigated FLAD-based linear regression analysis. A two stages approach based on FLAD proposed by Kim et al. [15] where in the first stage, LAD applied on their fuzzy given dataset are defuzzified whereas in their second stage the total deviations between the targeted and estimated output are minimized. Torabi and Behboodian

[16] introduced FLAD optimization method to estimate the parameter of their model with the help FIFO data. In bygone decade, many FLAD-based mathematical programming problems used to construct regression models in literature [17, 18, 19, 20, 21, 22, 23, 24, 25].

Under the concept of intuitionistic fuzzy number proposed by Atanassov [26] which express fuzzy information in the shade of membership and non-membership functions, Parvathi et al. [27] triangular initiated intuitionistic fuzzy regression model (IFRM) and the estimated coefficients are modeled as symmetric triangular intuitionistic fuzzy numbers (symmetric TIFNs). Parvathi et al. [27] proposed linear programming problem to estimate their model coefficients represented as symmetric TIFNs. Arefi and Taheri [28] proposed IFRM where they exploited FLSA to estimate their model parameters of full IFRM (the input and output given data as well as model parameters considered as symmetric TIFNs. Due to the multiplication of unknown parameters with known data inputs, i.e. unknown symmetric TIFN into known STIFN, IFRM introduced by Arefi and Taheri [28] lacked generalization and soundness where, Arefi and Taheri [28] used the approximation of product of two symmetric TIFNs proposed by Dubois and Prade [29] with which it is not necessary to get a symmetric TIFN resultant. Arefi and Taheri [28] formulate their solution to estimate their model parameters based on that limited multiplication approximation. Consequently, given data of the independent variables and model's parameters needed to be determined as positive symmetric TIFNs although in their example there is a negative parameter explains the negative relationship between the output variable and one of two input variables.

According to Zadeh's extension principle [30], fuzzy numbers operations including multiplication as well as their applications such as fuzzy regression models must have no conflict with mathematical and physical principles. One fundamental principle must be maintained in any computation and calculations on fuzzy numbers, is the homogeneity principle. The homogeneity in IFRM is that for full IFRM where the output and input variables as well as model's parameters, all of the three model's components must be Atanassov's intuitionistic fuzzy numbers. Moreover, with dataset of pairs, $(y_1, x_{1j}, \ldots, x_{1p}), (y_2, x_{2j}, \ldots, x_{2p}), \ldots, (y_2, x_{nj}, \ldots, x_{np})$ represented as symmetric TIFNs the parameters must be symmetric TIFNs either. So, IFRM can fit homogeneous datasets. Keeping in mind the homogeneity fuzzy regression models and especially for IFRM, Chen and Nien [31] proposed a mathematical programming approach to formulate IFRM based on least absolute of discrepancies as a generalized approach avoids the limitations and incontinent multiplication assumption of Arefi and Taheri [28]. To avoid the effect of the sign of the unknown model's parameters, Chen and Nien [31] setup two dummy TIFNs one is non-positive and another is nonnegative. Chen and Nien [31] claimed that their approach is general to fit symmetric and asymmetric TIFNs. Moreover, on claiming that their proposed approach overcomes the conflict made by Arefi and Taheri [28] with homogeneity principle, however the model proposed by Chen and Nien [31], as result on applying the example [[**31**], Section 4, Equation 35, pp. 205] is not symmetric IFRM although the data are represented as asymmetric TIFNs. Moreover, the assumptions of dummy variables set by Chen and Nien [31] to parametrize the unknown model's slopes are not always satisfied although the related constraints are made in Chen and Nien [31] mathematical programming formulation. Al-Qudaimi [32] pointed out the limitations of the two approaches [28, 31] and proposed a multiplication of an unrestricted TIFN with a restricted TIFN to be used in IFRM however, the proposed multiplication considered 5 cases, each case needs 16 multiplications. While the construction of full fuzzy regression models are reduced to find the optimal coefficients for various types of fuzzy data. The main concerning issue in literature came from the way of multiplying unrestricted fuzzy numbers, as coefficients are unknown, with related restricted ones, as input data values are known. Regardless the effect of multiplication operation of fuzzy numbers on the spread of models' output, the keystone of fuzzy regression components relationship undergo the sign of those component which, in turn, must be considered in the models formulation. Multiplication solutions have been proposed by Al-Qudaimi [32] nevertheless, they are pretty confounding and computationally complicated. Moreover Nien and Chen [28] rebutted Ishita approach proposed by Al-Qudaimi [32] and claim that Ishita approach violated the basic definitions of TIFN however that resulted from ordering spread terms of TIFN (advancing some and postponing others) in some cases involving the multiplication of a known variable by an unknown variable. Nien and Chen [28] proposed linearized an existing intuitionistic FLAD (IFLAD) [32] without considering a clear solution of multiplying the unrestricted coefficient(s) by related restricted input variable(s) to construct fully IFRM. Instead, they have served only to restrict the spreads of an estimated output in the constraints of their mathematical programming problem. Consequently, the model parameters expressed as vectors which, in turn, cannot be considered as TIFNs. Although their model gives five times degrees of freedom, their approach is statistical over-parameterization rather than giving theoretical improvement. Hence, their works can be considered as a 5-dimensional multivariate vector regression other than IFRM.

1.1 Research contributions of this work

Motivated by the general IFLAD and the limitations of the existing approach [32, 28, 31, 33], this paper is aimed to extend a fuzzy-LAD proposed to construct a full interval-valued triangular fuzzy regression model [134]. Contributions of This approach (named as Ishita approach) can be distinguished into follows:

- The exact multiplication of TIFN without need of approximation or shuffling the components of the TIFN resultant under the case of sign affect,

https://doi.org/10.57647/fomj.2026.0701.02

- Extending the existing fuzzy-LAD [34] to introduce an IFLAD metric for discrepancy between two TIFNs depending on the maximum points of uncertainty piecewise function of the two TIFNs such that by minimizing those points discrepancy, it implicitly maximizes the preciseness against the hesitation, based on the definition of TIFN. Moreover, the minimum discrepancy is performed with three points; two for degrees of hesitation as well as the main point of the two TIFNs rather than five points; membership and non-membership points of left and right as well as the main point of the two TIFNs. Therefore, this approach obviously reduces the computation complexity.

- The mathematical optimization problem with its linear form is formulated to minimize IFLAD between the estimated and the targeted dependent variable.

- The proposed approach is sound and better than the existing approaches [32, 28, 31, 33]. Furthermore, IFRM of the proposed approach is general to fit unrestricted data represented as TIFNs.

1.2 Organization

The Rest of the paper structure is briefly described as follows: section 2 is preliminarily devoted for necessary definitions. Section 3 is allocated for the introduced multiplication of an unknown TIFN into a known TIFN. In section 4, a general IFLAD metric is introduced. Section 5 devoted to formulate the proposed approach to construct IFRM by minimizing the intorduced IFLAD based discrepancy between the observed and the estimated values of the dependent variable. In section 6, an example is used to demonstrate the proposed approach. The proposed approach is used to construct IFRM for real life application and compared with the result of the existing approach in section 7. The paper is concluded in the last section.

2. Preliminaries

Let $X = \{x_1, x_2, \ldots, x_n\}$ denotes the discourse set, R is the set of all real numbers, $F(x)$ is for the set of all intuitionistic fuzzy numbers (IFNs) in R. $\tilde{A}$ $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ stand for intuitionistic fuzzy set (IFS), its degree of membership and non-membership functions in X, respectively.

Definition 1[27]: An IFS $\tilde{A}$ in X is expressed as in the form $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x))|x \in R\}$ where $\mu_{\tilde{A}}, \nu_{\tilde{A}}$: $X \longrightarrow [0,1]$ and for all $x \in X$, $0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1$. Additionally, the complement $\pi_{\tilde{A}}(x) = 1 - (\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x))$ denotes the degree of indecision of an element $x \in X$ to $\tilde{A}$.

Definition 2[27]: $\tilde{A}$ is IFS perceived by:

a. $\tilde{A}$ is defined on a real line.

b. There is only and only on value, $m \in X$ exists such that $\mu_{\tilde{A}}(m) = 1$ and $\nu_{\tilde{A}}(m) = 0$. m is called mean pointed value of $\tilde{A}$.

c. $\tilde{A}$ is convex, i.e., $\mu_{\tilde{A}}(\cdot)$ is defined as: $\mu_{\tilde{A}}(\theta x_1 + (1-\theta)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$, $x_1, x_2 x \in R$ and $\theta \in [0,1]$.

d. $\tilde{A}$ is concave, i.e., $\nu_{\tilde{A}}(\cdot)$ is defined as: $\nu_{\tilde{A}}(\theta x_1 + (1-\theta)x_2) \geq \max(\nu_{\tilde{A}}(x_1), \nu_{\tilde{A}}(x_2))$, $x_1, x_2 x \in R$ and $\theta \in [0,1]$.

Definition 3 [27]: A TIFN $\tilde{A}$ is an IFS with following $\mu_{\tilde{A}}(\cdot)$ and $\nu_{\tilde{A}}(\cdot)$:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a+\alpha_\mu}{\alpha_\mu} & \text{if} \quad a-\alpha_\mu \leq x \leq a \\ \frac{a+\beta_\mu - x}{\beta_\mu} & \text{if} \quad a \leq x \leq a+\beta_\mu \\ 0 & \text{elsewhere} \end{cases}$$

$$\nu_{\tilde{A}}(x) = \begin{cases} \frac{a-x}{\alpha_\nu} & \text{if} \quad a-\alpha_\nu \leq x \leq a \\ \frac{x-a}{\beta_\nu} & \text{if} \quad a \leq x \leq a+\beta_\nu \\ 1 & \text{elsewhere} \end{cases}$$

where $a \in R$ is the main value and $\alpha_\mu, \beta_\mu, \alpha_\nu, \beta_\nu \geq 0$ are called right and left spreads of $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$, respectively. The representation of a TIFN is symbolically, $\tilde{A} = \langle a; (\alpha_\mu, \alpha_\nu), (\beta_\mu, \beta_\nu)\rangle$ such that $\alpha_\mu \leq \alpha_\nu$ and $\beta_\mu \leq \beta_\nu$. The trigonometric representation of a TIFN is shown in Fig. 1.

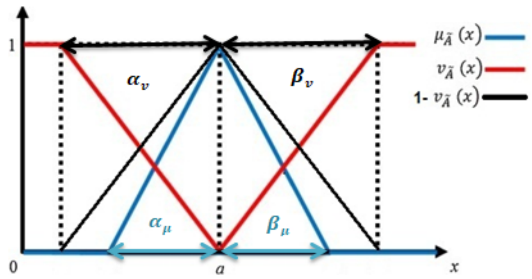

Figure 1. Trigonometric representation of a TIFN.

Definition 4 [29]: For a TIFN $\tilde{A} = \langle a; (\alpha_\mu, \alpha_\nu), (\beta_\mu, \beta_\nu)\rangle, \tilde{A} > 0 (< 0)$ when $a - \alpha_\nu \geq 0$ $(a + \beta_\nu \leq 0)$. Consequentially, neither $\tilde{A}$ is positive nor negative if $a - \alpha_\nu \leq 0$ and $a + \beta_\nu \geq 0$.

Definition 5: Based on Definition 3 and the philosophy of expressing the fuzziness in Trigonometry for a TIFN $\tilde{A} = \langle a; (\alpha_\mu, \alpha_\nu), (\beta_\mu, \beta_\nu)\rangle$, any value belongs to $\tilde{A}$ is either the main value a, a value in the left spread of a, i.e., $a - (\alpha_\mu - k)$ where $0 \leq k \leq \alpha_\mu$ and $a - (\alpha_\nu - k)$ where $0 \leq k \leq \alpha_\nu$, or a value in the left spread of m, i.e., $a + (\beta_\mu - k)$ where $0 \leq k \leq \beta_\mu$ and $a + (\beta_\nu - k)$ where $0 \leq k \leq \beta_\nu$. So, $\mu_{\tilde{A}}(k)$ and $\nu_{\tilde{A}}(k)$ are as follows:

$$\mu_{\tilde{A}}(k) = \begin{cases} \frac{k}{\alpha_\mu} & \text{if} \quad 0 \leq k \leq \alpha_\mu \\ \frac{k}{\beta_\mu} & \text{if} \quad 0 \leq k \leq \beta_\mu \end{cases}$$

$$\nu_{\tilde{A}}(k) = \begin{cases} \frac{\alpha_\nu - k}{\alpha_\nu} & \text{if} \quad 0 \leq k \leq \alpha_\nu \\ \frac{\beta_\nu - k}{\beta_\nu} & \text{if} \quad 0 \leq k \leq \beta_\nu \end{cases}$$

https://doi.org/10.57647/fomj.2026.0701.02

$\mu_{\tilde{A}}(k)$ and $\nu_{\tilde{A}}(k)$ can be called the " quantified membership function" and "quantified non-membership function", respectively.

Definition 6: A TIFN $\tilde{A} = \langle a; (\alpha_\mu, \alpha_\nu), (\beta_\mu, \beta_\nu)\rangle$ is Symmetric if a divides $\tilde{A}$ into two congruent sections, i.e., when $\alpha_\mu = \beta_\mu$ and $\alpha_\nu = \beta_\nu$. The symbolic representation of a symmetric TIFN $\tilde{A}$ can be tersely expressed as $\tilde{A} = \langle a; \gamma_\mu, \gamma_\nu\rangle$ where $\gamma_\mu = \alpha_\mu = \beta_\mu$ and $\gamma_\nu = \alpha_\nu = \beta_\nu$.

Definition 7: Consider $\tilde{A}_i = \langle a_i; (\alpha_{i\mu}, \alpha_{i\nu}), (\beta_{i\mu}, \beta_{i\nu})\rangle$. In the light of Zadeh's extension principle, for n TIFNs, $\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n$ we have:
$\sum_{i=1}^{n} \tilde{A}_i = \langle \sum_{i=1}^{n} a_i;\ (\sum_{i=1}^{n} \alpha_{i\mu},\ \sum_{i=1}^{n} \alpha_{i\nu}),\ (\sum_{i=1}^{n} \beta_{i\mu},\ \sum_{i=1}^{n} \beta_{i\nu})\rangle$ $\tilde{A}_1 - \sum_{i=2}^{n} \tilde{A}_i = \langle a_1 - \sum_{i=2}^{n} a_i;\ (|\alpha_{1\mu} - \sum_{i=2}^{n} \alpha_{i\mu}|,\ (|\alpha_{1\nu} - \sum_{i=2}^{n} \alpha_{i\nu}|),\ (|\beta_{1\mu} - \sum_{i=2}^{n} \beta_{i\mu}|,\ |\beta_{1\nu} - \sum_{i=2}^{n} \beta_{i\nu}|)\rangle$
$k \cdot \tilde{A}_i = \langle k a_i; (|k|\alpha_{i\mu}, |k|\alpha_{i\nu}), (|k|\beta_{i\mu}, |k|\beta_{i\nu})\rangle, k \in R$

$$\prod_{i=1}^{n} \tilde{A}_i = \langle \prod_{i=1}^{n} a_i; (\prod_{i=1}^{n} \alpha_{i\mu}, \prod_{i=1}^{n} \alpha_{i\nu}), (\prod_{i=1}^{n} \beta_{i\mu}, \prod_{i=1}^{n} \beta_{i\nu})\rangle \tag{1}$$

It is pertinent to mention that the direct multiplication in Expression (1), proposed by generalizing the existing multiplication [3], is suitable for TIFNs which are represented in terms of spreads such that $\alpha_\mu, \beta_\mu, \alpha_\nu, \beta_\nu \geq 0$.

3. Multiplication of unknown TIFN into known TIFN

As a statistical regression model, to find the fuzzy coefficients of an IFRM there is need to multiplying an unknown coefficient with known independent variable, both represented as TIFNs. This study proposes the multiplication according to Expression (1) in Definition 7 of the previous section. The multiplication conserves homogeneity principle such that for any two symmetric TIFNs, the resultant is symmetric TIFNs well.

Theorem 1: IF $\tilde{A}_i$ $(i = 1, 2)$ are symmetric TIFNs, then $\tilde{A} = \prod_{i=1}^{2} \tilde{A}_i$ is symmetric TIFN.

Proof: Let $\tilde{A}_1 = \langle a_1; (\alpha_{1\mu}, \alpha_{1\nu}), (\beta_{1\mu}, \beta_{1\nu})\rangle$ and $\tilde{A}_2 = \langle a_2; (\alpha_{2\mu}, \alpha_{2\nu}), (\beta_{2\mu}, \beta_{2\nu})\rangle$ are symmetric TIFNs and $\tilde{A} = \langle a; (\alpha_\mu, \alpha_\nu), (\beta_\mu, \beta_\nu)\rangle$, then using Expression (1), we have

$\tilde{A} = \langle a; (\alpha_\mu, \alpha_\nu), (\beta_\mu, \beta_\nu)\rangle = \tilde{A}_1 \otimes \tilde{A}_2 = \langle a_1 a_1; (\alpha_{1\mu}\alpha_{2\mu}, \alpha_{1\nu}\alpha_{2\nu}), (\beta_{1\mu}\beta_{2\mu}, \beta_{1\nu}\beta_{2\nu})\rangle$ so we have

$a = a_1 a_1, \alpha_\mu = \alpha_{1\mu}\alpha_{2\mu}, \alpha_\nu = \alpha_{1\nu}\alpha_{2\nu}, \beta_\mu = \beta_{1\mu}\beta_{2\mu}$ and $\beta_\nu = \beta_{1\nu}\beta_{2\nu}$. Hence, using quantified membership $(\mu_{\tilde{A}}(k))$ and quantified non-membership $(\nu_{\tilde{A}}(k))$ in Definition 5 We have

For the left $\mu_{\tilde{A}_1}(k).\mu_{\tilde{A}_2}(k) = \frac{k}{\alpha_{1\mu}}\frac{k}{\alpha_{2\mu}} = \frac{k^2}{\alpha_{1\mu}\alpha_{2\mu}}$ and $\nu_{\tilde{A}_2}(k).\nu_{\tilde{A}_2}(k) = \frac{(\alpha_{1\nu}-k)}{\alpha_{1\nu}}\frac{(\alpha_{2\nu}-k)}{\alpha_{2\nu}} = \frac{(\alpha_{1\nu}-k)(\alpha_{2\nu}-k)}{\alpha_{1\nu}\alpha_{2\nu}}$. Since $\tilde{A}_1$ and $\tilde{A}_2$ are symmetric and using Definition 6, $\tilde{A}_1$ and $\tilde{A}_2$ has two congruent sections, that means we have $\alpha_{1\mu} = \beta_{1\mu}, \alpha_{1\nu} = \beta_{1\nu}, \alpha_{2\mu} = \beta_{2\mu}$ and $\alpha_{2\nu} = \beta_{2\nu}$. Consequentially, $\alpha_{1\mu}\alpha_{2\mu} = \beta_{1\mu}\beta_{2\mu} = \alpha_\mu = \beta_\mu$ and $\alpha_{1\nu}\alpha_{2\nu} = \beta_{1\nu}\beta_{2\nu} = \alpha_\nu = \beta_\nu$

Thus, the proof of Theorem 1 is completed.

Moreover, the multiplication in Expression (1) is straightforward to conserve that for any TIFN $\langle m; (\alpha_\mu, \alpha_\nu), (\beta_\mu, \beta_\nu)\rangle$ the condition $\alpha_\mu \leq \alpha_\nu$ and $\beta_\mu \leq \beta_\nu$ is always satisfied. In the proof of Theorem 1, it obvious to notice that, while $\tilde{A}_1 = \langle a_1; (\alpha_{1\mu}, \alpha_{1\nu}), (\beta_{1\mu}, \beta_{1\nu})\rangle$, $\tilde{A}_2 = \langle a_2; (\alpha_{2\mu}, \alpha_{2\nu}), (\beta_{2\mu}, \beta_{2\nu})\rangle$ and $\alpha_{1\mu}$, $\beta_{1\mu}$, $\alpha_{1\nu}$, $\beta_{1\nu}$, $\alpha_{2\mu}$, $\beta_{2\mu}$, $\alpha_{2\nu}$, $\beta_{2\nu} \geq 0$ are symmetric that means $\alpha_{1\mu} \leq \alpha_{1\nu}, \beta_{1\mu} \leq \beta_{1\nu}, \alpha_{2\mu} \leq \alpha_{2\nu}$ and $\beta_{2\mu} \leq \beta_{2\nu}$ then $\alpha_{1\mu}\alpha_{2\mu} \leq \alpha_{1\nu}\alpha_{2\nu}$ and $\beta_{1\mu}\beta_{2\mu} \leq \beta_{1\nu}\beta_{2\nu}$. So, the multiplication expressed in Expression (1) is sound for all the cases of multiplication of two TIFNs including multiplying unknown TIFN with known TIFN.

4. Intuitionistic fuzzy least absolute of discrepancies (IFLAD)

This section introduces IFLAD to construct IFRM and estimate the model's parameters.

Initially, related theorem for non-fuzzy least absolute regression theory [35] is discussed.

Theorem 2 [35]: Consider $n > p$ given data $(Y_1, X_{1j}, \ldots, X_{1p})$, $(Y_2, X_{2j}, \ldots, X_{2p})$, $\ldots$, $(Y_i, X_{ij}, \ldots, X_{ip})$, $\ldots$, $(Y_n, X_{nj}, \ldots, X_{np})$, then the hyperplane $Y = A_0 + A_1 X_1 + A_2 X_2 + \ldots + A_p X_p, A_0, A_1, \ldots, A_p \in R$, estimated by IFLAD regression approach passes $p + 1$ given observed data. For $p = 1$ the line $Y = A_0 + A_1 X, A_0, A_1 \in R$, estimated by IFLAD passes two given observed data.

Definition 8: Let $\tilde{A}_1 = \langle a_1; (\alpha_{1\mu}, \alpha_{1\nu}), (\beta_{1\mu}, \beta_{1\nu})\rangle\rangle$ and $\tilde{A}_2 = \langle a_2; (\alpha_{2\mu}, \alpha_{2\nu}), (\beta_{2\mu}, \beta_{2\nu})\rangle$ be any two asymmetric TIFNs then, the IFLAD based discrepancy between $\tilde{A}_1$ and $\tilde{A}_2$ can be introduced as follows:

$$D(\tilde{A}_1, \tilde{A}_2) = |a_1 - a_2| + |\alpha_{1\mu} - \alpha_{2\mu}| + |\alpha_{1\nu} - \alpha_{2\nu}| + |\beta_{1\mu} - \beta_{2\mu}| + |\beta_{1\nu} - \beta_{2\nu}| \tag{2}$$

If $\tilde{A}_1$ and $\tilde{A}_2$ are symmetric TIFNs, i.e., $\tilde{A}_1 = \langle a_1; \gamma_{1\mu}, \gamma_{1\nu}\rangle$ and $\tilde{A}_2 = \langle a_2; \gamma_{2\mu}, \gamma_{2\nu}\rangle$, then the discrepancy $D(\tilde{A}_1, \tilde{A}_2)$ between $\tilde{A}_1$ and $\tilde{A}_2$ is $|a_1 - a_2| + |\gamma_{1\mu} - \gamma_{2\mu}| + |\gamma_{1\nu} - \gamma_{2\nu}|$.

Theorem 3: $D(\tilde{A}_1, \tilde{A}_2)$ is the discrepancy metric on any two TIFNs.

Proof:

1- Directly, known the expression (2); it is consistent, i.e., $D(\tilde{A}_1, \tilde{A}_2) = D(\tilde{A}_2, \tilde{A}_1)$ and nonnegative, i.e., $D(\tilde{A}_1, \tilde{A}_2) \leq 0$.

2- Triangular inequality: For three TIFNs $\tilde{A}_1 = \langle a_1; (\alpha_{1\mu}, \alpha_{1\nu}), (\beta_{1\mu}, \beta_{1\nu})\rangle$, $\tilde{A}_2 = \langle a_2; (\alpha_{2\mu}, \alpha_{2\nu}), (\beta_{2\mu}, \beta_{2\nu})\rangle$ and $\tilde{A}_3 = \langle a_3; (\alpha_{3\mu}, \alpha_{3\nu}), (\beta_{3\mu}, \beta_{3\nu})\rangle$, it necessary to prove the expression (2) satisfies the following inequality.

$D(\tilde{A}_1, \tilde{A}_3) \leq D(\tilde{A}_1, \tilde{A}_2) + D(\tilde{A}_2, \tilde{A}_3)$

we have

$D(\tilde{A}_1, \tilde{A}_3) = |a_1 - a_3| + |\alpha_{1\mu} - \alpha_{3\mu}| + |\alpha_{1\nu} - \alpha_{3\nu}| + |\beta_{1\mu} - \beta_{3\mu}| + |\beta_{1\nu} - \beta_{3\nu}|$
$= |a_1 - a_2 + a_2 - a_3| + |\alpha_{1\mu} - \alpha_{2\mu} + \alpha_{2\mu} - \alpha_{3\mu}| + |\alpha_{1\nu} - \alpha_{2\nu} + \alpha_{2\nu} - \alpha_{3\nu}| + |\beta_{1\mu} - \beta_{2\mu} + \beta_{2\mu} - \beta_{3\mu}| + |\beta_{1\nu} - \beta_{2\nu} +$

https://doi.org/10.57647/fomj.2026.0701.02

$\beta_{2\nu} - \beta_{3\nu}| \le |a_1 - a_2| + |\alpha_{1\mu} - \alpha_{2\mu}| + |\alpha_{1\nu} - \alpha_{2\nu}| + |\beta_{1\mu} - \beta_{2\mu}| + |\beta_{1\nu} - \beta_{2\nu}| + |a_2 - a_3| + |\alpha_{2\mu} - \alpha_{3\mu}| + |\alpha_{2\nu} - \alpha_{3\nu}| + |\beta_{2\mu} - \beta_{3\mu}| + |\beta_{2\nu} - \beta_{3\nu}| = D(\tilde{A}_1, \tilde{A}_2) + D(\tilde{A}_2, \tilde{A}_3)$

Hence, the proof of Theorem 3 is completed.

Expression (2) is generally characterized as a least absolute discrepancy metric of two TIFNs. It is practically productive to consider the discrepancy between two TIFNs based on the degree of hesitation which can be given as $\pi_{\tilde{A}}(x) = 1 - (\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x))$. For two TIFNs $\tilde{A}_1$ and $\tilde{A}_2$, the optimal discrepancy between them is the minimum distance between their a_1 and a_2 as well as the minimum distance between the maximum points of their degrees of the distance $\pi_{\tilde{A}_1}(x)$ and $\pi_{\tilde{A}_2}(x)$, respectively.

5. The proposed approach to construct IFRM

5.1 IFLAD

In this subsection IFLAD is introduced by extending the existing fuzzy-LAD between two interval-valued triangular fuzzy numbers proposed by Al-Qudaimi [34].

Definition 8: If $\tilde{A}_1 = \tilde{A}_1 = \langle a_1; (\alpha_{1\mu}, \alpha_{1\nu}), (\beta_{1\mu}, \beta_{1\nu})\rangle$ and $\tilde{A}_2 = \langle a_2; (\alpha_{2\mu}, \alpha_{2\nu}), (\beta_{2\mu}, \beta_{2\nu})\rangle$ two TIFNs, then using the Definition 10 and Definition 11 [[34], Section 5, pp. 4] the introduced IFLAD based metric expression is

$$D(\tilde{A}_1, \tilde{A}_2) = |a_1 - a_2| + D(\sup(\pi_{\tilde{A}_1}(x)), \sup(\pi_{\tilde{A}_2}(x))) \tag{3}$$

where

$D(\sup(\pi_{\tilde{A}_1}(x)), \sup(\pi_{\tilde{A}_2}(x)))$ is the distance between the two maximum points of degree of distance of $\tilde{A}_1$ and $\tilde{A}_2$.

Definition 9: According to $\pi_{\tilde{A}}(x) = 1 - (\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x))$ and based on Definition 3, $\pi_{\tilde{A}}(x)$ can be defined as piecewise function. Therefore,

$$\pi_{\tilde{A}}(x) = \begin{cases} \frac{(\alpha_\nu - \alpha_\mu)(a-x)}{\alpha_\nu \alpha_\mu} & x \in [a - \alpha_\nu, a] \\ \frac{(\beta_\nu - \beta_\mu)(x-a)}{\beta_\nu \beta_\mu} & x \in [a, a + \beta_\nu] \end{cases}$$

Now, tracking back to Expression (3), $D(\sup(\pi_{\tilde{A}_1}(x)), \sup(\pi_{\tilde{A}_2}(x)))$ as per Definition 9, can be defined as a sum of left and right spreads components of a TIFN as follows:

$$\begin{aligned} &D(\sup(\pi_{\tilde{A}_1}(x)), \sup(\pi_{\tilde{A}_2}(x))) = \\ &\left|\frac{(\alpha_{1\nu} - \alpha_{1\mu})}{\alpha_{1\nu}} - \frac{(\alpha_{2\nu} - \alpha_{2\mu})}{\alpha_{2\nu}}\right| + \\ &\left|\frac{(\beta_{1\nu} - \beta_{1\mu})}{\beta_{1\nu}} - \frac{(\beta_{2\nu} - \beta_{2\mu})}{\beta_{2\nu}}\right|, \\ &\alpha_{1\nu}, \alpha_{2\mu}, \beta_{1\nu}, \beta_{2\mu} > 0 \end{aligned} \tag{4}$$

by plugging Expression (4) into Expression (3), the distance between any two TIFNs $D(\tilde{A}_1, \tilde{A}_2)$ can be introduced as follows:

$$\begin{aligned} &D(\tilde{A}_1, \tilde{A}_2) = \\ &|a_1 - a_2| + \left|\frac{(\alpha_{1\nu} - \alpha_{1\mu})}{\alpha_{1\nu}} - \frac{(\alpha_{2\nu} - \alpha_{2\mu})}{\alpha_{2\nu}}\right| + \\ &\left|\frac{(\beta_{1\nu} - \beta_{1\mu})}{\beta_{1\nu}} - \frac{(\beta_{2\nu} - \beta_{2\mu})}{\beta_{2\nu}}\right|, \\ &\alpha_{1\nu}, \alpha_{2\mu}, \beta_{1\nu}, \beta_{2\mu} > 0 \end{aligned} \tag{5}$$

The optimal model's coefficients can be determined by minimizing the distance of Expression (5) which has two features over Expression (2) introduced in Definition 8:

- It focuses on reducing the hesitation and the more it is reduced, the more optimality in the constructed model is grasped.
- It clearly reduces the computational efforts especially with large dataset size since this distance has three sub-distances whereas; the distance of Expression (2) has five sub-distances.

5.2 IFRM formulation

Suppose that the patterns of given data $\{(\tilde{Y}_1, \tilde{X}_{1j}, \ldots, \tilde{X}_{1p}), (\tilde{Y}_2, \tilde{X}_{2j}, \ldots, \tilde{X}_{2p}), \ldots, (\tilde{Y}_i, \tilde{X}_{ij}, \ldots, \tilde{X}_{ip}), \ldots, (\tilde{Y}_n, \tilde{X}_{nj}, \ldots, \tilde{X}_{np})\}$, in which X_{ij} $(i = \overline{1, n}, j = \overline{1, p})$ is the j^{th} value of independent variable represented as TIFN, $\tilde{X}_{ij} = \langle x_{ij}; (\alpha_{x_{ij\mu}}, \alpha_{x_{ij\nu}}), (\beta_{x_{ij\mu}}, \beta_{x_{ij\nu}})\rangle$ and $\tilde{Y}_i(i = \overline{1, n})$ is a corresponding value of the dependent variable in i^{th} pattern. Y_i is represented as TIFN, $Y_i = \langle y_{ij}; (\alpha_{y_{i\mu}}, \alpha_{y_{i\nu}}), (\beta_{y_{i\mu}}, \beta_{y_{i\nu}})\rangle$. The prupose of IFRM is to fit the given patterns of data represented as TIFNs. The mathematical expression is a statistical relation in which an approximate value of the dependent variable is obtained by putting the value(s) of the independent variable(s). This model can be expressed as the following approximation

$$\tilde{Y}_i \approx \tilde{A}_0 \oplus (\tilde{A}_1 \otimes \tilde{X}_{1j}) \oplus \ldots \oplus (\tilde{A}_j \otimes \tilde{X}_{ij}) \oplus \ldots \oplus (\tilde{A}_p \otimes \tilde{X}_{np}) \tag{6}$$

Where $\tilde{A} = (\tilde{A}_0, \tilde{A}_1, \ldots, \tilde{A}_p)$ are the estimated model's parameters represented as TIFNs, i.e., $\tilde{A}_j = \langle a_j; (\alpha_{a_{j\mu}}, \alpha_{a_{j\nu}}), (\beta_{a_{j\mu}}, \beta_{a_{j\nu}})\rangle, j = 0, 1, 2, \ldots, p$ and $a_j, (\alpha_{a_{j\mu}}, \alpha_{a_{j\nu}}), (\beta_{a_{j\mu}}, \beta_{a_{j\nu}})$ are the main value, membership and non-membership left and right spreads, respectively. As regression is supervised learning approach, the given patterns of data are used to estimate the model's parameters and the actual response of the dependent variable $\tilde{Y}_i$,

$$\hat{\tilde{Y}}_i = \tilde{A}_0 \oplus (\tilde{A}_1 \otimes \tilde{X}_{1j}) \oplus \ldots \oplus (\tilde{A}_j \otimes \tilde{X}_{ij}) \oplus \ldots \oplus (\tilde{A}_p \otimes \tilde{X}_{np}) = \tilde{A}_0 \oplus \sum_{j=1}^{p} (\tilde{A}_j \otimes \tilde{X}_{ij}), i = 1, 2, \ldots, n. \tag{7}$$

While the given data patterns, estimated parameters and actual response of the dependent variable $\hat{\tilde{Y}}_i$ are represented as TIFNs the full IFRM can be transformed into IFRM (8)

$$\langle\hat{y}_i;(\alpha_{\hat{y}_{i_\mu}},\alpha_{\hat{y}_{i_\nu}}),(\beta_{\hat{y}_{i_\mu}},\beta_{\hat{y}_{i_\nu}})\rangle = \langle a_0;(\alpha_{a_{0_\mu}},\alpha_{a_{0_\nu}}),(\beta_{a_{0_\mu}},\beta_{a_{0_\nu}})\rangle\oplus \sum_{j=1}^{p}\langle a_j;(\alpha_{a_{j_\mu}},\alpha_{a_{j_\nu}}),(\beta_{a_{j_\mu}},\beta_{a_{j_\nu}})\rangle\otimes \langle x_{ij};(\alpha_{x_{ij_\mu}},\alpha_{x_{ij_\nu}}),(\beta_{x_{ij_\mu}},\beta_{x_{ij_\nu}})\rangle, i=1,2,\dots,n. \tag{8}$$

Using Definition 7, IFRM (8) can be transformed into its equivalent IFRM (9)

$$\langle\hat{y}_i;(\alpha_{\hat{y}_{i_\mu}},\alpha_{\hat{y}_{i_\nu}}),(\beta_{\hat{y}_{i_\mu}},\beta_{\hat{y}_{i_\nu}})\rangle = \langle a_0+\sum_{j=1}^{p}a_jx_{ij}, (\alpha_{a_{0_\mu}}+\sum_{j=1}^{p}\alpha_{a_{j_\mu}}\alpha_{x_{ij_\mu}},\alpha_{a_{0_\nu}}+\sum_{j=1}^{p}\alpha_{a_{j_\nu}}\alpha_{x_{ij_\nu}}), (\beta_{a_{0_\mu}}+\sum_{j=1}^{p}\beta_{a_{j_\mu}}\beta_{x_{ij_\mu}},\beta_{a_{0_\nu}}+\sum_{j=1}^{p}\beta_{a_{j_\nu}}\beta_{x_{ij_\nu}})\rangle \tag{9}$$

On decomposing IFRM (9), we have

$$\hat{y}_i = a_0+\sum_{j=1}^{p}a_jx_{ij},\ (\alpha_{\hat{y}_{i_\mu}},\alpha_{\hat{y}_{i_\nu}}) = (\alpha_{a_{0_\mu}}+\sum_{j=1}^{p}\alpha_{a_{j_\mu}}\alpha_{x_{ij_\mu}},\alpha_{a_{0_\nu}}+\sum_{j=1}^{p}\alpha_{a_{j_\nu}}\alpha_{x_{ij_\nu}}),$$
$$(\beta_{\hat{y}_{i_\mu}},\beta_{\hat{y}_{i_\nu}}) = (\beta_{a_{0_\mu}}+\sum_{j=1}^{p}\beta_{a_{j_\mu}}\beta_{x_{ij_\mu}},\beta_{a_{0_\nu}}+\sum_{j=1}^{p}\beta_{a_{j_\nu}}\beta_{x_{ij_\nu}})$$

Using Expression (5) for $\tilde{Y}_i$ and $\hat{\tilde{Y}}_i$,i.e., $D(\tilde{Y}_i,\hat{\tilde{Y}}_i)$, the optimal value of each estimated model's parameter $\tilde{A}_j$ can be determined by minimizing the total absolute discrepancies according to the proposed absolute discrepancy metric. Therefore, the objective function is as follows:

$$\min\sum_{i=1}^{n}D(\tilde{Y}_i,\hat{\tilde{Y}}_i) = \frac{1}{3n}\sum_{i=1}^{n}|y_{ij}-(a_0+\sum_{j=1}^{p}a_jx_{ij})|+ \left|\frac{(\alpha_{y_{i_\nu}}-\alpha_{y_{i_\mu}})}{\alpha_{y_{i_\nu}}}-(\frac{(\alpha_{a_{0_\nu}}-\alpha_{a_{0_\mu}})}{\alpha_{a_{0_\nu}}}+\sum_{j=1}^{p}\frac{(\alpha_{a_{j_\nu}}-\alpha_{a_{j_\mu}})}{\alpha_{a_{j_\nu}}}\frac{(\alpha_{x_{ij_\nu}}-\alpha_{x_{ij_\mu}})}{\alpha_{x_{ij_\nu}}})\right|+ \left|\frac{(\beta_{y_{i_\nu}}-\beta_{y_{i_\mu}})}{\beta_{y_{i_\nu}}}-(\frac{(\beta_{a_{0_\nu}}-\beta_{a_{0_\mu}})}{\beta_{a_{0_\nu}}}+\sum_{j=1}^{p}\frac{(\beta_{a_{j_\nu}}-\beta_{a_{j_\mu}})}{\beta_{a_{j_\nu}}}\frac{(\beta_{x_{ij_\nu}}-\beta_{x_{ij_\mu}})}{\beta_{x_{ij_\nu}}})\right|$$

Subject to

$0\le\alpha_{a_{j_\mu}},\alpha_{a_{j_\nu}}\beta_{a_{j_\mu}},\beta_{a_{j_\nu}}, j=0,1,\dots,p.$

The abovementioned expression can be converted to the linear form as follows:

$\min\sum_{i=1}^{n}D(\tilde{Y}_i,\hat{\tilde{Y}}_i) = \frac{1}{3n}\sum_{i=1}^{n}[(\tau_{1_i}+\tau_{2_i})+(\sigma_{1_i}+\sigma_{2_i})+(\delta_{1_i}+\delta_{2_i})$

Subject to

$$\begin{aligned}
&y_{ij}-(a_0+\sum_{j=1}^{p}a_jx_{ij}) = \tau_{1_i}-\tau_{2_i}\\
&\frac{(\alpha_{y_{i_\nu}}-\alpha_{y_{i_\mu}})}{\alpha_{y_{i_\nu}}}-(\frac{(\alpha_{a_{0_\nu}}-\alpha_{a_{0_\mu}})}{\alpha_{a_{0_\nu}}}+\sum_{j=1}^{p}\frac{(\alpha_{a_{j_\nu}}-\alpha_{a_{j_\mu}})}{\alpha_{a_{j_\nu}}}\frac{(\alpha_{x_{ij_\nu}}-\alpha_{x_{ij_\mu}})}{\alpha_{x_{ij_\nu}}}) = \sigma_{1_i}-\sigma_{2_i}\\
&\frac{(\beta_{y_{i_\nu}}-\beta_{y_{i_\mu}})}{\beta_{y_{i_\nu}}}-(\frac{(\beta_{a_{0_\nu}}-\beta_{a_{0_\mu}})}{\beta_{a_{0_\nu}}}+\sum_{j=1}^{p}\frac{(\beta_{a_{j_\nu}}-\beta_{a_{j_\mu}})}{\beta_{a_{j_\nu}}}\frac{(\beta_{x_{ij_\nu}}-\beta_{x_{ij_\mu}})}{\beta_{x_{ij_\nu}}}) = \delta_{1_i}-\delta_{2_i}\\
&\tau_{k_i}\ge 0,\sigma_{k_i}\ge 0,\delta_{k_i}\ge 0,\\
&0\le\alpha_{a_{j_\mu}},\alpha_{a_{j_\nu}}\beta_{a_{j_\mu}},\beta_{a_{j_\nu}},\\
&i=1,2,\dots,n,\quad k=1,2,\quad j=0,1,\dots,p.
\end{aligned} \tag{10}$$

On solving (10) the estimated model's parameters components, $a_0,\alpha_{a_{0_\mu}},\alpha_{a_{0_\nu}},\beta_{a_{0_\mu}},\beta_{a_{0_\nu}},a_j,\alpha_{a_{j_\mu}},\alpha_{a_{j_\nu}},\beta_{a_{j_\mu}}$ and $\beta_{a_{j_\mu}}$ are determined. Putting the determined values in Expression (7), IFRM is constructed.

The above-mentioned expression is for asymmetric TIFNs. Specifically, if a given dataset is represented as symmetric TIFNs, i.e., $\alpha_{\blacksquare_\mu}=\beta_{\blacksquare_\mu}$, and $\alpha_{\blacksquare_\nu}=\beta_{\blacksquare_\nu}$, where $\blacksquare\in\{x_i,y_i\}$, IFRM (8) can be rewritten as follows:

$$\langle\hat{y}_i;\alpha_{\hat{y}_{i_\mu}},\alpha_{\hat{y}_{i_\nu}}\rangle = \langle a_0;\alpha_{a_{0_\mu}},\alpha_{a_{0_\nu}}\rangle\oplus \sum_{j=1}^{p}\langle a_j;\alpha_{a_{j_\mu}},\alpha_{a_{j_\nu}}\rangle\otimes\langle x_{ij};\alpha_{x_{ij_\mu}},\alpha_{x_{ij_\nu}}\rangle, i=1,2,\dots,n. \tag{11}$$

The linear objective function for symmetric IFRM (11) is a reduction of (8), as follows:

$\min\sum_{i=1}^{n}D(\tilde{Y}_i,\hat{\tilde{Y}}_i) = \frac{1}{3n}\sum_{i=1}^{n}[(\tau_{1_i}+\tau_{2_i})+(\sigma_{1_i}+\sigma_{2_i})]$

subject to

$$\begin{aligned}
&y_{ij}-(a_0+\sum_{j=1}^{p}a_jx_{ij}) = \tau_{1_i}-\tau_{2_i}\\
&\frac{(\alpha_{y_{i_\nu}}-\alpha_{y_{i_\mu}})}{\alpha_{y_{i_\nu}}}-(\frac{(\alpha_{a_{0_\nu}}-\alpha_{a_{0_\mu}})}{\alpha_{a_{0_\nu}}}+\sum_{j=1}^{p}\frac{(\alpha_{a_{j_\nu}}-\alpha_{a_{j_\mu}})}{\alpha_{a_{j_\nu}}}\frac{(\alpha_{x_{ij_\nu}}-\alpha_{x_{ij_\mu}})}{\alpha_{x_{ij_\nu}}}) = \sigma_{1_i}-\sigma_{2_i}\\
&\tau_{k_i}\ge 0,\sigma_{k_i}\ge 0,\alpha_{a_{j_\mu}},\alpha_{a_{j_\nu}}\ge 0,\\
&i=1,2,\dots,n,k=1,2,j=0,1,\dots,p.
\end{aligned} \tag{12}$$

On solving (12) the estimated model's parameters components, $a_0,\alpha_{a_{0_\mu}},\alpha_{a_{0_\nu}},a_j,\alpha_{a_{j_\mu}}$ and $\alpha_{a_{j_\nu}}$ are determined. Putting the determined values in IFRM (11), symmetric IFRM is constructed.

The proposed approach can flexibly manipulate different types of datasets represented as asymmetric and/ or symmetric unrestricted TIFNs (negative and/or positive) even neither negative nor positive

https://doi.org/10.57647/fomj.2026.0701.02

data. Non-TIFNs of independent and dependent variables also can be manipulated by considering $Y_i = \langle y_{ij}; (\alpha_{y_{i\mu}}, \alpha_{y_{i\nu}}), (\beta_{y_{i\mu}}, \beta_{y_{i\nu}})\rangle$ with $\alpha_{x_{ij\nu}}, \beta_{x_{ij\nu}}, \alpha_{y_{i\nu}}$ and $\beta_{y_{i\nu}}$ equal to zero.

The optimization problems (10) and (12) can be solved using professional software, e.g., Maple.

6. Demonstrative example

To demonstrate the proposed approach an IFRM is obtained by considering the following given data

$X_{11} = \langle 3; (1,2), (1,2)\rangle, Y_1 = \langle 10; (5,9), (5,9)\rangle$

$X_{21} = \langle 2; (1,2), (1,2)\rangle, Y_2 = \langle 11; (4,7), (4,7)\rangle$

Assuming that $\tilde{A}_j = \langle a_j; (\alpha_{a_{j\mu}}, \alpha_{a_{j\nu}}), (\beta_{a_{j\mu}}), \beta_{a_{j\nu}})\rangle$ and $\hat{\tilde{Y}}_i = \langle \hat{y}_i; (\alpha_{\hat{y}_{i\mu}}, \alpha_{\hat{y}_{i\nu}}), (\beta_{\hat{y}_{i\mu}}\beta_{\hat{y}_{i\nu}})\rangle, j = 0, 1, i = 1, 2$ we have

$\langle \hat{y}_1; (\alpha_{\hat{y}_{1\mu}}, \alpha_{\hat{y}_{1\nu}}), (\beta_{\hat{y}_{1\mu}}\beta_{\hat{y}_{1\nu}})\rangle =$
$\langle a_0; (\alpha_{a_{0\mu}}, \alpha_{a_{0\nu}}), (\beta_{a_{0\mu}}, \beta_{a_{0\nu}})\rangle \oplus$
$\langle a_1; \alpha_{a_{1\mu}}, \alpha_{a_{1\nu}}), (\beta_{a_{1\mu}}, \beta_{a_{1\nu}})\rangle \otimes \langle 3; (1,2), (1,2)\rangle$

$\langle \hat{y}_2; (\alpha_{\hat{y}_{2\mu}}, \alpha_{\hat{y}_{2\nu}}), (\beta_{\hat{y}_{2\mu}}\beta_{\hat{y}_{2\nu}})\rangle =$
$\langle a_0; (\alpha_{a_{0\mu}}, \alpha_{a_{0\nu}}), (\beta_{a_{0\mu}}, \beta_{a_{0\nu}})\rangle \oplus$
$\langle a_1; \alpha_{a_{1\mu}}, \alpha_{a_{1\nu}}), (\beta_{a_{1\mu}}, \beta_{a_{1\nu}})\rangle \otimes \langle 2; (1,2), (1,2)\rangle$

Using Definition 7, we get

$\langle \hat{y}_1; (\alpha_{\hat{y}_{1\mu}}, \alpha_{\hat{y}_{1\nu}})), (\beta_{\hat{y}_{1\mu}}\beta_{\hat{y}_{1\nu}})\rangle$
$= \langle a_0 + 3a_1; (\alpha_{a_{0\mu}} + \alpha_{a_{1\mu}}, \alpha_{a_{0\nu}} + 2\alpha_{a_{1\nu}}), (\beta_{a_{0\mu}} + \beta_{a_{1\mu}}, \beta_{a_{0\nu}} + 2\beta_{a_{1\nu}})\rangle$

$\langle \hat{y}_2; (\alpha_{\hat{y}_{2\mu}}, \alpha_{\hat{y}_{2\nu}})), (\beta_{\hat{y}_{2\mu}}\beta_{\hat{y}_{2\nu}})\rangle$
$= \langle a_0 + 2a_1; (\alpha_{a_{0\mu}} + \alpha_{a_{1\mu}}, \alpha_{a_{0\nu}} + 2\alpha_{a_{1\nu}}), (\beta_{a_{0\mu}} + \beta_{a_{1\mu}}, \beta_{a_{0\nu}} + 2\beta_{a_{1\nu}})\rangle$

Using Expression (5) for $\tilde{Y}_i$ and $\hat{\tilde{Y}}_i$,i.e., $D(\tilde{Y}_i, \hat{\tilde{Y}}_i)$, and the linear form of mathematical programming problem of Expression (10), we have

$\min\, [D(\tilde{Y}_1, \hat{\tilde{Y}}_1) + D(\tilde{Y}_2, \hat{\tilde{Y}}_2))] = \frac{1}{6}[(\tau_{1_1} + \tau_{2_1}) + (\sigma_{1_1} + \sigma_{2_1}) + (\delta_{1_1} + \delta_{2_1}) + (\tau_{1_2} + \tau_{2_2}) + (\sigma_{1_2} + \sigma_{2_2}) + (\delta_{1_2} + \delta_{2_2})]$

Subject to

$10 - a_0 - 3a_1 = \tau_{1_1} - \tau_{2_1}$

$11 - a_0 - 2a_1 = \tau_{1_2} - \tau_{2_2}$

$\frac{4}{9} - (\frac{(\alpha_{a_{0\nu}} - \alpha_{a_{0\mu}})}{\alpha_{a_{0\nu}}} + \frac{1}{2}(\frac{(\alpha_{a_{1\nu}} - \alpha_{a_{1\mu}})}{\alpha_{a_{1\nu}}}) = \sigma_{1_1} - \sigma_{2_1}$

$\frac{3}{7} - (\frac{(\alpha_{a_{0\nu}} - \alpha_{a_{0\mu}})}{\alpha_{a_{0\nu}}} + \frac{1}{2}(\frac{(\alpha_{a_{1\nu}} - \alpha_{a_{1\mu}})}{\alpha_{a_{1\nu}}}) = \sigma_{1_2} - \sigma_{2_2}$

$\frac{4}{9} - (\frac{(\beta_{a_{0\nu}} - \beta_{a_{0\mu}})}{\beta_{a_{0\nu}}} + \frac{1}{2}(\frac{(\beta_{a_{1\nu}} - \beta_{a_{1\mu}})}{\beta_{a_{1\nu}}}) = \delta_{1_1} - \delta_{2_1}$

$\frac{3}{7} - (\frac{(\beta_{a_{0\nu}} - \beta_{a_{0\mu}})}{\beta_{a_{0\nu}}} + \frac{1}{2}(\frac{(\beta_{a_{1\nu}} - \beta_{a_{1\mu}})}{\beta_{a_{1\nu}}}) = \delta_{1_2} - \delta_{2_2}$

$\tau_{k_i} \geq 0, \sigma_{k_i} \geq 0, \delta_{k_i} \geq 0,$

$0 \leq \alpha_{a_{j\mu}}, \alpha_{a_{j\nu}} \beta_{a_{j\mu}}, \beta_{a_{j\nu}},$

$i = 1, 2, \ldots, n, k = 1, 2, j = 0, 1, \ldots, p.$

On solving this mathematical linear programming problem, the obtained optimal solution is $a_0 = 13, a_1 = -1, \alpha_{a_{0\mu}} = \alpha_{a_{0\nu}} = \beta_{a_{0\mu}} = \beta_{a_{0\nu}} = \alpha_{a_{1\mu}} = \beta_{a_{1\mu}} = 1$ and $\alpha_{a_{1\nu}} = \beta_{a_{1\nu}} = 2$ using the obtained values, we get $\tilde{A}_0 = \langle a_0; (\alpha_{a_{0\mu}}, \alpha_{a_{0\nu}}), (\beta_{a_{0\mu}}, \beta_{a_{0\nu}})\rangle = \langle 13; (1,1), (1,1)\rangle$ and $\tilde{A}_1 = \langle a_1; (\alpha_{a_{1\mu}}, \alpha_{a_{1\nu}}), (\beta_{a_{1\mu}}, \beta_{a_{1\nu}})\rangle = \langle -1; (1,2), (1,2)\rangle$ so, IFRM of this example is $\langle \hat{y}; \alpha_{\hat{y}_\mu}, \alpha_{\hat{y}_\nu}, \beta_{\hat{y}_\mu}, \alpha_{\hat{y}_\nu})\rangle = \langle 13; (1,1), (1,1)\rangle \oplus \langle -1; (1,2), (1,2)\rangle \otimes \langle x; (\alpha_{x_\mu}, \alpha_{x_\nu}), (\beta_{x_\mu}, \beta_{x_\nu})\rangle$

7. Application

The proposed approach constructs linear IFTRM. One of the existing approaches is very limited; Parvathi et al. [27] approach cannot be compared here because it is for crisp given data. Four approaches in literature [2, 28, 31, 33] can be compared with the approach of this study.

The dataset adopted in Table 1 given patterns represented as symmetric TIFNs, collected from Arefi and Taheri [28].

The proposed IFTRM compared with existing models, Chen and Nien [31] and Arefi and Taheri [28] based on discrepancy metric propounded in Definition 8. The discrepancy between the given and estimated outputs of existing models, Chen and Nien [31] (named D_{CN}), Nien and Chen [33] (named D_{ND}), Al-Qudaimi [34] (named D_{Ishita}) and Arefi and Taheri [28] (named D_{AT}) with the proposed model of this study (named D_{AS}) is shown in Table 2.

The discrepancy of any two TIFNs is taken as the distance of the components of the two TIFNs. The better performance decided by the smaller distance between the given output and the actual one which is obtained by the estimated model.

Chen and Nien [31] obtained the following IFRM.

$$\begin{aligned}\hat{\tilde{Y}}_{CN} =& \langle \hat{y}; (\alpha_{\hat{y}_\mu}, \alpha_{\hat{y}_\nu}), (\beta_{\hat{y}_\mu}, \beta_{\hat{y}_\nu})\rangle \\ =& \langle 21.0663; (0.0000, 0.3537), (0.1043, 0.6842)\rangle \oplus \\ & (\langle -0.1969; (0.0117, 0.0118), (0,0)\rangle \otimes \\ & \langle x_1; (\alpha_{x_{1\mu}}, \alpha_{x_{1\nu}}), (\beta_{x_{1\mu}}, \beta_{x_{1\nu}})\rangle) \oplus \\ & (\langle 2.6922; (0,0), (0,0)\rangle \otimes \langle x_2; (\alpha_{x_{2\mu}}, \alpha_{x_{2\nu}}), \\ & (\beta_{x_{2\mu}}, \beta_{x_{2\nu}})\rangle)\end{aligned} \tag{13}$$

The intercept $\tilde{A}_0 = \langle 21.0663; (0, 0.3537), (0.1043, 0.6842)\rangle$ in IFRM (10) is obviously asymmetric, i.e., $\alpha_{a_{0\mu}} = 0 \neq \beta_{a_{0\mu}} = 0.1043$ and $\alpha_{a_{0\nu}} = 0.3537 \neq \beta_{a_{0\nu}} = 0.6842$ whereas, all the patterns of given dataset are symmetric TIFNs, i.e., model (10) is not necessarily conserve the homogeneity principle. Therefore, the model obtained by Chen and Nien [31] faces inexact results on using it for prediction.

Arefi and Taheri [28] obtained the following IFRM.

$$\begin{aligned}\hat{\tilde{Y}}_{AT} =& \langle \hat{y}; (\alpha_{\hat{y}_\mu}, \alpha_{\hat{y}_\nu}) \\ =& \langle 21.9811; 0.8933, 1.9882\rangle \oplus \\ & (\langle -0.2221; 0.0117, 0.0118\rangle \otimes \\ & \langle x_1; \alpha_{x_{1\mu}}, \alpha_{x_{1\nu}}\rangle) \oplus (\langle 2.4701; 0, 0\rangle \otimes \\ & \langle x_2; \gamma_{x_{2\mu}}, \gamma_{x_{2\nu}}\rangle)\end{aligned} \tag{14}$$

Nein and Chen [[33], Section 3, pp. 6] express TIFN in spread-wise as $\langle a; \alpha_\mu, \beta_\mu, \alpha_\nu, \beta_\nu\rangle$ for input/ output variables where α_μ and β_μ are the left and right membership spreads, and α_ν and β_ν are the left and right non-membership spreads around the main value a. Based on their representation, they obtained their IFRM coefficients as vectors. On considering the first slope as a TIFN —

Table 1. given dataset patterns used in Example 6.1 represented as symmetric TIFNs.

i	$\langle y_i; \alpha_{y_{i\mu}}, \alpha_{x_{1\nu}} \rangle$	$\langle x_{i1}; \alpha_{x_{i1\mu}}, \alpha_{x_{i1\nu}} \rangle$	$\langle x_{i2}; \alpha_{x_{i2\mu}}, \alpha_{x_{i2\nu}} \rangle$
1	⟨16.5; 0.82, 1.65⟩	⟨35; 3.5, 5.25⟩	⟨0.88; 0.09, 0.13⟩
2	⟨18.6; 0.93, 1.86⟩	⟨37; 3.7, 5.55⟩	⟨1.13; 0.11, 0.17⟩
3	⟨19.3; 0.97, 1.93⟩	⟨27; 2.7, 4.05⟩	⟨1.31; 0.13, 0.20⟩
4	⟨20.3; 1.02, 2.03⟩	⟨29; 2.9, 4.35⟩	⟨1.98; 0.20, 0.30⟩
5	⟨17.3; 0.87, 1.73⟩	⟨38; 3.8, 5.70⟩	⟨1.02; 0.10, 0.15⟩
6	⟨20.4; 1.02, 2.04⟩	⟨32; 3.2, 4.80⟩	⟨1.29; 0.13, 0.19⟩
7	⟨19.3; 0.97, 1.93⟩	⟨29; 2.9, 4.35⟩	⟨1.52; 0.15, 0.23⟩
8	⟨21.9; 1.09, 2.19⟩	⟨18; 1.8, 2.70⟩	⟨1.33; 0.13, 0.20⟩
9	⟨15.9; 0.80, 1.59⟩	⟨40; 4.0, 6.00⟩	⟨1.71; 0.17, 0.26⟩
10	⟨18.3; 0.92, 1.83⟩	⟨28; 2.8, 4.20⟩	⟨2.00; 0.20, 0.30⟩
11	⟨22.6; 1.13, 2.26⟩	⟨13; 1.3, 1.95⟩	⟨1.68; 0.17, 0.25⟩
12	⟨23.7; 1.19, 2.37⟩	⟨19; 1.9, 2.85⟩	⟨2.15; 0.22, 0.32⟩
13	⟨24.4; 1.22, 2.44⟩	⟨31; 3.1, 4.65⟩	⟨3.52; 0.35, 0.53⟩
14	⟨21.8; 1.09, 2.18⟩	⟨31; 3.1, 4.65⟩	⟨2.33; 0.23, 0.35⟩
15	⟨23.8; 1.19, 2.38⟩	⟨17; 1.7, 2.55⟩	⟨1.71; 0.17, 0.26⟩
16	⟨20.8; 1.04, 2.08⟩	⟨14; 1.4, 2.10⟩	⟨1.14; 0.11, 0.17⟩
17	⟨17.5; 0.88, 1.75⟩	⟨19; 1.9, 2.85⟩	⟨0.99; 0.10, 0.15⟩
18	⟨17.8; 0.89, 1.78⟩	⟨28; 2.8, 4.20⟩	⟨1.14; 0.11, 0.17⟩
19	⟨20.2; 1.01, 2.02⟩	⟨26; 2.6, 3.90⟩	⟨1.46; 0.15, 0.22⟩
20	⟨20.0; 1.00, 2.00⟩	⟨32; 3.2, 4.80⟩	⟨1.81; 0.18, 0.27⟩
21	⟨22.8; 1.14, 2.28⟩	⟨10; 1.0, 1.50⟩	⟨1.38; 0.14, 0.21⟩
22	⟨19.1; 0.96, 1.91⟩	⟨38; 3.8, 5.70⟩	⟨0.84; 0.08, 0.13⟩
23	⟨12.1; 0.60, 1.21⟩	⟨49; 4.9, 7.35⟩	⟨1.48; 0.15, 0.22⟩
24	⟨12.8; 0.64, 1.28⟩	⟨42; 4.2, 6.30⟩	⟨1.08; 0.11, 0.16⟩

$\tilde{A}_1 = \langle -0.1982; -0.0091, -0.0091, -0.1321, -0.1321 \rangle$ which as per the symmetric TIFN representation of the proposed approach equals $\langle -0.1982; -0.0091, -0.1321 \rangle$ which, in turn, violates the preliminary definition of TIFN since the spreads are negative. However, they kept the parameters as vectors so, their model can be considered a vector regression since what they have constructed is [28]:

$$\langle \hat{y}_i; \alpha_{\hat{y}_{i\mu}}, \beta_{\hat{y}_{i\mu}}, \alpha_{\hat{y}_{i\nu}}, \beta_{\hat{y}_{i\nu}} \rangle$$

$$[a_j, \alpha_{a_{j\nu}}, \beta_{a_{j\mu}}, \alpha_{a_{j\nu}}, \beta_{a_{j\nu}}] \begin{bmatrix} x_{ij} \\ \alpha_{x_{ij\mu}} \\ \beta_{x_{ij\mu}} \\ \alpha_{x_{ij\nu}} \\ \beta_{x_{ij\nu}} \end{bmatrix}$$

Parameters' vectors according to Nein and Chen approach [33] have obtained as:

$\tilde{A}_0 = [21.0663, 1.0553, 1.0553, 2.1066, 2.1066]$,

$\tilde{A}_1 = [-0.1982; -0.0091, -0.0091, -0.1321, -0.1321]$, and

$\tilde{A}_2 = [2.6922, 1.3461, 1.3461, 1.7947, 1.7947]$

It is pertinent to notice that to get a valid estimated output $\hat{\tilde{Y}}_{NC} = \tilde{A}_0 \oplus (\tilde{A}_1 \otimes \tilde{X}_1) \oplus (\tilde{A}_1 \otimes \tilde{X}_2)$ as a TIFN the spreads of the estimated output must satisfy the follows:

$\alpha_{\hat{y}_{i\mu}} = 1.0553 - 0.0091\alpha_{x_{i1\mu}} + 1.3461\alpha_{x_{i2\mu}} \geq 0$
$\beta_{\hat{y}_{i\mu}} = 1.0553 - 0.0091\beta_{x_{i1\mu}} + 1.3461\beta_{x_{i2\mu}} \geq 0$
$\alpha_{\hat{y}_{i\nu}} = 2.1066 - 0.1321\alpha_{x_{i1\nu}} + 1.7947\alpha_{x_{i2\nu}} \geq 0$
$\beta_{\hat{y}_{i\nu}} = 2.1066 - 0.1321\beta_{x_{i1\nu}} + 1.7947\beta_{x_{i2\nu}} \geq 0$

However, for some cases such that any spread of $\tilde{X}_1$ is larger enough, that will lead to have related negative spread of $\hat{\tilde{Y}}$. For example on predicting an output related to $\tilde{X}_1 = \langle 50; 13.7, 25.55, 13.7, 25.55 \rangle$ and $\tilde{X}_2 = \langle 1.13; 0.11, 0.17, 0.11, 0.17 \rangle$

$\beta_{\hat{y}_\nu} = 2.1066 - 0.1321 \times 25.55 + 1.7947 \times 0.17 = -0.9635 < 0$ and $\alpha_{\hat{y}_\nu} = 2.1066 - 0.1321 \times 25.55 + 1.7947 \times 0.17 = -0.9635 < 0$, hence, the predicted output is invalid since $\beta_{\hat{y}_\nu}$ and $\alpha_{\hat{y}_\nu} < 0(-0.9635)$. While for any spread of $\hat{\tilde{Y}}$ we have $\blacksquare_{\hat{y}_*} = \blacksquare_{\tilde{A}_{0_*}} + (-\blacksquare_{\tilde{A}_{1_*}} \times \blacksquare_{x_{1_*}}) + (\blacksquare_{\tilde{A}_{2_*}} \times \blacksquare_{x_{2_*}})$, to ensure non-negativity of $\blacksquare_{\hat{y}_*}$ the term $(-\blacksquare_{\tilde{A}_{1_*}} \times \blacksquare_{x_{1_*}})$ must be greater than $\blacksquare_{\tilde{A}_{0_*}} + (\blacksquare_{\tilde{A}_{2_*}}) \times \blacksquare_{x_{2_*}})$, where $\blacksquare \in \{\alpha, \beta\}$ and $* \in \{\mu, \nu\}$

The data of Table 1 used to validate Nein and Chen

https://doi.org/10.57647/fomj.2026.0701.02

Table 2. Comparison with existing IFTRMs.

i	$(D_{CN})_i$	$(D_{AT})_i$	$(D_{Ishita})_i$	$(D_{NC})_i$	$(D_{AS})_i$
1	3.8517	1.1375	0.4243	0.0000	0.1206
2	6.2146	2.4459	1.8549	1.8233	0.6313
3	4.6710	0.37110	0.1533	0.0571	0.0190
4	5.3346	0.3256	0.4679	0.3503	0.1246
5	5.0177	2.0356	1.2906	1.0176	0.4309
6	7.1394	2.5279	2.6604	2.2017	0.8219
7	4.7961	0.2998	0.20310	0.1119	0.0576
8	6.2251	1.3326	1.6957	0.8199	0.4587
9	5.5218	2.6492	3.05610	1.8437	0.8567
10	6.9853	2.8401	3.2975	2.6023	1.0036
11	6.0673	1.5841	1.2398	0.4132	0.3438
12	6.5544	1.8634	1.4342	0.6104	0.4352
13	6.2167	1.98310	0.0314	0.0000	0.0000
14	5.9626	1.5434	0.7251	0.6037	0.2468
15	7.4751	2.6478	2.6544	1.4987	0.7828
16	5.6766	1.2823	1.2506	1.250608	0.3080
17	6.5983	3.3313	2.6772	0.5612	0.9024
18	5.0200	1.3659	0.9958	2.4666	0.3344
19	5.2403	0.5882	0.6085	0.7871	0.1744
20	5.2194	0.8834	0.4754	0.3551	0.1356
21	5.7103	1.3870	1.0891	0.4018	0.2456
22	7.8423	3.8092	3.6852	0.0000	1.1225
23	5.7905	5.0751	5.4167	3.2411	1.4584
24	5.6019	4.6906	4.5273	2.8513	1.2674
total	140.7329	48.0019	41.9164	27.9204	12.2821

[28] has tiny spreads of input variables so, their vector regression model is:

$$\hat{\tilde{Y}}_{NC} = \langle \hat{y}_i; \alpha_{y_{i_\mu}}, \beta_{y_{i_\mu}}, \alpha_{y_{i_\nu}}, \beta_{y_{i_\nu}} \rangle =$$
$$[21.0663, 1.0553, 1.0553, 2.1066, 2.1066] +$$
$$[-0.1982; -0.0091, -0.0091, -0.1321, -0.1321] \begin{bmatrix} x_{i1} \\ \alpha_{x_{i1_\mu}} \\ \beta_{x_{i1_\mu}} \\ \alpha_{x_{i1_\nu}} \\ \beta_{x_{i1_\nu}} \end{bmatrix}$$
$$+ [2.6922, 1.3461, 1.3461, 1.7947, 1.7947] \begin{bmatrix} x_{i2} \\ \alpha_{x_{i2_\mu}} \\ \beta_{x_{i2_\mu}} \\ \alpha_{x_{i2_\nu}} \\ \beta_{x_{i2_\nu}} \end{bmatrix}$$

Al-Qudaimi [32] obtained the following IFRM.

$$\hat{\tilde{Y}}_{Ishita} = \langle \hat{y}; \alpha_{\hat{y}_\mu}, \alpha_{\hat{y}_\nu} \rangle = \langle 21.0663; 0.7971, 1.5861 \rangle \oplus (\langle -0.1982; 0, 0 \rangle \otimes \langle x_1; \alpha_{x_{1_\mu}}, \alpha_{x_{1_\nu}} \rangle) \oplus (\langle 2.6922; 1.2083, 1.6111 \rangle \otimes \langle x_2; \alpha_{x_{2_\mu}}, \alpha_{x_{2_\nu}} \rangle)$$

Using the approach proposed in this study, the estimated model's parameters are as follows:

The obtained model's intercept is $\tilde{A}_0 = \langle 21.0663, (1.0253, 2.0481), (1.0253, 2.0481) \rangle$.

The first slope is, $\tilde{A}_1 = \langle -0.1982, (0, 0), (0, 0) \rangle$ and the second slope is $\tilde{A}_2 = \langle 2.6922, (2.9577, 5.9080), (2.9577, 5.9080) \rangle$. Therefore, IFRM is

$$\hat{\tilde{Y}}_{As} = \langle \hat{y}; \alpha_{\hat{y}_\mu}, \alpha_{\hat{y}_\nu} \rangle = \langle 21.0663; 1.0253, 2.0481 \rangle \oplus (\langle -0.1982; 0, 0 \rangle \otimes \langle x_1; \alpha_{x_{1_\mu}}, \alpha_{x_{1_\nu}} \rangle) \oplus (\langle 2.6922; 2.9577, 5.9080 \rangle \otimes \langle x_2; \alpha_{x_{2_\mu}}, \alpha_{x_{2_\nu}} \rangle))$$

The proposed approach of this study is compared with the existing approaches [32, 33, 36] in the basis of discrepancy metric introduced in Definition 8. The best performance matches the smallest total of discrepancies. The outcomes are listed out in Table 2. Total discrepancies of the proposed approach $\sum_{i=1}^{24} (D_{AS})_i$ is the minimum among the existing approaches.

https://doi.org/10.57647/fomj.2026.0701.02

8. Conclusion

This paper is devoted to introduce the discrepancy between two TIFNs, apply IFLAD based estimators to construct robust IFTRM for data represented as TIFNs either symmetric or asymmetric and positive/negative or neither negative nor positive, conserving the homogeneity principle in models such that if all the patterns in a dataset are symmetric, they must be fitted by symmetric model, i.e., the nature of the model and given dataset representation are homogeneous. The soundness and generality comes from the silent property of a straightforward introduced multiplication and use the IFLAD metric of TIFNs to evaluated fitting performance of the given and the estimated output values. Although there is a lack of intuitionistic data, an application has demonstrated the proposed approach. Comparatively, the application illustrates how far the proposed approach better than the existing IFRMs.

Authors contributions
All authors contributed equally to the conception, design, execution, and writing of this work. All authors read and approved the final manuscript.

Availability of data and materials
The authors declare that the data supporting the findings of this study are available within the paper.

Conflict of interests
The authors assert that they do not have any identifiable conflicting financial interests or personal relationships that might be perceived to influence the work presented in this paper.

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