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A robust model for the Best-Worst Method (BWM) based on goal programming

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Abstract:

The utilization of effective decision-making methodologies has historically captured the attention of decision-makers. The Best-Worst Method (BWM) is a prominent and extensively utilized technique in Multi Attribute Decision Making (MADM), owing to its simplicity, reduced number of pairwise comparisons, and high consistency of results. However, the original BWM operates under the assumption that decision-makers' judgments are precise and certain. In real-world conditions, uncertainty is prevalent. Furthermore, when addressing MADM problems that are not fully consistent and involve more than three criteria, the BWM technique may not result in a unique optimal solution. To address these limitations, this study proposes a Robust BWM approach based on goal programming and formulated as a linear optimization model. This approach incorporates interval preferences to model uncertainty in judgments. Additionally, a methodology is put forward to assess the Consistency Ratio (CR) of input data in the presence of uncertainty, thereby ensuring the reliability of the results obtained. The performance of the model has been examined through six numerical examples, and its validity has been evaluated. The findings suggest that the proposed method exhibits adequate efficiency and performance in addressing MADM problems and managing uncertain data. Consequently, the proposed method can serve as an effective solution for decision-making problems in real-world and uncertainty-based environments.

Keywords: Best-Worst Method (BWM); Multi Attribute Decision Making (MADM); Robust optimization; Uncertainty; Goal programming; Consistency Ratio (CR)

1. Introduction

Decision-making is a critical responsibility of management in organizations, playing a pivotal role in their success, productivity, and sustainable development. In the context of decision-making environments characterized by increasing complexity and the presence of diverse criteria, the utilization of scientific and structured methods has become an indisputable necessity. Multi-criteria decision making (MCDM) is a subfield of operations research (OR) that aims to evaluate and select the optimal alternative by considering multiple criteria simultaneously [1]. MCDM techniques have been shown to possess a high capability to reduce decision-making complexity, integrate both quantitative and qualitative criteria, provide a structured framework for decision problems, and offer ease of use. This makes them practical tools for managers and researchers [2]. The MCDM methods are generally classified into two main categories: Multi-Objective Decision Making (MODM) is employed for continuous problems, while Multi-Attribute Decision Making (MADM) is utilized for discrete problems [3].

The Best-Worst Method (BWM) is a highly effective and extensively utilized technique for addressing multi-attribute decision-making (MADM) problems. The original BWM model, a nonlinear model introduced by Rezaei in 2015, is the basis for this study [4]. In comparison with other multi-attribute decision-making (MADM) methods, BWM is more straightforward and demands a smaller number of pairwise comparisons. Specifically, it is sufficient to identify only the best and worst criteria, and the criterion weights can be calculated with a minimal number of pairwise comparisons ($2n-3$ comparisons, where n is the number of criteria [5]). This advantage becomes particularly apparent when the number of criteria is large [6]. To ensure the acceptability of the weights calculated by BWM, the Consistency Ratio (CR) must be computed [4]. The utilization of standard threshold tables for the Consistency Ratio Thresholds (CRT) [7] proves advantageous in this context. Notwithstanding its myriad advantages, the original BMW possesses certain limitations. For instance, if a problem is not fully consistent (CR $\neq 0$) and involves more than three criteria, the original model may yield multiple optimal solutions. In such circumstances, rather than a distinct optimal weight for each criterion, the weights fall within an optimal interval [8]. While multiple optimal solutions may be sought in certain instances, in the majority of cases, decision-makers prefer a singular optimal solution [9]. In addressing this limitation, researchers have proposed versions of the BWM that yield unique solutions. One notable example is the linear BWM model proposed by Rezaei in 2016, which is regarded as equivalent to the nonlinear BWM model (the original BWM model) [10]. Subsequently, Amiri and Emamat presented linear and nonlinear BWM models based on goal programming. The objective of these models is to obtain a unique optimal solution, even in problems that are not fully consistent and involve more than three criteria. A comparison of these two models with the original model reveals that they possess a reduced number of constraints, consequently yielding a lower computational complexity [9]. Subsequent studies have been undertaken that propose models based on analytical relations to determine the weights of criteria and to achieve unique optimal solutions [11, 12]. However, given the characteristics of the goal programming-based BWM model proposed by Amiri and Emamat, when the objective is to obtain unique optimal solutions, the use of these goal programming-based BWM models would be an appropriate and efficient approach.

In mathematical programming models, problems are typically solved under the assumption of data certainty. However, in the real world, most data are subject to uncertainty [13]. As demonstrated by Ben-Tal et al., the solution to an optimization problem frequently exhibits high sensitivity to variations in input data [14]. Consequently, disregarding uncertainty may yield a solution that is suboptimal or impractical [15]. Consequently, a significant concern in numerous decision-making scenarios pertains to the uncertainty associated with input parameters (e.g., pairwise comparisons). The presence of uncertainty has the potential to yield results that are not aligned with the prevailing circumstances, thereby introducing complexity to the decision-making process. Consequently, uncertainty should be regarded as an inherent component of real-world systems, and its incorporation into input parameters serves to augment the applicability of the model to real-world problems [16]. In the original BWM, the pairwise preference comparisons between different criteria are expressed as crisp values on a scale from 1 to 9. However, in complex and uncertain decision-making conditions, experts often find themselves unable to provide precise and unequivocal preferences due to limitations in personal knowledge, individual preferences, ambiguity, and the complexity of human cognition [17]. Therefore, another limitation of the original BWM, as well as the goal programming models [9], is the assumption of deterministic decision-maker judgments. However, in real-world environments, data are often accompanied by uncertainty, ambiguity, or incomplete information. In order to confront this challenge and to advance both the theoretical and practical development of BWM, various types of fuzzy sets have been successfully integrated into BWM. These include interval numbers [18], triangular fuzzy numbers, trapezoidal fuzzy numbers, interval fuzzy numbers, and others [17]. Furthermore, the utilization of fuzzy logic to enhance the goal programming-based BWM method has received considerable attention. This combination enables the achievement of unique optimal solutions under uncertainty conditions [19].

On the other hand, the robust optimization approach offers a robust framework for modeling and addressing decision-making problems under uncertainty [15]. However, the number of studies employing this approach in the context of BWM remains limited. Among these studies, one of the most significant contributions was made by Sad ozi and Karimi, who proposed three BWM solution models based on the robust optimization approach. The development of these models was informed by the prior work of Rezaei, who introduced the linear BWM model. According to the report, all the proposed models yield the same results [16]. However, it should be noted that the solutions obtained from the linear BWM are approximations of those from the nonlinear BWM. In some cases, these approximations may differ significantly from the nonlinear BWM solutions. Consequently, the solutions derived from the linear BWM [20] cannot be relied upon in all cases. Conversely, the linear goal programming BWM model proposed by Amiri and Emamat has demonstrated superior performance in comparison to Rezaei's linear BWM model and has addressed some of its limitations [9]. Furthermore, the utilization of interval data in pairwise comparisons-particularly in methodologies such as BWM-facilitates the enhancement of decision-making models' flexibility and adaptability under uncertainty, without the computational complexity typically associated with fuzzy methods. This type of extension is significant in the development of robust models for criteria weighting [21]. Consequently, in real-world settings, where decision-makers frequently lack the capacity to express preferences as precise numerical values, interval preferences emerge as a suitable methodology for representing uncertainty and ambiguity in judgments. The utilization of intervals as opposed to fixed values facilitates the articulation of a multitude of perspectives and fosters augmented flexibility and convenience in the decision-making process [18]. Accordingly, in this study, to address uncertainty in the input data, a robust optimization approach combined with interval data in pairwise comparisons is employed. Given that implementing the robust optimization approach necessitates a base linear model, and considering the merits of the linear goal programming BWM model [9], this model is adopted as the foundation for developing the robust model in this research. Additionally, the robust models proposed in [16] did not provide a suitable mechanism for evaluating the reliability or confidence of model results under uncertainty. This issue has also been identified as a significant research gap in the present study.

A review of the extant literature reveals several significant research gaps in the field. These include the inability of the original BWM and goal programming-based models to handle uncertainty; the occurrence of multiple optimal solutions in both the original BWM and the robust BWM models proposed; and the absence of a coherent method for ensuring the uniqueness and reliability of the robust BWM models' results under real-world conditions. The primary innovation of this research lies in the development of a novel robust optimization model for BWM using linear goal programming, which aims to address the aforementioned research gaps. In this model, interval preferences are employed to represent uncertainty in pairwise comparisons. Additionally, an algorithm is proposed to assess the CR of the data under uncertainty, ensuring the reliability of the results. The efficacy of the model is substantiated through a series of six numerical illustrations, which underscore its proficiency in generating dependable resolutions in authentic settings.

The structure of the paper is organized as follows: Section 2 presents the theoretical background and a review of relevant literature. Section 3 introduces the proposed robust BWM model, and the steps involved in its development. In section 4, six numerical examples are solved under both deterministic and uncertain input conditions, followed by analysis of the results. Finally, section 5 concludes the study and provides suggestions for future research.

2. Theoretical foundations and literature review

This section first examines theoretical concepts related to uncertainty and robust optimization, the BWM and its consistency, as well as the extended BWM models. Then, the related literature on combining various extended BWM models with optimization approaches under uncertainty are reviewed.

2.1 Uncertainty and robust optimization

Uncertainty refers to a phenomenon that arises from the lack of sufficient knowledge or incomplete information regarding the likelihood of future events. Over the past five decades, this concept has become one of the most prominent and frequently discussed topics in scientific studies and decision-making [22]. It has found wide application in fields such as economics, engineering, and management. Different types of uncertainty in the real world can be classified into several categories based on the nature of the problem and their specific characteristics, such as ambiguity, randomness, data incompleteness, and so on [23].

Optimization approaches are generally divided into two main categories: deterministic and non-deterministic programming. Despite the role deterministic optimization approaches play in understanding the concept of problem optimization, they often ignore the inherent uncertainty of some parameters, which raises concerns about the validity of their results. For this reason, non-deterministic optimization has been introduced as a new paradigm to overcome the limitations of deterministic approaches [24].

Non-deterministic optimization approaches have evolved into three main branches: 1) Stochastic optimization, 2) Fuzzy optimization, and 3) Robust optimization [25].

The robust optimization approach has been developed to solve problems where data are subject to uncertainty. This approach seeks near-optimal solutions that are highly plausible with high probability. In other words, a robust decision is one that is resilient to uncertainty in the environment, with minimal variation in its resulting performance. Robust optimization involves two distinct constraints: Structural constraints and control constraints. Structural constraints follow the concept of linear programming, with their input data remaining unchanged; while control constraints are auxiliary constraints influenced by the input data [13]. This approach generally includes two main steps. In the first step, an uncertainty set is defined within the problem space to determine the range of permissible values for uncertain parameters; Then, in the second step, the optimal solution is obtained such that it remains feasible and performs satisfactorily across all possible states within this set [15]. Therefore, robust optimization is a risk-averse approach for tackling optimization problems under uncertainty; This approach aims to provide a solution that remains feasible across various scenarios. In the field of robust optimization, Soyster (1973) developed a pessimistic robust programming method to address imprecise linear programming problems. Several years later, Ben-Tal and Nemirovski made significant progress in the development of robust programming theory by extending Soyster's method to handle linear programming problems under uncertainty with various convex uncertainty sets. Bertsimas and Sim also provided a flexible approach for robust optimization, in which a parameter can vary within a defined percentage, allowing uncertainty to be modeled [26]. The mathematical model of Soyster will be examined next.

Suppose there exists a linear optimization model as given in model (1), where l, u, and c are n-dimensional vectors, b is of dimension m, and A is an $m \times n$ matrix.

$$\begin{aligned} &\max\ c^T x \\ &s.t: \\ &Ax \leq b \\ &l \leq x \leq u \end{aligned} \tag{1}$$

In the Soyster approach for addressing uncertainty in model (1), it is assumed that the right-hand side coefficients and the objective function coefficients are deterministic, while the coefficients of the variables in the constraints are random variables that independently lies within a symmetric range. Soyster's robust optimization model presented model (1) as model (2) [27].

$$\begin{aligned} &\max\ c^T x \\ &s.t: \\ &\sum_j a_{ij}x_j + \sum_{j\in J_i} \hat{a}_{ij}y_j \leq b_i \qquad \forall_i \\ &-y_j \leq x_j \leq y_j \qquad \forall_j \\ &l \leq x \leq u \\ &y \geq 0 \end{aligned} \tag{2}$$

All variables in the Soyster model are allowed to deviate within the range $[-1, 1]$. Therefore, the obtained solution remains feasible for every possible value of the uncertain coefficients [15].

2.2 BWM and its consistency

BWM was introduced by Rezaei as a nonlinear model (model (3)) for solving MADM problems, and it is considered the original BWM model. The steps for constructing and solving the original BWM model are as follows:

Step 1: The set of decision criteria is defined as $\{C_1, C_2, \ldots, C_n\}$.

Step 2: The Best (B) and the Worst (W) criteria are identified.

Step 3: The preference of the best criterion over all other criteria (a_{Bj}) is determined using a scale from 1 to 9.

Step 4: The preference of all criteria over the worst criterion (a_{jW}) is determined using a scale from 1 to 9.

Step 5: By formulating and solving model (3), the optimal weights of the criteria (w_1^*, w_2^*, …, w_n^*) are calculated.

$$\begin{aligned} &\min \xi \\ &s.t: \\ &|\frac{w_B}{w_j} - a_{Bj}| \leq \xi, \forall_j \\ &|\frac{w_j}{w_w} - a_{jw}| \leq \xi, \forall_j \\ &\sum_j w_j = 1 \\ &w_j \geq 0, \forall_j \end{aligned} \tag{3}$$

To ensure the reliability of BWM results, the CR is calculated based on equation (4); where ξ^* is the optimal value of the objective function in model (3), and the Consistency Index (CI), or $\xi_{\max}$, is obtained from Table 1. The CR is a numerical value between zero and one. The closer the CR is to zero, the higher the consistency; and the closer it is to one, the lower the consistency [4].

$$\mathrm{CR} = \frac{\xi^*}{\mathrm{CI}} \tag{4}$$

Regarding the assessment of the CR in the conventional BWM model, two important points should be noted:
If the CR is zero, the results are consistent and reliable. However, if the CR is not zero, no clear acceptable threshold has been proposed, and it remains unclear how to accurately judge the consistency of the results. It has only been stated that the closer the CR is to zero, the higher the consistency, and the closer it is to one, the lower the consistency.
Consistency assessment is only possible after the formation and solution of model (3) [28].
Subsequently, Liang et al., in order to ensure the consistency of the BWM data and results, proposed methods for calculating the Input-based Consistency Ratio ($\mathrm{CR^I}$), the Output-based Consistency Ratio ($\mathrm{CR^O}$), and the Consistency Ratio Thresholds (CRT); The $\mathrm{CR^O}$ is calculated based on equation (4). The $\mathrm{CR^I}$ is computed according to the number of criteria in each problem, as shown in equation (5).

$$\begin{aligned} &\mathrm{CR^I} = \max_j \mathrm{CR}_j{}^{\mathrm{I}} \\ &\mathrm{CR}_j{}^{\mathrm{I}} = \begin{cases} \frac{|a_{Bj} \times a_{jw} - a_{Bw}|}{a_{Bw} \times a_{Bw} - a_{Bw}} & a_{Bw} > 1 \\ 0 & a_{Bw} = 1 \end{cases} \end{aligned} \tag{5}$$

The use of $\mathrm{CR^I}$ offers several advantages. One of the main benefits of $\mathrm{CR^I}$ is that it provides immediate feedback; meaning that its calculation does not require solving model (3). Another advantage is that it is independent of the model type; in other words, $\mathrm{CR^I}$ can be used to assess consistency levels in other BWM models (in addition to model (3). CRT tables for $\mathrm{CR^I}$ and $\mathrm{CR^O}$ have been provided for up to 9 criteria. Table 2 presents the CRT for $\mathrm{CR^I}$. To evaluate the reliability of the results, the CR of the given problem is first calculated using equations (4) and (5), and then compared with the CRT obtained from the CRT tables. If the CR is lower than the CRT, the results are deemed reliable; otherwise, it is recommended to revise the initial pairwise comparisons. It is worth noting that a high correlation between $\mathrm{CR^I}$ and $\mathrm{CR^O}$ has been reported [7]. therefore, $\mathrm{CR^I}$ can be used to verify the consistency of the results with high probability even before formulating and solving the BWM model. In this regard, if $\mathrm{CR^I}$ indicates inconsistency, it is not necessary to repeat all the steps of BWM, and it is sufficient to revise the input data. This feature can lead to a reduction in computational load; additionally, using $\mathrm{CR^O}$ enhances confidence in the results [28].

2.3 Developed BWM models

In this section, the BWM models developed under deterministic conditions are first reviewed; These models include linear models, goal programming models, and methods developed to derive unique optimal solutions for BWM based on deterministic data. Then, the BWM models proposed under uncertainty conditions are examined; these models have employed fuzzy, robust, and interval-based approaches to deal with imprecise data and uncertain environments.

2.3.1 BWM models under deterministic conditions

Given that solving linear programming models is generally simpler than nonlinear programming, efforts have been made to develop methods based on linear programming modeling and solution approaches. The linear BWM model was first introduced by Rezaei as model (6). For CR calculation, the closer ξ^L is to zero, the more reliable the results

Table 1. Consistency indices (CI) [4].

a_{BW}	1	2	3	4	5	6	7	8	9
CI ($\xi_{\max}$)	0.00	0.44	1.00	1.63	2.30	3.00	3.73	4.47	5.23

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Table 2. CRT for CR[I] [7].

		Criteria						
		3	**4**	**5**	**6**	**7**	**8**	**9**
Scales	**3**	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667
	4	0.1121	0.1529	0.1898	0.2206	0.2527	0.2577	0.2683
	5	0.1354	0.1994	0.2306	0.2546	0.2716	0.2844	0.2960
	6	0.1330	0.1990	0.2643	0.3044	0.3144	0.3221	0.3262
	7	0.1294	0.2457	0.2819	0.3029	0.3144	0.3251	0.3403
	8	0.1309	0.2521	0.2958	0.3154	0.3408	0.3620	0.3657
	9	0.1359	0.2681	0.3062	0.3337	0.3517	0.3620	0.3662

are considered [10]. Moreover, other studies such as Dehghani and Abbasi [29] and Abbasi and Dehghani [30] have proposed algorithms that, using linear programming models, can reach the solutions of the nonlinear model. These methods can be applied in step 5.

$$\begin{aligned}
&\min \xi^L \\
&s.t: \\
&|w_B - a_{Bj}w_j| \leq \xi^L, \forall_j \\
&|w_j - a_{jw}w_w| \leq \xi^L, \forall_j \\
&\sum_j w_j = 1 \\
&w_j \geq 0, \forall_j
\end{aligned} \tag{6}$$

On the other hand, if the problem is not fully consistent (CR $\neq 0$), and involves more than three criteria, the likelihood of multiple optimal solutions for model (3) increases. In such situations, two linear programming models (models (7) and (8)) are used to determine the acceptable bounds for each criterion's weight $[{w_j}^{\min *}, {w_j}^{\max *}]$; and the average of the interval is considered an estimate of a single representative solution [8, 10].

$$\begin{aligned}
&\min w_j \\
&s.t: \\
&|w_B - a_{Bj}w_j| \leq \xi^* w_j, \forall_j \\
&|w_j - a_{jw}w_w| \leq \xi^* w_w, \forall_j \\
&\sum_j w_j = 1 \\
&w_j \geq 0, \forall_j
\end{aligned} \tag{7}$$

$$\begin{aligned}
&\max w_j \\
&s.t: \\
&|w_B - a_{Bj}w_j| \leq \xi^* w_j, \forall_j \\
&|w_j - a_{jw}w_w| \leq \xi^* w_w, \forall_j \\
&\sum_j w_j = 1 \\
&w_j \geq 0, \forall_j
\end{aligned} \tag{8}$$

To obtain unique solutions in cases where the problem is not fully consistent, Amiri and Emamat developed two BWM models—linear and nonlinear-based on goal programming. The linear goal programming BWM model was introduced in model (9).

$$\begin{aligned}
&\min Z = \sum_j (y_j^+ + y_j^-) + \sum_j (z_j^+ + z_j^-) \\
&s.t: \\
&w_B - a_{Bj}w_j = y_j^+ - y_j^-, \forall_j \\
&w_j - a_{jw}w_w = z_j^+ - z_j^-, \forall_j \\
&\sum_j w_j = 1 \\
&w_j, y_j^+, y_j^-, z_j^+, z_j^- \geq 0, \forall_j
\end{aligned} \tag{9}$$

Equation (10) is used to calculate the CR of the corresponding model.

$$\begin{aligned}
&\xi = \max{}_j \{y_j^+ - y_j^-; z_j^+ - z_j^-\} \\
&\mathrm{CR} = \frac{\xi}{\mathrm{CI}}
\end{aligned} \tag{10}$$

The number of constraints in these models is $2n-2$, whereas the number of constraints in the previous models is $4n-5$. Therefore, the proposed models have fewer constraints compared to the previous ones; as a result, these models involve lower computational complexity and thus, provide more reliable results. On the other hand, the linear goal programming BWM model proposed produces better results compared to Rezaei's linear BWM model [9].
Since obtaining unique optimal solutions in the BWM method has always been a concern for decision-makers; in addition to linear model [10] and the goal programming-based models [9], other studies have also been conducted in this area. For example, Wu et al. proposed a comprehensive analytical framework for the BWM method; the original nonlinear BWM model (the main model) may result in multiple optimal solutions, whereas its linearized version, although yielding a unique solution, differs in the feasible solution space from the nonlinear model and also alters the objective function. In this study, they developed an optimization model equivalent to the original BWM and introduced a secondary objective function, enabling the derivation of analytical forms for both unique and interval-based solutions; unlike previous models that required optimization software to determine the optimal weights of

criteria, in the proposed model of this study, the optimal weights of the criteria can be calculated solely based on analytical relations; this innovation represents a significant step toward understanding the mathematical structure of the BWM method [12]. In the same vein, Ratandhara and Kumar (2024), aiming to reduce computational complexity and improve time efficiency, proposed an analytical model for BWM that derives interval-based optimal weights without requiring numerical solving; an approach that also enables generalization to other versions of BWM, including fuzzy BWM [11].

2.3.2 BWM models under uncertainty

Since data in the real world is often not available in a deterministic form, numerous studies aiming to enhance BWM have attempted to integrate it with non-deterministic approaches such as fuzzy, robust, and interval models. In this regard, fuzzy-based approaches integrated with BWM are first reviewed; for instance, Mohammadi and Rezaei proposed a Bayesian BWM model for group decision-making [31]. Amiri et al. introduced a triangular fuzzy BWM model based on alpha-cut (α) analysis; in this model, the decision-maker can determine the alpha value between 0.1 and 0.9 depending on the level of uncertainty. A higher alpha indicates lower uncertainty, while a lower alpha reflects greater uncertainty in decision-making [32]. Bonab et al. proposed an extended version of BWM in a spherical fuzzy (SFS) environment [33]. Further efforts were made to improve the classical fuzzy BWM model proposed by Zhao and Guo [34] to address the limitation of neglecting the full shape of fuzzy numbers. To this end, Ratandhara and Kumar developed a new model based on α-cut intervals for all α values in the range [0, 1]. In this model, the optimal weights are approximated and calculated based on the midpoints of the resulting intervals. Moreover, to enhance the accuracy and reliability of the results in MADM problems, the CR was employed [35].
In line with improving the goal programming-based BWM model [9] using fuzzy logic, Amirghodsi et al. integrated the BWM method with goal programming for the purpose of supplier selection. In their study, BWM was first employed to assess the weights of the criteria. Then, three key factors involved in the decision-making process-product/technology selection, selection of the technology/product transfer method, and supplier selection-were analyzed using goal programming. The objective of the model was to minimize cost and failure risk while maximizing the level of service and compliance with environmental requirements [36]. However, in the proposed model, no mathematically formulated integration between BWM and goal programming was established; instead, the two methods were applied sequentially and independently. Subsequently, Rostami et al. proposed a fuzzy BWM method based on goal programming to determine the criteria weights in supplier selection problems. By employing fuzzy logic and triangular fuzzy numbers, they enhanced the goal programming-based BWM model introduced by Emamat and Amiri; a model that was originally designed for fully deterministic data and could not accommodate uncertainty. As a result, their developed model is capable of handling real-world uncertainty while still producing unique solutions [19].
In recent years, the use of interval data has expanded as an effective approach for addressing uncertainty in MADM problems. For example, Torkan et al. in an applied study, used interval values to model the performance indicators of bank branches and compared them using the PROMETHEE II, TOPSIS, and EDAS methods. This type of modeling allows for capturing data fluctuations over time and serves as a suitable alternative to fuzzy methods in situations with incomplete or uncertain information. This approach can also be effectively extended to weighting methods such as robust BWM; in which pairwise comparisons are performed using interval values instead of precise or fuzzy numbers to ensure the stability and consistency of results against data fluctuations [21]. Qu et al. also proposed a novel three-stage method in the field of MADM; in which an integration of the interval BWM model and the normal distribution is employed. In the second stage of this method, the interval BWM model combined with the normal distribution is used to derive the weights of the criteria. This method creatively utilizes interval fuzzy numbers [18]. On the other hand, deriving the weights of criteria and value functions in MADM problems often involves challenges such as ambiguity in judgments and uncertainty. Since decision-makers are often unable to express their preferences precisely, the use of interval preferences in pairwise comparisons can be an effective solution for representing uncertainty and flexibility in evaluations. Accordingly, Brunelli et al. proposed the Best–Worst Disaggregation (BWD) method; which, by utilizing interval pairwise comparisons, provides a framework that reduces the cognitive burden on the decision-maker while enabling the realistic and robust derivation of value functions and criterion weights [37].
In the context of solving BWM under uncertainty using robust optimization approaches, limited research has been conducted. For instance, Sadjadi and Karimi proposed three robust optimization models for solving BWM based on Rezaei's linear model. The core idea of their study was the incorporation of uncertainty into pairwise comparisons; they argued that it is more realistic to ask the decision-maker (DM) to assign an interval rather than a single precise number for the comparison between two criteria [16]. Considering some of the drawbacks associated with Rezaei's linear model, it can be concluded that the results from Sadjadi and Karimi's study serve as approximations of the actual BWM solutions. Moreover, their study did not provide a mechanism for validating or ensuring the reliability of the results under uncertainty.
Hendalianpour et al. developed a robust BWM model using interval-valued rough fuzzy numbers [38]. Qu et al. proposed a robust BWM optimization method. They introduced four robust counterpart constraints instead of the two linear constraints in the original optimization model. In their model, the weight of each criterion is obtained as an optimal interval [39]. Omrani et al. proposed a new BWM model based on robust Data Envelopment Analysis (DEA). In their proposed model, they aimed to leverage the strengths of BWM for capturing expert opinions and robust

DEA for modeling uncertainty, in order to provide more reliable and realistic results [40]. Huang proposed a robust group decision-making framework based on Multi-Actor Multi-Criteria Analysis (MAMCA) and BWM. The integration of these two methods results in a robust framework that reduces ambiguity and conflicts among stakeholders and leverages optimization models to achieve consensus thereby improving the quality and reliability of decisions [41]. Luo et al. employed a robust approach within the BWM framework to handle the ambiguity and uncertainty present in expert judgments. In this study, they utilized the Probabilistic Linguistic Term Set (PLTS) environment to model expert evaluations. This approach enables the expression of preferences in linguistic terms with varying degrees of probability [42].

According to the reviewed studies, obtaining unique and reliable optimal solutions under conditions where the problem is not fully consistent and data are accompanied by uncertainty, using interval data and robust optimization approaches, has received less attention; whereas most research on uncertainty has focused on developing fuzzy BWM models. Moreover, the reviewed studies have not provided a specific mechanism for assessing the consistency of input data under uncertainty and ensuring the reliability of the obtained solutions. Therefore, considering the advantages of the linear goal programming-based BWM model proposed [9], the present study develops a robust BWM model based on a goal programming framework that not only provides unique and valid solutions under uncertainty, but also incorporates a mechanism for controlling the consistency of input data. This capability has been less addressed in previous studies.

3. Proposed robust BWM model algorithm

The proposed robust BWM model, developed based on the goal programming framework for weighting the criteria in MADM problems, is implemented through the following stages:

Stage 1: In this stage, similar to the original BWM steps, the set of decision criteria is defined as $\{C_1, C_2, \ldots, C_n\}$.

Stage 2: In this stage, the best (B) and the worst (W) criteria must be identified.

Stage 3: In this stage, the preference of the best criterion over all others (a_{Bj}) is determined.

In the original steps of BWM, the preference values a_{Bj} are assigned using integer numbers ranging from 1 to 9; however, due to the presence of uncertainty in real-world data, a range is assigned to the preference value a_{Bj} instead of a single crisp value. This range is defined as $\tilde{a}_{Bj} = [\tilde{a}'_{Bj}, \tilde{a}''_{Bj}]$, or equivalently, $\tilde{a}_{Bj} = \dot{a}_{Bj} \pm \hat{a}_{Bj}$ $\dot{a}_{Bj}$ represents the actual preference value of the best criterion over the other criteria, which is calculated as the average of the interval, i.e $\dot{a}_{Bj} = \frac{\tilde{a}'_{Bj}+\tilde{a}''_{Bj}}{2}$. $\hat{a}_{Bj}$ denotes the estimation error of the actual preference value $\dot{a}_{Bj}$, which is calculated as half the width of the interval, i.e., $\hat{a}_{Bj} = \frac{\tilde{a}''_{Bj}-\tilde{a}'_{Bj}}{2}$. Instead of directly receiving an interval for the preference values, it is also possible to obtain the actual value $\dot{a}_{Bj}$ along with its corresponding absolute or percentage error $\hat{a}_{Bj}$. Moreover, the self-comparison of the best criterion is defined as $\tilde{a}_{BB} = [1, 0]$, or equivalently $\tilde{a}_{BB} = 1 \pm 0$.

Stage 4: The preference of other criteria over the worst criterion (a_{jW}) is determined.

Similar to the previous stage, instesd of using a single integer (ranging from 1 to 9) for the preference value a_{jw}, an interval is used. This interval is defined as $\tilde{a}_{jw} = [\tilde{a}'_{jw}, \tilde{a}''_{jw}]$ or equivalently $\tilde{a}_{jw} = \dot{a}_{jw} \pm \hat{a}_{jw}.\dot{a}_{jw}$ represents the actual preference of other criteria over the worst criterion and is calculated as $\dot{a}_{jw} = \frac{\tilde{a}'_{jw}+\tilde{a}''_{jw}}{2}$. Moreover, $\hat{a}_{jw}$ denotes the estimation error of the actual value $\dot{a}_{jw}$ and is obtained by $\hat{a}_{jw} = \frac{\tilde{a}''_{jw}-\tilde{a}'_{jw}}{2}$. At this stage as well, instead of receiving a direct interval for the preferences, one can provide the actual value $\dot{a}_{jw}$ and its associated absolute or relative error $\hat{a}_{jw}$. Furthermore, the self-comparison of the worst criterion is assumed as $\tilde{a}_{ww} = [1, 0]$ or equivalently $\tilde{a}_{ww} = 1 \pm 0$.

Stage 5: Ensuring data consistency using CR^{I}.

To ensure the consistency of the data and the reliability of the robust BWM optimization model results under uncertainty, a systematic approach must be adopted. Therefore, at this stage, a method is proposed to verify the consistency of the data and, consequently, to ensure the validity of the robust BWM model outcomes under uncertain conditions. In this process, the CR^{I}, is first calculated using equation (5) for three categories of values-namely the actual values, lower bounds, and upper bounds-separately; which are denoted respectively as $\dot{\mathrm{CR}}^{\mathrm{I}}$, $\mathrm{CR}^{\mathrm{I}'}$, and $\mathrm{CR}^{\mathrm{I}''}$. Then, the maximum of these three values is considered as the total input-based CR under uncertainty and is denoted by $\mathrm{TCR}^{\mathrm{I}}$ (as defined in equation (11)).

$$\begin{aligned}
&\dot{\mathrm{CR}}^{\mathrm{I}} = \mathrm{CR}^{\mathrm{I}}\{\dot{a}_{Bj}, \dot{a}_{jw}\} \\
&\mathrm{CR}^{\mathrm{I}'} = \mathrm{CR}^{\mathrm{I}}\{\tilde{a}'_{Bj}, \tilde{a}'_{jw}\} \\
&\mathrm{CR}^{\mathrm{I}''} = \mathrm{CR}^{\mathrm{I}}\{\tilde{a}''_{Bj}, \tilde{a}''_{jw}\} \\
&\mathrm{TCR}^{\mathrm{I}} = \mathrm{MAX}\{\dot{\mathrm{CR}}^{\mathrm{I}}, \mathrm{CR}^{\mathrm{I}'}, \mathrm{CR}^{\mathrm{I}''}\}
\end{aligned} \tag{11}$$

Then, if the $\mathrm{TCR}^{\mathrm{I}}$ of the problem is smaller than the CRT corresponding to the CR^{I}, as extracted from Table 2 the data are considered consistent, and the solutions obtained from the model are also likely to be consistent and reliable. Otherwise, the pairwise comparison preference data need to be revised. The value of a_{BW} used to determine the acceptable threshold is obtained from the rounded value of $\min(\tilde{a}'_{BW}, \dot{a}_{BW}, \tilde{a}''_{BW})$. Obviously,the maximum value of a_{BW} is equal to 9.

Proof: To ensure the consistency of the data, three influencing factors must be considered: the number of criteria, the value of a_{BW}, and CR^{I}. Since the number of criteria is fixed for the problem, the remaining two factors are considered in their most conservative (pessimistic) values to enhance confidence in the consistency of the data. Therefore, the smaller the value of a_{BW}, the lower the CRT extracted from the corresponding table. On the other hand, the greater the $\mathrm{CR^I}$ value of the problem, the higher the confidence in the consistency of the data. For this reason, the maximum $\mathrm{CR^I}$ value ($\mathrm{TCR^I}$) obtained for the problem is used in the assessment. Similarly, the minimum value of a_{BW} is considered as the reference value for consistency evaluation.

Step 6: In this step, based on the robust optimization model of Soyster (equation (2)) and the linear goal programming BWM model (equation (9)), a robust mathematical programming model for BWM was developed using the goal programming approach, as formulated in equation (12).

$$
\begin{aligned}
&\min Z = \sum_j (y_j^+ + y_j^-) + \sum_j (z_j^+ + z_j^-) \qquad (12)\\
&s.t:\\
&\xi \geq y_j^+ - y_j^-, \forall_j\\
&\xi \geq z_j^+ - z_j^-, \forall_j\\
&w_B - \dot{a}_{Bj} w_j + \hat{a}_{Bj} y_b - (y_j^+ - y_j^-) \leq 0, \forall_{j,b}\\
&\dot{a}_{Bj} w_j - w_B - \hat{a}_{Bj} y_{b'} + (y_j^+ - y_j^-) \leq 0, \forall_{j,b'}\\
&w_j - \dot{a}_{jw} w_{\dot{w}} + \hat{a}_{jw} y_w - (z_j^+ - z_j^-) \leq 0, \forall_{j,w}\\
&\dot{a}_{jw} w_{\dot{w}} - w_j - \hat{a}_{jw} y_{w'} + (z_j^+ - z_j^-) \leq 0, \forall_{j,w'}\\
&-y_b \leq w_j \leq y_b, \forall_{b,j}\\
&-y_{b'} \leq w_j \leq y_{b'}, \forall_{b',j}\\
&-y_w \leq w_w \leq y_w, \forall_{w,j}\\
&-y_{w'} \leq w_w \leq y_{w'}, \forall_{w',j}\\
&0 \leq w_j \leq 1\\
&\sum_j w_j = 1, \forall_j\\
&w_j, y_j^+, y_j^-, z_j^+, z_j^- \geq 0, \forall_j
\end{aligned}
$$

The proposed model can be modeled and solved using optimization software, such as Lingo, for operations research problems, and the optimal values of the criteria weights $(w_1^*, w_2^*, \ldots, w_n^*)$ can be calculated.

Proof of the proposed model: To prove the proposed model, we first describe the steps of converting the constraints and utilizing Soyster's robust optimization model, and then examine how to incorporate uncertainties into the goal programming model. The constraints in the linear goal programming BWM model (model (9)) are defined as equalities. To apply Soyster's robust optimization method, the equalities must be converted into inequalities. Additionally, $(y_j^+ - y_j^-)$ and $(z_j^+ - z_j^-)$ are positive values. Therefore, it can initially be assumed that the equality constraints of the goal programming model can be written as two inequality constraints ($\geq$ and $\leq$), i.e.:

$$
w_B - a_{Bj} w_j = y_j^+ - y_j^- \approx \begin{cases} w_B - a_{Bj} w_j \leq y_j^+ - y_j^-, \forall_j \\ w_B - a_{Bj} w_j \geq y_j^+ - y_j^-, \forall_j \end{cases}
$$

$$
w_B - a_{Bj} w_j = z_j^+ - z_j^- \approx \begin{cases} w_B - a_{Bj} w_j \leq z_j^+ - z_j^-, \forall_j \\ w_B - a_{Bj} w_j \geq z_j^+ - z_j^-, \forall_j \end{cases}
$$

Then, the greater-than-or-equal constraints are converted into less-than-or-equal constraints, and based on Soyster's robust optimization model, the uncertainties are incorporated into the constraints of the goal programming model. Therefore, the constraints of the goal programming model, considering the uncertainties, can be rewritten as follows.

$$
w_B - a_{Bj} w_j = y_j^+ - y_j^- \approx \begin{cases} w_B - \dot{a}_{Bj} w_j + \hat{a}_{Bj} y_b - (y_j^+ - y_j^-) \\ \qquad \leq 0, \forall_{j,b} \\ \dot{a}_{Bj} w_j - w_B - \hat{a}_{Bj} y_{b'} + (y_j^+ - y_j^-) \\ \qquad \leq 0, \forall_{j,b} \end{cases}
$$

$$
w_B - a_{Bj} w_j = z_j^+ - z_j^- \approx \begin{cases} w_j - \dot{a}_{jw} w_{\dot{w}} + \hat{a}_{jw} y_w - (z_j^+ - z_j^-) \\ \qquad \leq 0, \forall_{j,w} \\ \dot{a}_{jw} w_{\dot{w}} - w_j - \hat{a}_{jw} y_{w'} + (z_j^+ - z_j^-) \\ \qquad \leq 0, \forall_{j,w} \end{cases}
$$

On the other hand, considering that ξ is equal to the maximum of $(y_j^+ - y_j^-)$ and $(z_j^+ - z_j^-)$, the corresponding constraints can be added to the model as follows.

$$
\begin{aligned}
\xi \geq y_j^+ - y_j^-, \forall_j\\
\xi \geq z_j^+ - z_j^-, \forall_j
\end{aligned}
$$

Therefore, the robust BWM mathematical model based on the goal programming approach was derived using Soyster's method, as shown in equation (12). As a result, this robust BWM model comprehensively incorporates all uncertainties and preserves the characteristics of the goal programming model. It is worth mentioning that a step can be included to calculate the $\mathrm{CR^O}$, as defined in equation (10), and the results can be compared with the CRT values provided by Liang et al. to ensure the reliability of the obtained solutions. However, since there is a high correlation between $\mathrm{CR^I}$ and $\mathrm{CR^O}$, and input data consistency was conservatively verified in stage 4, this stage is not essential, and thus the computational burden is reduced.

4. Research findings

In this section, the performance of the proposed robust BWM model based on the goal programming approach is evaluated under both deterministic and uncertain conditions regarding parameters or input data (i.e., pairwise comparison values). First, the proposed model is applied to solve three problems under deterministic conditions (i.e., with no errors in input data). Next, the model is employed to solve three problems under uncertain conditions (i.e., with errors present in the input data); and through solving six different problems, the proposed model is analyzed and validated. Finally, the obtained results are compared with this of existing methods.

Throughout various parts of the study, different notations are used to represent the computational results and the weights of the criteria. These notations are specifically used in the computational tables to indicate the criteria weights and the calculated rankings from different methods. To facilitate the understanding of the tables and results, the used notations are defined as follows:

LP: Linear BWM model by Rezaei [10]

GP: Linear goal programming BWM model by Amiri and Emamat [9]

R: Robust BWM model by Sadjadi and Karimi [16]

RGP: Proposed robust BWM model based on the goal programming approach developed in the present study

W_{LP}: Criteria weights based on LP model

W_{GP}: Criteria weights based on GP model

W_R: Criteria weights based on R model

W_{RGP}: Criteria weights based on RGP model

R_{GP}: Criteria rankings based on the computed W_{GP}

R_R: Criteria rankings based on the computed W_R

R_{RGP}: Criteria rankings based on the computed W_{RGP}

4.1 Problem solving under deterministic conditions

In this section, three problems from Rezaei's study [10] are solved under the condition that the problem parameters are deterministic and the input errors are equal to zero ($\hat{a}_{Bj} = \hat{a}_{jw} = 0$). These problems pertain to decision-making in the context of car selection, where the buyer considers five main criteria: Quality (C_1), price (C_2), comfort (C_3), safety (C_4), and style (C_5). Here, since the pairwise comparison values have been predetermined (as shown in Table 3), stage 1 to 4 of the proposed method have been implemented. According to stage 5, the consistency of the input data must be assessed. Since the input errors are zero, to determine the consistency of the input data, it is only necessary to calculate the CR^I for the actual values and compare it with the CRT extracted from Table 2. Based on the results obtained, Problem 1 is fully consistent. Problems 2 and 3 are also consistent, though the consistency level of Problem 2 is

Table 3. Solving examples 1 to 3 from Rezaei [10] under deterministic conditions (zero error).

Problem No.		C_1	C_2	C_3	C_4	C_5	**Consistency Status**
1	a_{Bj}	2	1	4	2	8	$\dot{CR}^I = TCR^I = 0.0000$ $CRT = 0.2958$ Fully consistent
	a_{jw}	4	8	2	4	1	
	Max-W	0.2105	0.4211	0.1053	0.2105	0.0526	
	min-W	0.2105	0.4211	0.1053	0.2105	0.0526	
	W_{LP}	0.2105	0.4211	0.1053	0.2105	0.0526	
	W_{GP}	0.2105	0.4211	0.1053	0.2105	0.0526	
	W_{RGP}	0.2105	0.4211	0.1053	0.2105	0.0526	
	Difference W_{RGP} - W_{GP}	0.0000	0.0000	0.0000	0.0000	0.0000	
2	a_{Bj}	2	1	4	3	8	$\dot{CR}^I = TCR^I = 0.0179$ $CRT = 0.2958$ Consistent
	ajw	4	8	2	3	1	
	Max-W	0.2289	0.4571	0.1176	0.1602	0.0561	
	min-W	0.2145	0.4461	0.1085	0.1563	0.0548	
	W_{LP}	0.2295	0.4481	0.1148	0.1530	0.0546	
	W_{GP}	0.2264	0.4528	0.1132	0.1509	0.0566	
	W_{RGP}	0.2264	0.4528	0.1132	0.1509	0.0566	
	Difference W_{RGP} - W_{GP}	0.0000	0.0000	0.0000	0.0000	0.0000	
3	a_{Bj}	2	1	4	3	8	$\dot{CR}^I = TCR^I = 0.1429$ $CRT = 0.2958$ Consistent but less than Problem 2
	ajw	4	8	4	2	1	
	Max-W	0.2469	0.4932	0.1644	0.1579	0.0548	
	min-W	0.1579	0.4286	0.1429	0.1111	0.0476	
	W_{LP}	0.2462	0.4308	0.1231	0.1538	0.0462	
	W_{GP}	0.2264	0.4528	0.1132	0.1509	0.0566	
	W_{RGP}	0.2264	0.4528	0.1132	0.1509	0.0566	
	Difference W_{RGP} - W_{GP}	0.0000	0.0000	0.0000	0.0000	0.0000	

higher than that of Problem 3. Then, in Stage 6, the problem was formulated using equation (12) and solved accordingly. The weights of the criteria (W_{RGP}) were calculated and are presented in Table 3.
To verify the reliability of the solutions obtained from the proposed model, the maximum and minimum values of the criteria weights (i.e., the weight intervals) were calculated as shown in Table 3. Since the mathematical formulation of the proposed method is based on the linear goal programming model of BWM, and robust optimization methods also require the model to be linear, the criteria weights were also calculated using two linear methods: LP model (W_{LP}) and GP model (W_{GP}). The results were then compared with these two methods, and the inclusion of the calculated weights within the derived intervals was also examined.
Problem 1 is fully consistent and has a unique solution. Therefore, the solutions obtained from all three methods are identical. Moreover, since the solution is unique, the maximum and minimum values of the criteria weights are equal and match the obtained results. These results indicate that the proposed mathematical model performs effectively and acceptably when the problem is fully consistent and input data are deterministic. It is worth noting that when the problem is fully consistent or the number of criteria is less than three, the robust optimization methods proposed in this study and by Sadjadi and Karimi yield identical results.
Problems 2 and 3 are not fully consistent and therefore have multiple solutions. The criteria weight obtained from the proposed method are exactly the same as those from the GP model. Moreover, the weights from the proposed method fall within the acceptable range of criteria weights. The slight differences observed between the weights obtained from the proposed method and those from LP model are due to the multiplicity of solutions. Furthermore, the rankings of the criteria weights in both problems are identical across all three solution methods. These findings confirm the appropriate and reliable performance of the proposed mathematical model under conditions where the problem is consistent, and input data are deterministic.

4.2 Problem solving under uncertainty

In this section, the solution of three problems from the study by Sadjadi and Karimi ([16]), which are presented under uncertainty with specified errors in the input parameters, is addressed. First, the first two problems from that study, which are adapted from first and second in the study by Rezaei [10] under uncertainty with specified errors, are examined. Considering that the pairwise comparison values are provided (as shown in Table 4), stage 1 to 4 of the proposed method are fully implemented. According to stage 5, the consistency of the input data was evaluated using the proposed method in this study. The results indicated that the input data for both problems met a reliable level of consistency. Next, following stage 6, the problem was modelled according to equation (12) and solved. Finally, the weights of the criteria were calculated using the proposed model (W_{RGP}) and presented in Table 4.
To validate the results of the proposed model, a comparison was made between the solutions obtained from this model and the results derived from the robust BWM method by

Table 4. Example 1 and 2 of Sadjadi and Karimi [16] under Uncertainty Conditions.

Problem No.		C_1	C_2	C_3	C_4	C_5	**Consistency Status**
4	a_{Bj}	2±0.25	1	4±0.5	3±0.3	8±0.65	
	a_{jw}	4±0.25	8±0.45	2±0.15	3±0.25	1	$\dot{CR}^{I} = 0.0179$
	Max-W	0.2458	0.4761	0.1252	0.1721	0.0596	$CR^{I'} = 0.0329$
	min-W	0.2031	0.4233	0.1000	0.1463	0.0519	$CR^{I''} = 0.0487$
	W_R	0.2251	0.4503	0.1126	0.1535	0.0586	$TCR^{I} = 0.0487$
	W_{RGP}	0.2399	0.4198	0.1199	0.1555	0.0648	$CRT = 0.2958$
	Difference W_{RGP} - W_R	0.0	0.0	0.0	0.0	0.0	Consistent
	R_R	2	1	4	3	5	
	R_{RGP}	2	1	4	3	5	
5	a_{Bj}	2±0.1	1	4±0.08	2±0.06	8±0.04	
	a_{jw}	4±0.08	8±0.4	2±0.02	4±0.08	1	$\dot{CR}^{I} = 0.0000$
	Max-W	0.2177	0.4295	0.1076	0.2159	0.0538	$CR^{I'} = 0.0153$
	min-W	0.2036	0.4124	0.1030	0.2053	0.0515	$CR^{I''} = 0.0101$
	W_R	0.2090	0.4180	0.1076	0.2131	0.0523	$TCR^{I} = 0.0153$
	W_{RGP}	0.2165	0.4113	0.1049	0.2120	0.0552	$CRT = 0.2958$
	Difference W_{RGP} - W_R	0.0	0.0	0.0	0.0	0.0	Consistent
	R_R	3	1	4	2	5	
	R_{RGP}	2	1	4	3	5	

Sadjadi and Karimi (W_R). Additionally, the rankings of the criteria were compared between the two methods. It is worth noting that neither of the two problems examined his fully consistent; therefore, both problems have multiple optimal solutions. Due to the multiple solutions for the problem, slight differences were observed in the final weights; however, the ranking of the criteria in the proposed method of this study and the method presented by Sadjadi and Karimi are almost identical.

To ensure the validity and robustness of the results obtained from the proposed model, the maximum and minimum values of the criteria weights (weight intervals), as shown in Table 4, were approximately calculated based on the concepts presented in studies [10] and [8] These were calculated according to the equations in model (13). Using these equations, the maximum and minimum weights for each criterion were derived; thereby determining the confidence interval for each criterion's weight. These intervals can serve as a basis for more precise sensitivity analysis to better assess potential variations due to uncertainty in the input data.

$$\begin{aligned}
&\max(\min)\ w_j \qquad (13)\\
&s.t:\\
&w_B \leq \tilde{a}''_{Bj} w_j, \forall_j\\
&w_B \geq \tilde{a}'_{Bj} w_j, \forall_j\\
&w_j \leq \tilde{a}''_{jw} w_w, \forall_j\\
&w_j \geq \tilde{a}''_{jw} w_j, \forall_j\\
&\sum_j w_j = 1\\
&w_j \geq 0, \forall_j
\end{aligned}$$

In conclusion, it is worth mentioning that Problem 5 was analyzed as fully consistent under deterministic conditions, but under uncertain conditions, due to the presence of errors in the comparisons, it has turned into a consistent problem with multiple optimal solutions.

Subsequently, based on the algorithm and method presented in this research, the fourth example from the paper by Sadjadi and Karimi [16], which has 8 criteria, was addressed as the sixth problem of this research. It was solved under uncertainty conditions, taking into account specific errors (as shown in Table 5). This problem is also defined in the context of car selection. In this case, the decision maker considers eight criteria: quality (C_1), price (C_2), comfort (C_3), safety (C_4), style (C_5), speed (C_6), fuel consumption (C_7), and after-sales service (C_8). Then, the calculated ranks and weights of the criteria were compared with the results from that paper. The results obtained demonstrate the suitable and reliable performance of the mathematical model proposed in this research under conditions where the problem is consistent and the data are uncertain (have uncertainty). Therefore, the proposed model is of suitable and reliable validity.

4.3 Discussion

Based on the results obtained from the comparison of the proposed method with other decision-making models in sections 4.1 and 4.2, a preliminary validation was conducted using the weights and final rankings of the criteria. This validation demonstrated that the proposed model performs effectively and acceptably under various data conditions, including both certainty and uncertainty. Subsequently, to conduct a more in-depth and multidimensional evaluation of the model's reliability, the Sum of Squared Errors (SSE) index, sensitivity analysis with respect to changes in input parameters, and the interpretation of managerial implications resulting from the use of the proposed method were examined. This section not only reaffirms the accuracy and efficiency of the proposed model, but also provides practical insights for decision-makers operating in real-world uncertain environments.

As previously stated, in this study, six numerical examples involving both deterministic and interval data were analyzed to validate the innovation of the proposed method. In the first three examples, the pairwise comparisons were entered into the model in a deterministic form. Whereas in the other three examples, the data were designed in an uncertain form using intervals. To evaluate the performance of the proposed method, the SSE was employed. However, to avoid oversimplification and ignoring the uncertainty in interval data, the SSE_{final} formula was defined in a combined form according to the mathematical relations in equation (14).

$$\mathrm{SSE} = \sum_i \sum_j \left(a_{ij} - \frac{w_i}{w_j} \right)^2 \quad ,(i \neq j) \qquad (14)$$
$$\mathrm{SSE}_{\text{final}} = \alpha \times \mathrm{SSE}_\mu + (1-\alpha) \times \mathrm{SSE}_\delta$$

In this formula, SSE_μ is calculated based on the mean of the interval pairwise comparisons ($\dot{a}_{ij}$) and SSE_δ is computed based on the deviation from the midpoint of the intervals, which represents the error due to uncertainty in the decision-

Table 5. Example 4 of Sajadi and Karimi [16] under uncertainty conditions (Problem No. 6).

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	Consistency Status
a_{Bj}	3 ± 0.11	4 ± 0.40	6 ± 0.02	1	5 ± 1	2 ± 0.005	9 ± 0.33	7 ± 0.91	$C\dot{R}^I = 0.2222$
a_{jw}	7 ± 0.95	6 ± 0.96	4 ± 0.15	9 ± 0.97	5 ± 0.09	8 ± 0.48	1	3 ± 0.16	$CR^{I'} = 0.1460$
W_R	0.137	0.097	0.071	0.328	0.071	0.213	0.028	0.054	$CR^{I''} = 0.3003$
W_{RGP}	0.106	0.085	0.051	0.306	0.077	0.287	0.038	0.050	$TCR^I = 0.3003$
R_R	3	4	6	1	5	2	8	7	$CRT = 0.3620$
R_{RGP}	3	4	6	1	5	2	8	7	Consistent

maker's preferences ($\hat{a}_{ij}$). The coefficient α specifies the relative importance weight of each component of the SSE; such that it allows adjusting the contribution of central data and uncertainty in the model. In this study, α is set to 0.5 to assign equal weight to both sources of error. However, this parameter can be adjusted depending on the nature of the problem or the decision-maker's preferences. Employing this combined formula ensures that both the numerical accuracy of judgments and the degree of confidence or instability are simultaneously taken into account. This approach provides a suitable method for fair and comprehensive comparison between deterministic and interval models, enabling more precise analysis of the effects of uncertainty in the decision-making process of MADM problems.

The results obtained from applying this approach to the six numerical examples analyzed in this study revealed that, in all cases-particularly those involving interval data-the proposed model yielded a lower SSE_{final} compared to other methods. The notable reduction in SSE following the integration of both error components (interval mean and deviation from the midpoint) reflects the model's capacity to preserve numerical accuracy and stability under uncertainty. Given that real-world decision problems often involve human judgments with inherent ambiguity, this advantage of the proposed model is also operationally significant.

To better illustrate the differences in model performance under deterministic data conditions, Table 6 presents all computed SSE values for the three problems solved using deterministic data. The results in this table indicate that the proposed model yields the lowest error compared to the benchmark models, demonstrating its superior performance under deterministic data settings. Moreover, in the case of a fully consistent problem (Problem 1), the SSE_{final} is zero across all three models. This result confirms that under fully consistent conditions, the proposed model produces a unique solution that perfectly aligns with the input data. Therefore, a zero SSE value may be regarded as an indicator of solution uniqueness and numerical precision under ideal conditions.

Table 6. SSE values for the three Problems solved with deterministic data.

Problem No.	LP	GP	RGP
1	0.00	0.00	0.00
2	0.19	0.11	0.11
3	9.24	4.44	4.44

Figure 1 below presents a visual comparison of the SSE_{final} values between the proposed model and a benchmark model under uncertain, interval-based data conditions. The relative superiority of the proposed model-particularly in examples involving interval data-is clearly illustrated in this chart. These results indicate that the proposed approach demonstrates high flexibility, generalizability, and robustness when dealing with uncertainty, which distinguishes it from other comparable existing methods.

Given that Problems 2 and 3 were obtained by modifying the pairwise comparison values a_{24} and a_{45} relative to Problem

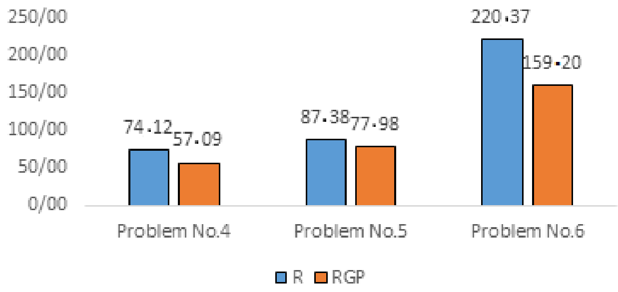

Figure 1. Comparison of SSE_{final} values between the proposed and robust model [16] in three interval-based problems.

1, it can be assumed that two input parameters have been altered. Therefore, comparing the results from Problems 1 to 3 enables sensitivity analysis and evaluation of the models' stability under deterministic input data conditions. Thus, in the continuation of sensitivity analysis, to assess the robustness of the proposed model against possible fluctuations in input data, the weighted average of the percentage changes in criteria weights after applying modifications in the pairwise comparisons have been numerically calculated. In this analysis, the initial weight of each criterion (from Problem 1) is considered as the importance coefficient in calculating the weighted average of changes. This approach ensures that variations in more important criteria have a greater impact on the final sensitivity assessment of the model. The formula used is defined according to mathematical relation in equation (15):

$$\text{Weighted Average of Changes} = \frac{\sum_{i=1}^{n} w_i \times \Delta_i}{\sum_{i=1}^{n} w_i} \tag{15}$$

where w_i is the weight of criterion i in the original problem (before the changes), and Δ_i is the percentage change in the weight of the same criterion i after the input modifications. Under deterministic data conditions, the results of this analysis and comparison of the proposed model with two GP models and LP model, according to Table 7 and figure 2 (using data from Table 3), indicate that the proposed model has the lowest average percentage change in weights. The obtained value for the proposed model is 2.38%; while this value is calculated as 2.38% and 2.89% for the GP models and LP model, respectively. These differences indicate that the proposed model exhibits greater stability against input variations and, consequently, higher tolerance to fluctuations in pairwise comparisons.

Also, Problems 4 and 5 were generated by modifying the pairwise comparison values a_{24} and a_{45} relative to each other, as well as altering the error levels of each pairwise comparison. Therefore, it can be assumed that the two main input parameters-pairwise comparison values and the uncertainty or errors associated with each comparison-have been modified. This provides a suitable context for evaluating the stability of the models under interval data conditions. Accordingly, the comparison of the proposed model's results with those of the robust BWM model by Sajadi and Karimi, as presented in Table 8, showed that the proposed model has the lowest weighted average sum of percentage changes. Furthermore, figure 2 provides a clear visual representation

Table 7. Sensitivity analysis of models under deterministic data (Problems 1 – 3).

Criteria	Model	Weights (Problem 1)	Weights after Change (Problem 2)	% Change Weights (1 – 2)	Weights after Change (Problem 3)	% Change Weights (1 – 3)	Maximum Weight Change
C_1	LP	0.2105	0.2295	1.90%	0.2462	3.56%	3.56%
	GP	0.2105	0.2264	1.59%	0.2264	1.59%	1.59%
	RGP	0.2105	0.2264	1.59%	0.2264	1.59%	1.59%
C_2	LP	0.4211	0.4481	2.70%	0.4308	0.97%	2.70%
	GP	0.4211	0.4528	3.18%	0.4528	3.18%	3.18%
	RGP	0.4211	0.4528	3.18%	0.4528	3.18%	3.18%
C_3	LP	0.1053	0.1148	0.95%	0.1231	1.78%	1.78%
	GP	0.1053	0.1132	0.79%	0.1132	0.79%	0.79%
	RGP	0.1053	0.1132	0.79%	0.1132	0.79%	0.79%
C_4	LP	0.2105	0.1530	5.75%	0.1538	5.67%	5.75%
	GP	0.2105	0.1509	5.96%	0.1509	5.96%	5.96%
	RGP	0.2105	0.1509	5.96%	0.1509	5.96%	5.96%
C_5	LP	0.0526	0.0546	0.20%	0.0462	0.65%	0.65%
	GP	0.0526	0.0566	0.40%	0.0566	0.40%	0.40%
	RGP	0.0526	0.0566	0.40%	0.0566	0.40%	0.40%
					Weighted Average of Changes -RGP		2.38%
					Weighted Average of Changes -GP		2.38%
					Weighted Average of Changes -LP		2.89%

Note: Changes correspond to variations in pairwise comparisons a_{24} and a_{45}.

of the percentage changes in criteria weights across the two models, clearly illustrating the relative superiority of the proposed model in controlling fluctuations.

These findings confirm that the proposed model demonstrates higher stability and robustness not only under deterministic conditions but also in the presence of data uncertainty compared to similar models. In other words, smaller fluctuations in the criteria weights lead to more stable and reliable decision-making outcomes. The comparison of percentage changes in criteria weights across different models (as presented in Tables 7 and 8 and illustrated in figure 2) indicates that the proposed model generally exhibits smaller variations than the other methods.

On the other hand, analyzing the upper and lower bounds of the criteria weights in Problems 1 through 3 provides important insights into the stability of each criterion. The results indicate that in Problem 3 (with the lowest level of consistency), the weight of criterion C_5 (style) shows the narrowest range of variation, while the weights of C_1 (quality) and, subsequently, C_3 (comfort) exhibit the largest ranges. This suggests that these criteria are more sensitive to changes in input parameters. In Problem 2, criterion C_5 again exhibits the smallest range of variation, whereas the weights of C_4 (safety) and then C_1 show the greatest fluctuations. In contrast, Problem 1, which is fully consistent, yields unique solutions, and naturally, no variation in criteria weights is observed. These findings reveal that as the level of input consistency decreases, the sensitivity of criteria weights increases, and the model's stability against variations declines. In other words, under conditions of lower consistency in pairwise comparisons, the weights of certain criteria are highly affected by minor variations in

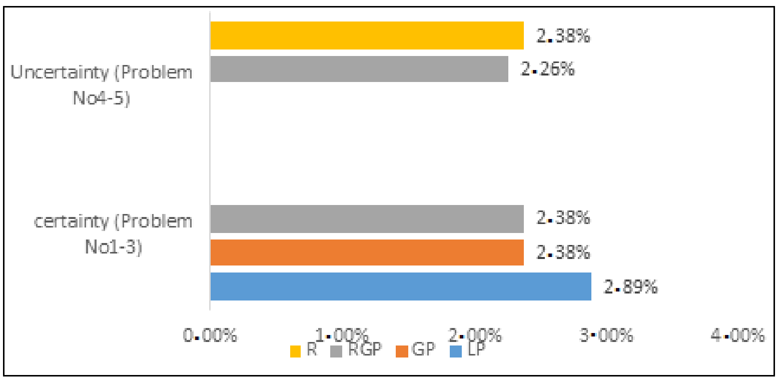

Figure 2. Sensitivity analysis: weighted % change in criteria weights (deterministic vs. interval).

Table 8. Sensitivity analysis under interval data (Problems 4 – 5).

Criteria	Model	Weights (Problem 4)	Weights after Change (Problem 5)	% Change Weights (4 – 5)
C_1	LP	0.2251	0.2090	1.61%
	RGP	0.2399	0.2165	2.34%
C_2	LP	0.4503	0.4180	3.22%
	RGP	0.4198	0.4113	0.85%
C_3	LP	0.1126	0.1076	0.50%
	RGP	0.1199	0.1049	1.50%
C_4	LP	0.1535	0.2131	5.96%
	RGP	0.1555	0.2120	5.65%
C_5	LP	0.0586	0.0523	0.63%
	RGP	0.0648	0.0552	0.96%
			Weighted Average of Changes -RGP	2.26%
			Weighted Average of Changes -R	2.28%

Note: Changes correspond to variations in pairwise comparisons a_{24} and a_{45}.

input parameters. Figure 3 graphically illustrates the ranges of criteria weight variations for Problems 2 and 3, offering a clear depiction of the weight fluctuations under such conditions.

In the continuation of the sensitivity analysis under uncertainty conditions, Problems 4 and 5 were used as the basis for evaluation. Problem 4 is derived from a low-consistency problem, formed by introducing uncertainty into the pairwise comparisons (adding errors); while Problem 5 originates from a fully consistent problem with added specified levels of uncertainty. The results of this analysis indicate that in Problem 4, criterion C_5 (style), considered the worst criterion, exhibited the lowest sensitivity or weight changes against input variations. In contrast, in Problem 5, criterion C_3 (comfort) showed the least sensitivity or weight changes in response to input data variations, thus demonstrating the most stable behavior against fluctuations in input data. Furthermore, in both problems, criterion C_2 (price) experienced the highest sensitivity and weight fluctuations, indicating a high sensitivity to changes in input data. Therefore, a trend similar to deterministic data conditions was observed; namely, the higher the initial consistency of the problem (as in Problem 5), the lower the sensitivity of the criteria weights to input data variations. This finding suggests that the initial level of consistency can be considered a key factor in the stability of models under uncertainty conditions. The ranges of criteria weight variations in the two interval problems are comparatively illustrated in figure 4, which clearly demonstrates the relative stability of the proposed model under uncertainty conditions.

From a managerial perspective, the proposed model has significant implications for improving the decision-making process under uncertainty conditions. The model helps decision-makers enhance the reliability of decision-making outcomes in real-world uncertain conditions, and enables organizations to ensure the consistency and validity of expert judgments before making critical decisions such as project selection, supplier evaluation, and similar cases. This leads

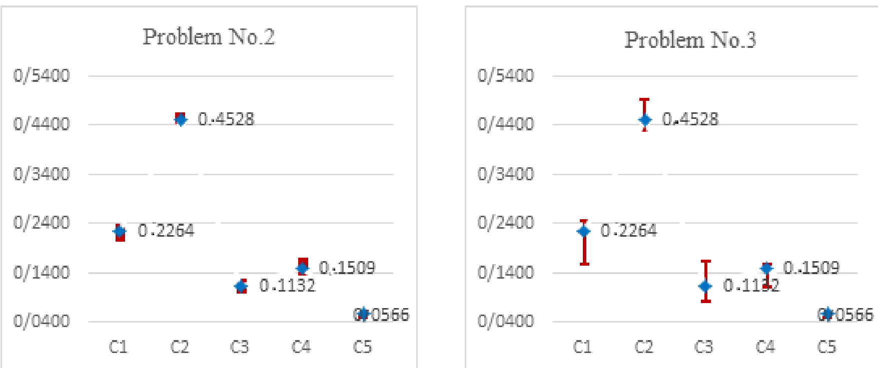

Figure 3. Ranges of criteria weight variations in deterministic data Problems (Problems 2 and 3)

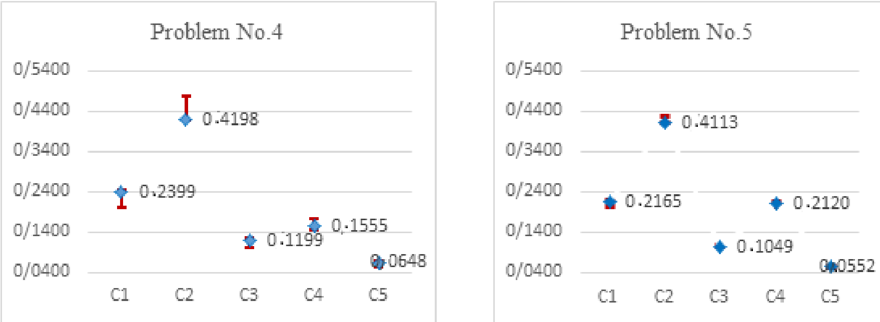

Figure 4. Ranges of criteria weight variations in uncertain Problems (Problems 4 and 5).

to stronger, more documented, and defensible decisions in complex managerial environments. Moreover, instead of requiring exact pairwise comparison values from decision-makers and experts, the proposed model allows the use of preference intervals; a feature that reduces complexity and facilitates decision-makers' tasks in the decision-making process. Furthermore, employing a data consistency checking mechanism prior to modeling saves computational time and reduces the likelihood of human errors, ultimately enhancing the efficiency and quality of the decision-making process. Overall, the proposed model can serve as a reliable and practical tool for decision-making in MADM problems within the real and ambiguous organizational contexts; a model that, while improving the quality of choices, enables more documented, transparent, and stable decisions for managers and stakeholders, ultimately leading to enhanced decision effectiveness and risk reduction.

5. Conclusion

In this research, to improve the accuracy, reliability, and efficiency of the BWM method in real-world conditions where decision-making data often involve uncertainty, a novel robust optimization model based on linear goal programming for BWM was developed. In the proposed model, interval preferences were used to represent uncertainty in pairwise comparisons. Additionally, a new method was introduced to calculate the CR of input data based on CR^I under uncertainty, in order to ensure the validity and consistency of the data before entering the modelling process.

To evaluate the performance of the proposed model, six numerical problems were solved and analysed under both certain and uncertain pairwise comparison data. The results demonstrated that the model is capable of generating unique, valid weights with acceptable levels of consistency. In contrast, under similar conditions, the original BWM and existing robust models occasionally yield multiple optimal solutions. This highlights the superior performance and higher accuracy of the proposed model. Furthermore, the proposed consistency-checking mechanism applied prior to the modelling phase led to savings in computational time and reduced the likelihood of errors. Furthermore, the calculation and analysis of SSE, along with the sensitivity assessment of criteria weights to input data fluctuations, demonstrated that the proposed model exhibits greater stability compared to other methods.

Key advantages of the proposed approach include the ability to assess the quality of decision-making data before modelling, the generation of unique solutions, and enhanced reliability of results under uncertainty. On the other hand, the disadvantages include increased computational burden of the proposed model in large-scale problems and the dependence of the model performance on the accuracy of determining the preference intervals.

Despite its strengths, this study has some limitations. First, the model is designed for individual decision-making problems and has not been extended to group decision-making contexts. Second, it only considers a specific type of uncertainty (interval preferences), while other forms-such as fuzzy or probabilistic uncertainty-have not yet been explored. Third, expert judgments still play a crucial role in the proposed model; therefore, the validity and accuracy of the results largely depend on the quality and reliability of expert opinions.

For future research, it is recommended to extend the proposed model to group decision-making environments or integrate it with other robust optimization techniques. Furthermore, applying this algorithm to real-world MADM problems such as project prioritization, performance evaluation, supplier selection, productivity improvement could expand its practical applicability across various managerial domains.

Authors contributions
Authors have contributed equally in preparing and writing the manuscript.

Availability of data and materials
The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflict of interests
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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