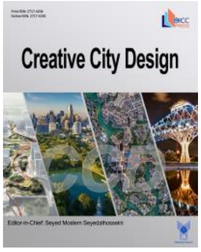

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Research Article

Deciphering the Hidden Geometry of the Hasht Behesht Palace Plan (Isfahan): An Analytical Study Based on the Square-Growth System

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Abstract

Aims: Architectural design in every era relies on precisely planned geometry to achieve compositional unity. In Iran, the Safavid dynasty emphasized geometric principles and proportional systems, embedding them into architectural layouts. Hasht Behesht Palace in Isfahan, the only remaining Safavid palace, features a unique plan that diverges from its contemporaries. Its design logic remains unexplored. To date, no specific research has been conducted to decode the concealed geometric system underlying the palace's planimetric. This study is essential to uncover the architectural principles governing the palace's planimetric organization and to investigate the relationship between theoretical geometry and Safavid-era architecture. This study explores the geometric framework of the palace to demonstrate how its Planimetric order is structured through a system of geometric proportions, the square-growth system. After introducing the organizing principles, it conducts a planimetric analysis of the palace to reveal the underlying logic of its geometric composition.

Methodology: The research adopts a historical-analytical approach, integrating literature review, historical maps, field observations, and planimetric analysis using AutoCAD. The study examines the spatial logic of the palace plan through the 1:√2 proportional system.

Finding: The findings demonstrate that the palace plan is organized using a square-growth system based on concentric squares and the recurrent 1:√2 ratio. These findings confirm that the palace's planimetric divisions, including the central hall and porticos, resulted from conscious design and geometric reasoning, establishing geometry as a fundamental organizing principle in Safavid architecture.

Conclusion: This insight deepens understanding of how mathematical precision informed the architectural structure and Planimetric order of the Safavid era.

Keywords: Geometric System; Hidden geometry; Proportions; Square Growth; Planimetric structure; Hasht Behesht Palace (Isfahan)

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1. Introduction

The recorded evolution of geometry dates back millennia before the Common Era. As a subset of mathematical science [1-6], geometry has fundamentally shaped architectural creation since antiquity. Within Islamic civilization, geometry was more than mathematics; it was a design philosophy that bridged the gap between abstract order and material form. Conversely, the precise role of mathematicians in Islamic architectural development remains a contentious nexus within interdisciplinary studies [7]. Contemporary scholarship on the interconnection between mathematical history and Islamic architecture reflects two divergent perspectives: either affirming it [8, 9] or disputing it [10-15]. Architects' strategic application of geometry in design, ornamentation, and construction has yielded enduring monuments. Consequently, analyzing artistic artifacts from specific historical periods reveals critical insights into civilizations' trajectories [16-18].

This study examines Hasht Behesht Palace (Isfahan, Safavid era, c. 1669 CE), [19] characterized by four symmetrical façades and classified as a central pavilion type structure within its garden complex [20]. While earlier palaces shared similar geometric configurations, Hasht Behesht remains the sole extant exemplar of this architectural typology [21-23] and indeed, its most complete manifestation [23-25]. This unique status positions it as an ideal subject for analyzing geometric organization rather than mere descriptive interpretation.

To bridge this gap, the study employs a historical-analytical methodology, drawing upon diverse sources, including literature reviews, historical maps, and field surveys. The analytical approach centers on reverse-engineering the architectural plan through geometric analysis in AutoCAD, with a view to uncovering the use of the 1:√2 proportional system and its manifestation through square-based growth patterns. Despite extensive scholarship on Safavid architecture, a critical lacuna persists: no systematic study deciphers the geometric systems.[1] Underlying Safavid palaces' Planimetric Structure. Moreover, while Hasht Behesht awaits UNESCO World Heritage nomination [26][2], this research advances its documentation, particularly its geometric schema, thereby strengthening its case for cultural preservation and global recognition. The present work thus innovates by reconstructing the applied geometric order in Persian palace architecture.

The main research question is in what ways does the design of the Hasht Behesht Palace incorporate the square-growth geometric system?

This question directly aligns with the central objective of the study: to investigate the palace's Planimetric structure by identifying the codified systems of proportion and geometry, including the square-growth system, that inform its planimetric logic and formal organization. This study aims to decode the geometric system underlying the plan of the Hasht Behesht Palace and to reveal its organization based on geometric proportions and the square-growth system.

2. Literature review

Geometry, conceptually defined as the science of quantities and measurements, is a branch of mathematics [27, 28]. Through the collaboration of mathematicians and architects especially scholars of ilm al hiyal (mechanical sciences) a strong connection was established between theoretical and practical sciences. This relationship led to a clearer definition of the nature of applied geometry in architecture [29, 30]. As a result, the comprehensive use of proportional[3] systems in ancient Iranian architecture observable in plan designs, elevations, geometric motifs, and mechanical and structural characteristics can be proven through geometric analyses of historical Iranian buildings [31]. Therefore, research on geometry in ancient Iranian architecture is impossible without familiarity with geometry itself, as geometry is one of the principal elements shaping Iranian architectural formation [32].

Aligned with the variables of this study, initial attention is given to studies addressing the relationship between geometry, mathematics, and architecture and their historical interactions [2, 32-37]. In this field, two groups of research emerge: one group denies any relationship between theoretical sciences and the practical application of mathematical knowledge in crafts and industries [8, 9]. The other emphasizes the existence of such a relationship between mathematical and geometric theory and the structure of Islamic cities and buildings [10, 11, 13-15, 38-46]. Subsequently, research focusing on the application of systematic geometric frameworks has been reviewed. Among these, several studies by Rempel, Pugachenkova, Mankovskaia, Nemtseva, Baklanov, and most comprehensively Bulatov, demonstrate the use of precise proportional rules such as the square root of two ratio or isosceles triangles inscribed within circles, which have been applied in Islamic architectural design since its early periods [47]. Some studies have briefly addressed geometric analyses based on square-based systems [28, 48-50]. A limited number of investigations have examined the square-based frameworks in notable architectural monuments from other historical periods [15, 22, 51].

[1] As noted, given that Hasht Behesht Palace in Isfahan lacks UNESCO World Heritage inscription (as documented within the boundaries delineated on UNESCO's World Heritage properties maps), this research holds significance through its systematic documentation particularly regarding the palace's planimetric schema.

[2] This status is documented within the boundaries delineated on UNESCO's World Heritage properties maps.

[3] Proportion refers to the harmonious relationship between components and their integration with the architectural composition as a whole [70].

In addition, several studies have analyzed mosques and squares of the Safavid period using this system [28, 52]. In this regard, many studies have defined proportional systems in architecture, particularly the golden ratio [22, 31, 53-58]. Considering ancient Iranian architecture and its valuable monuments, analytical studies have focused on golden proportions, divine ratios, and modular systems (peymon) present in works from different historical periods [59-61].

Other studies have geometrically analyzed proportions and golden ratios in various case studies [12, 28, 51, 52, 54, 62-68], including some studies emphasizing regular pentagonal geometries [69-71]. Notably, a study has been conducted that specifically examines the proportional systems employed in the Hasht Behesht Palace [20].

Given the existing body of research, it becomes evident that the subject under study has not yet been the focus of a detailed geometric analysis aimed at interpreting the governing planimetric order, specifically elaborating the principles of the square-growth system. Therefore, this research can be considered the first systematic attempt to investigate this framework.

3. Research Methodology

Previous research has emphasized the importance of understanding the geometric principles underlying historical plan layouts, particularly those of the Safavid period. Geometric patterns were predominantly based on the square and its derivatives, with fundamental shapes, especially squares and circles, serving as the primary elements in the design process. Theorists such as Durand, Krier, and Hrdlicka focused on recognizing these basic geometric forms and their combinations, rotation, symmetry, and repetition, to identify earlier architectural patterns [52, 66, 72-79]. Accordingly, this study adopts the square-growth system as its analytical framework to examine the plan of Hasht Behesht Palace. Building on this theoretical foundation, methodological steps were designed to conduct a systematic analysis of the palace's geometric structure.

Data collection employed library research[4] and field surveys[5], with the study population comprising Safavid era historical palaces and Hasht Behesht Palace in Isfahan selected as the complete purposive sample[6], being the sole surviving Safavid palace along the Chahar Bagh axis with this specific typological configuration [23]; this palace represents one of the most renowned Safavid monuments and has received the most extensive documentation among Isfahan's palaces [23, 24, 82-84].

The case study has undergone successive deterioration and restoration phases over centuries [19, 83, 85, 87, 88]; consequently, alongside available archival and visual resources, authoritative travel accounts [19, 20, 83, 85, 88, 89- 98] were incorporated to enable deductive validation of spatial descriptions and facilitate comprehensive understanding of longitudinal transformations. Additionally, field investigations[7] including observational mapping[8], photography, and metric surveying were conducted to verify documentation consistency and address methodological limitations regarding measurement discrepancies in published planimetric records.

The analytical basis for this study is a new, researcher-generated digital plan created from a 2024 field survey using laser distance meters and measuring tapes. This plan was rigorously validated against archival drawings from the Isfahan Cultural Heritage Organization and previous studies [20, 23, 85, 99], confirming a high degree of dimensional accuracy with discrepancies of less than 2%. Thus, this verified AutoCAD plan served as the definitive foundation for the geometric analysis in this research.

This research employs a quantitative approach[9] coupled with inductive reasoning[10] for geometric analysis[11].

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[4] For library research, Central Library of Isfahan and Sheikh Bahai Specialized Art Library were consulted, utilizing historical/contemporary books, journals, doctoral dissertations, and master's theses relevant to this study.

[5] Field methodology employs the researcher's observational intuition and expertise to comprehensively gather data [80].

[6] Purposive sampling selection is based on researcher judgment and specialization [81], ensuring the sample optimally represents the study population [86].

[7] Fieldwork methodology involved accessing Hasht Behesht Palace with official permits, during which extensive photogrammetric documentation was conducted; however, selective photographic restrictions applied to certain sections. Comprehensive understanding of the palace complex was consequently achieved through triangulation of: metric surveys; archival and contemporary imagery; textual records; and spatial documentation from the Isfahan Cultural Heritage Organization archives.

[8] Observation entails "identification, classification, comparison, description, and systematic documentation of phenomena" [81].

[9] Quantitative research employs predefined measurement protocols where concepts, variables, and hypotheses remain constant. Hypothesis validation concludes the study, utilizing objective mathematical analysis of empirical data to measure variables [80].

[10] Inductive reasoning verifies theories through observation [80], identifying patterns from specific phenomena to general principles [100]. This process examines causal relationships through phenomenological immersion, progressing from particular instances to universal conclusions [81].

[11] Geometric analysis employs spatial decomposition into elementary geometric forms, while dimensional ratio analysis systematically examines planimetric measurements, angular relationships, and numerical proportions between spatial boundaries [66, 101].

The methodological workflow proceeded as follows: First, the plan was digitized in AutoCAD software.Subsequently, through typological comparison with analogous structures, the central core and primary axes were established as geometric benchmarks. This facilitated analysis of proportional systems manifested through iterative shape replication, diagonal proportional relationships, and rotational transformations of base geometries within the palace's planimetric configuration, decoded via the regulatory geometric framework of square growth progression and its secondary ratios. Consequently, the square circle dyad was identified as the generative geometric pattern governing the overall planimetric logic.
The Ikhwan al-Safa are recognized among the scholars who identified this geometric system as a foundational principle for architectural design. However, the precise origins of this system remain undocumented in authoritative sources. Therefore, its drafting method was identified through a review of diverse sources and applied to analyze the plan of Hasht Behesht Palace. A detailed explanation of this procedure is presented in the subsequent sections.
This study identifies the square-growth geometric system (and its 1:√2 & √2 ratios) as the independent variable governing the palace's planimetric organization the dependent variable. The scope was deliberately limited to the overall plan structure, excluding decorative details, materials, and later additions to enhance analytical precision and maintain focus on plan geometry. Disruptive variables (historical changes, measurement errors) were also controlled by integrating field data with validated archival maps from the Cultural Heritage Organization of Isfahan and verifying measurement accuracy (error less than 2%). To further ensure that the plan used in the analysis closely corresponds to the original palace plan, the plan employed was cross-checked against descriptions provided by travelers in historical travelogues, confirming its similarity [83, 88]. The reverse engineering-based analysis method directly addresses the hidden geometric concept within the plan's structure (conceptual validity), as it is grounded in theoretical geometric principles. Researcher impartiality was maintained throughout data collection and analysis. Disruptive variables were identified and controlled to ensure internal validity. The validity of the findings was confirmed through consistency between the derived ratios and prior geometric studies [15, 51, 63, 65-68]. The reliability of the analytical process is supported by its repeatability and its grounding in established research on Iranian-Islamic geometry [20, 22, 28, 49, 50, 52, 103]. Consequently, this framework is generalizable to other historic plans with similar geometries.

4. Theoretical framework

4.1. Geometry and architecture

Geometry etymologically denoting land measurement and surveying [104] evolved into a universal paradigm that shapes built forms through proportional systems defined by calibrated lines [105].
Persian architecture employed proportional geometry, wherein dimensions derived not from absolute measurements but from interrelated coefficients obtained through geometric subdivisions [106]. Safavid architects notably prioritized such proportional systems, embedding geometric principles into their designs. These systems, grounded in specific geometric progressions, offered flexibility while unifying compositional integrity [107].
Research on Persian architectural geometry remains inseparable from geometric knowledge itself, as geometry constitutes a primary generative element in Iranian architecture [32].
Architectural design materializes through geometric frameworks [108], reinstating terrestrial order via mathematical regulation [109].

4.2. Hidden geometric order

Islamic architecture is fundamentally governed by a hidden geometric order that underlies both its physical structure and spatial logic. This order results from the systematic application of basic geometric shapes[12], an emphasis on centrality, attention to axes and orientations, and the governing principles of symmetry and repetition [111].
In most Iranian Islamic monuments, the architectural plan is generated through the repetition of a single modular unit, enabling the integration of all spatial components within a coherent framework [112].
According to Bulatov, traditional architects employed a generative unit as the basis of the geometric system, from which all significant dimensions of the building were derived.
For instance, in structures with a central hall, the side length of this space served as the generative unit [22].
Thus, the underlying geometry of this architecture is primarily based on fundamental shapes and their combinations. This reveals a hidden geometric logic that unifies the parts into an ordered whole [111], commonly referred to as “hidden geometry.”

[12] The foundation of geometric systems in Iranian arts is primarily formed by the square and the circle [110].

4.3. Regulatory geometric frameworks

Historical geometric practices are partially documented. Among the oldest extant sources are gridded plans from Islamic Central Asia, indicating continuity of proportional systems in Islamic architecture through the Safavid era [47, 83, 113]. The use of gridded underlays likely originated during the Timurid Turkmen period, adapting regionally, though its precise genesis remains unclear. Surviving documents (e.g., Topkapı scrolls, Tashkent manuscripts) reveal architecture reliant on gridded plans and geometric patterns [12].

Regulatory geometric frameworks constitute design systems based on precise subdivisions and mathematical ratios, extensively applied in architecture and ornamentation. Canonical proportions and principles by mathematicians like al Buzjani determined angles, dimensions, and spatial relationships [12, 114] (Table 1).

Table 1. General Classification of Proportional Systems in Architecture

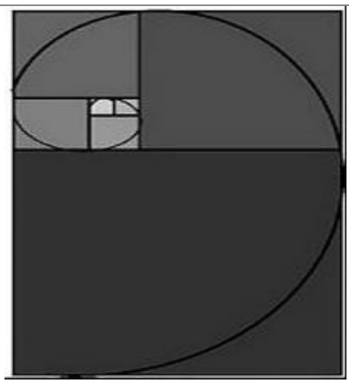
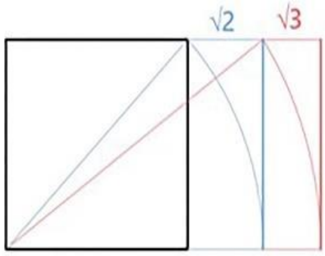
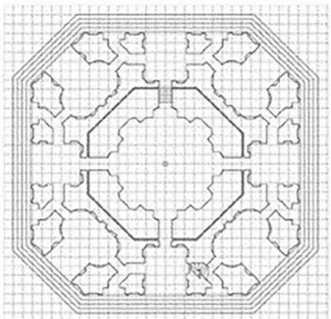
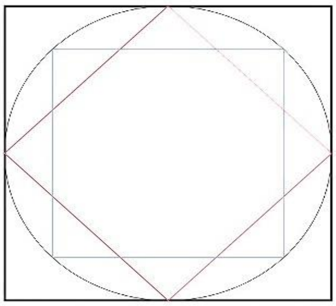
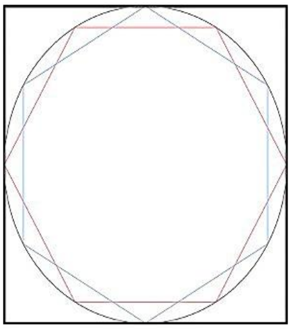
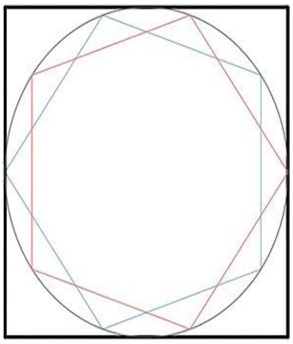
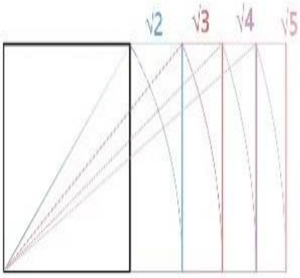
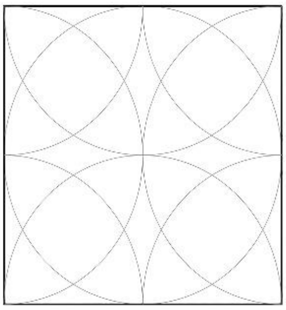
General Category	System Name	Shape of Each System	Description
Golden Ratio	Golden Ratio	 [120]	One of the proportions consistently used since antiquity, known as the Golden Ratio [115]. It manifests in geometric structures such as squares and derivatives (rectangles, spirals[a], triangles, regular pentagons) [15, 116]. Algebraically expressed as: (a+b)/ a= a/b [115], yielding Phi (φ ≈ 0.618 or 1.618) [117-119].
	Persian Golden Ratio		The Persian Golden Ratio relate to √2 and √3 [49, 55]. Construction begins with a unit square, generating Platonic rectangles (√3 rectangles) [121], later extending to dynamic rectangles [122].
	Module System (Peymon)	 [66]	Through the application of Peymon[b], requisite proportions were achieved while establishing multilateral harmony among architectural elements and components [123]. This core principle of Persian architecture served as a modular benchmark for design calculations and execution, employing Gaz and Gereh measurement units in its dimensional framework [61]. Gaz: A unit of measurement [123]. In Iran: Shah Abbasi Gaz ≈ 1.10m; Shirazi Gaz (Zera) ≈ 1.04m [60]. Gereh: Equivalent to 6.66 cm, divisible into two Bahr [123].
Core Proportional Frameworks (Triadic Proportional Systems)	Square-Based Geometry		This system is based on fundamental patterns formed by squares inscribed within circles. By overlapping two squares, an eight pointed star is created, where the ratio of the side length of the inscribed square to that of the circumscribed square is 1/√2 [44, 49, 50]. This geometric system served as an efficient and practical method for measurements during a historical period when access to advanced numerical calculation methods was not available [49].

Table 1. General Classification of Proportional Systems in Architecture (Continued)

General Category	System Name	Shape of Each System	Description
	Hexagonal-Based Geometry		A geometrical system based on regular hexagons[c] and circles. The core ratio includes 1/2 and √3/2 as seen in hexagonal star patterns. This system is used for radial symmetry and tiling patterns [49].
	Pentagonal -Based Geometry		Based on the golden ratio and includes the combination of two inverted regular pentagons inscribed within a circle [49]. In this regard, the regular decagon will follow the same principle.
Dynamic (Root) Rectangles			In constructing a rectangle, a reference square is used in such a way that, with the help of a compass and the diagonal of the reference square, a √2 rectangle is obtained. This process continues to create dynamic rectangles, resulting in √2, √3, √4, and so on these are referred to as dynamic rectangles [122, 126].
Compass Geometry			This technique, first explored by Al Sufi and later by Buzjani, involves drawing regular polygons using only a compass with a fixed span [127, 128]. Italian mathematician Mascheroni proved in 1797 that all constructions can be achieved with a compass alone [129, 130].

[a] Subdividing a rectangle generates a logarithmic spiral rotating from an initial point at specific angles, known as the golden spiral, formed through the Phi (φ) factor or golden division [124].
[b] According to Pirniya, Peymon is equivalent to door width and is classified into two main types: small and large, measuring 14 Gereh (93 cm) and 18 Gereh (125 cm) respectively [123].
[c] The foundational scheme employs two nested hexagons [125].
Authors retrieved from: [15, 44, 49, 50, 55, 60-61, 66, 115-123, 126-130].

In all regulatory geometric frameworks, the generative order of base pattern repetition underlies comprehensive compositions.
Through circular arrangement, most methodologies generate localized patterns by reiterating a foundational motif. This process further manifests systematic proportional divisions within pattern replication [49]. The geometric analysis approach adopted in this study was formulated in accordance with the theoretical framework presented earlier.

As summarized in Table 1, the square-based geometry and the 1:√2 proportional ratio were identified as the theoretical foundations of the analysis.
Accordingly, the geometric examination of the Hasht Behesht Palace plan was conducted through reverse-engineering in AutoCAD, based on the square-growth system. This approach allows the theoretical concepts of hidden geometric order to interpreted within the architectural plan structure.

5. Case Study: Hasht Behesht Palace, Isfahan

Hasht Behesht Palace ranks among the most celebrated royal buildings of the late Safavid era, consistently attracting architectural scholars, art historians, and tourists [20]. Completed in 1669 CE (1080 AH; third regnal year of Shah Soleiman)[13] [19], It remains the sole preserved pavilion along Chahar Bagh Avenue, currently situated within Shahid Rajaei Park[14]. Classified typologically as a central pavilion type structure [20], it features four symmetrical façades oriented cardinal, with the principal north facing elevation.
The northern portico distinguished by its flat ceiling serves as the main entrance, while the southern portico occupies the rear [85].
Eastern and western porticos mirror one another. This extroverted, two story structure rests on a raised plinth (kursi) [85].
Each elevation accesses an ornamented hall centered around a pool (hawz), crowned by a wind catcher (badgir) for passive ventilation [83].
The octagon in square central hall forms the geometric and spatial nucleus. Cruciform organization emphasizes the north south axis bisecting the complex [92] constituting its primary symmetry axis[15] [85] (Figure 1).

6. Geometric analysis of hasht behesht palace (Plan Based)

6.1. Square- growth geometry

To determine the planimetric organization, guidance drawings are employed to identify the key points, lines, and surfaces that define the architectural framework. One of the commonly applied methods is the base circle, or Parhoon, which is divided into four equal parts to establish the Square Growth Geometry System [52].
In this process, a square rotated by 45 degrees is inscribed within the circle, and lines drawn from the center toward the intersection points of the surrounding geometric forms generate the guiding lines, providing a structured foundation for planimetric analysis [132] (Figure 2a,d). This system reflects both the well-known Parhoon division and the static geometric principles rooted in ancient Iranian architecture [114], while also corresponding to the square-growth system based on the √2 ratio described by El-Said and Parman (1976). The geometric framework is based on a core pattern of concentric squares inscribed within a circle. Each successive square maintains a side length ratio of 1 to √2 between the inscribed and circumscribed squares. By drawing the diagonals of the circumscribed square and connecting their intersection points with the inner circle, an eight pointed star grid is generated (Figure 2a). This stellar grid forms the foundational organizational matrix, imparting order and structural harmony to the overall design (Figure 2b,c). The pattern's concentric stellar expansions and nodal connections produce a subdivision system that harmonizes with the base pattern. The same principle applies to the squares with vertical sides as well (Figure 2e,f). Historically, this method functioned as an efficient and practical technique for geometric measurement during eras lacking advanced computational tools [49].
The √2 ratio occupies a significant position among proportional systems in traditional Persian art and architecture. Its origins trace back to Al Buzjani, a prominent 10th century mathematician, who articulated the fundamental principles of this geometric system in his treatises [49]. Al Buzjani described the system as a replicating proportion that maintains constant area, perimeter, and side length relationships [102]. In essence, the system establishes a coherent and regulated structure through the use of concentric squares and geometric progression based on the √2 ratio. The regulating lines of this system are anchored in the geometry of the eight-pointed star (Octagonal).
Each successive side length exhibits a √2 relationship [28, 49, 51]. The efficiency of this system lies in its capacity to preserve the legibility of the base grid throughout expansion, making the √2 ratio an ideal instrument in traditional architectural design [28, 133, 134] (Figure 2).

[13]Joseph Grelot (1076-1077 AH/1665-1666 CE) documented Hasht Behesht Palace through architectural drawings (Vol. II, European Pictorial Documents of Iran, Gholam-ʿAli Humayun). Scholarly consensus thus affirms construction commenced late in Shah ʿAbbas II's reign and concluded under Shah Sulayman [19].
[14]Hasht Behesht Palace was erected within Bulbul Garden. Chardin references it as "Bagh-e Bolbol" due to nightingale nests historically inhabiting the grounds [91], presently designated Shahid Rajaei Park.
[15]Architects intentionally incorporated compositional variations in palace and pavilion designs through asymmetrical layouts. This is exemplified by the peripheral chambers and inter-level relationships in Hasht Behesht, where localized asymmetries coexist within an overarching symmetrical framework [53].

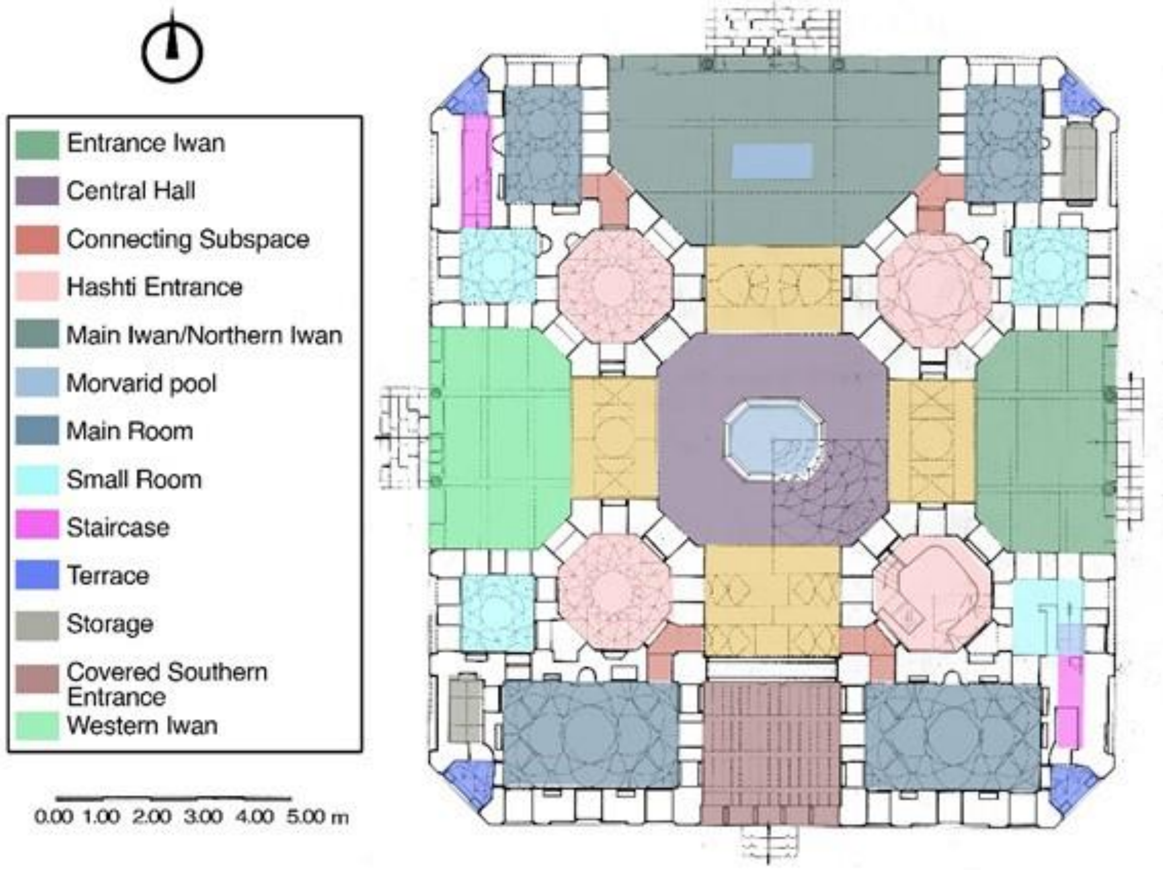


Figure 1. Ground floor plan of Hasht Behesht Palace [85, 131, Authors]

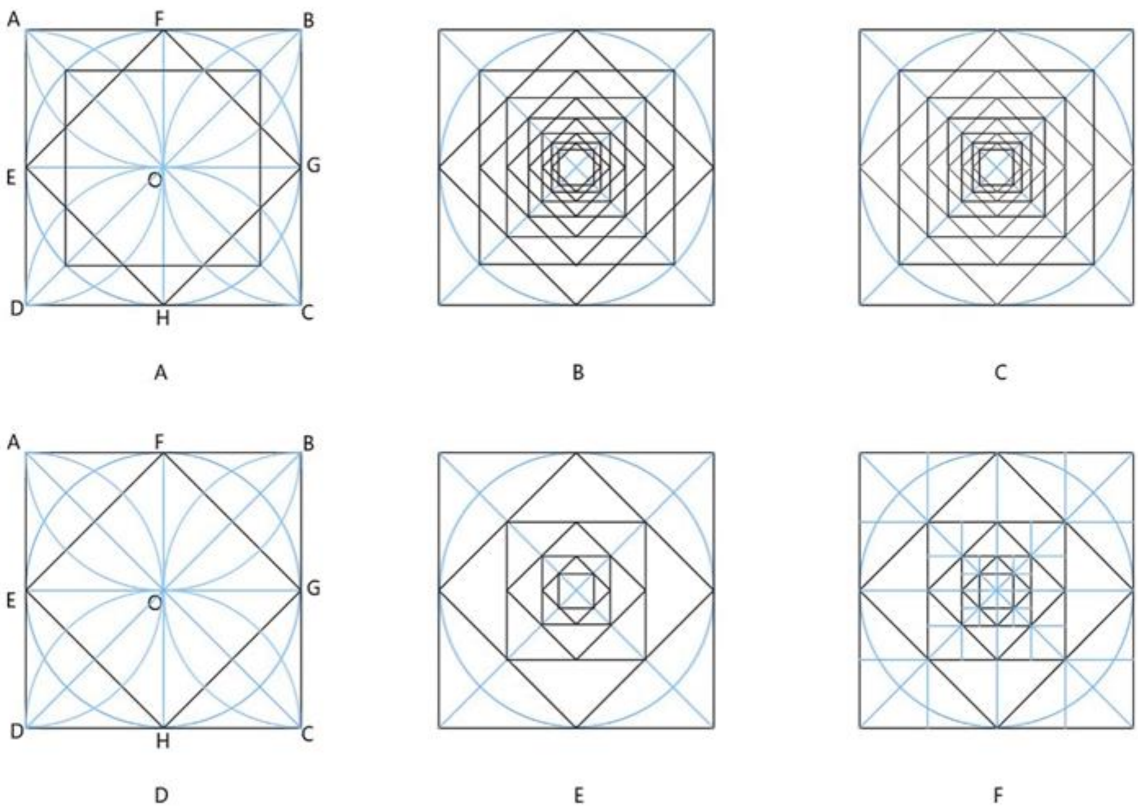


Figure 2. Regulatory geometric framework based on squares and square root ratios: a. Eight-pointed star derived from the base square, b. Concentric eight-pointed star, c. Concentric eight-pointed star with vertical sides, d. Base pattern square - AD/ad = 1/√2, e. Sequential concentric squares, f. Sequential concentric squares with vertical sides [28, 49, 64, Authors]

6.2. Geometric analysis

The research focuses on the objective of analyzing the spatial logic of the Hasht Behesht Palace plan through the 1:√2 proportional system. This study employs reverse engineering to reconstruct the original geometric logic underlying the architect's design. It aims to demonstrate how the palace's planimetric order is structured through a system of geometric proportions known as the square-growth system. Geometric analysis of any structure involves two phases: identifying key planimetric points that form the architect's organizational basis for spatial layout, and recognizing base geometries and ratios determining spatial and component dimensions. A square or rectangular domed chamber forms the fundamental basis of this type of design, to which additional spaces are attached. According to previous studies [15, 22, 114], the shorter side serves as the primary reference. The error rate in the geometric system analysis conducted in this study is below 1.5%, indicating that it is negligible and can be disregarded. In this study, the central hall[16] of the Hasht Behesht Palace was identified as a key planimetric point, where Circle 1 was drawn from focal point O with radius OE (diameter equaling the centeral hall's width, establishing it as the key point); circumscribing square ABCD around this circle defined the centeral hall's totality; within this regulatory geometric framework, the side ratio of inscribed square EFGH to circumscribed square ABCD is 1:√2; through this ratio, squares expand concentrically in geometric progression, with areas doubling at each expansion stage (Figure 3); vertices E, F, G, H of the inscribed square mark midpoints of primary corridors, while vertices A, B, C, D of the circumscribed square indicate depth centers of entry vestibule corridors connecting spatial units. This procedural sequence continued iteratively: four semicircles were first drawn centered at points A, B, C, D with radii OA, OB, OC, OD respectively; connecting their intersection points generated square IJKL (Figure 4), whereby the sides of this derived square align with the diagonal axes of the vestibules.

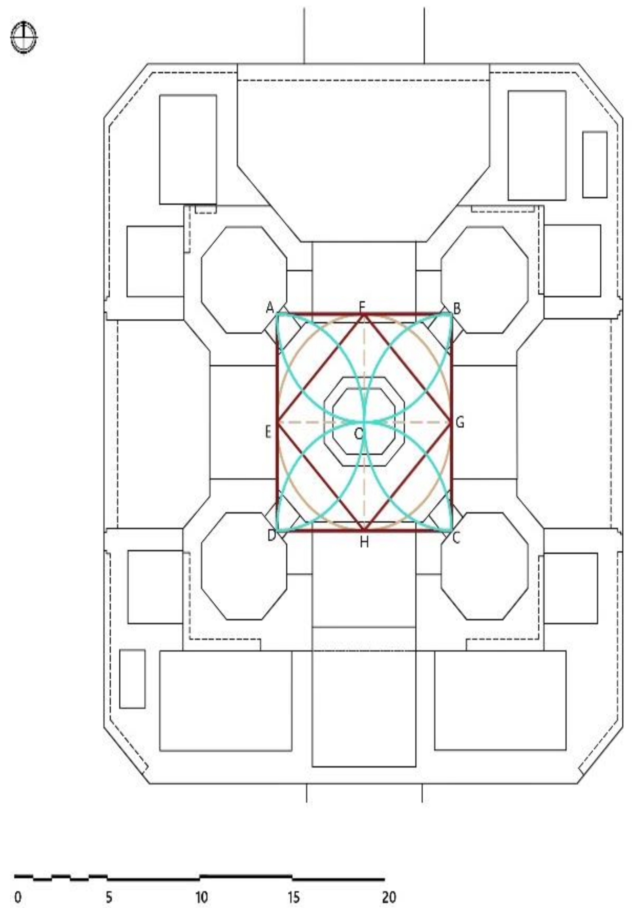


Figure 3. Squares in geometric progression with 1:√2 ratio Entry vestibule (Hashti)

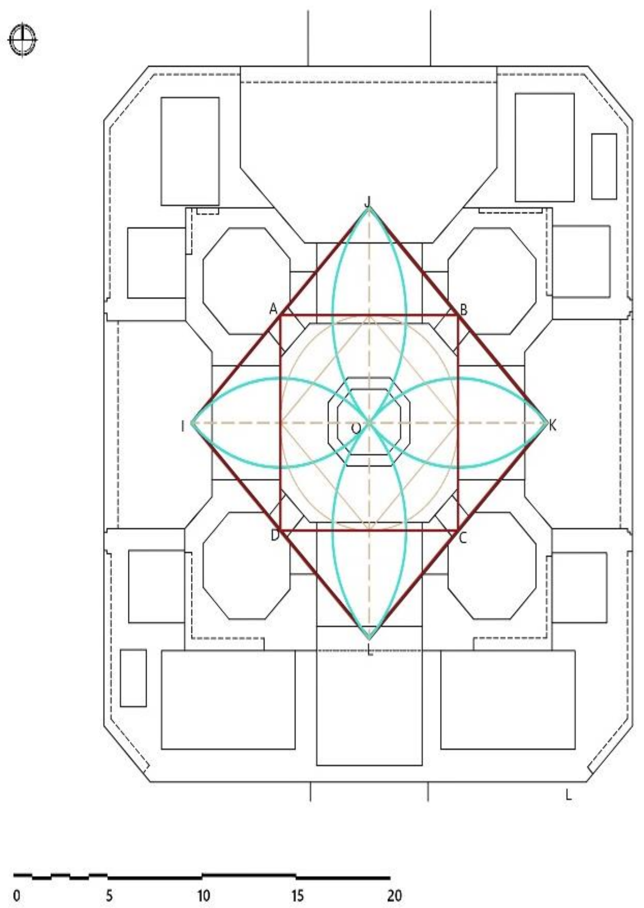


Figure 4. Expansion of the regulatory geometric framework based on squares and square root ratios Central vestibule (Hashti markazi)

[16] The focal point of this key geometric locus is demonstrable and constructible through both Mascheroni compass-only methods [135] and al- Buzjani's classical geometric techniques.

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Subsequently, semicircles were drawn centered at points I, J, K, L with radii OI, OJ, OK, OL, respectively, generating square MNPQ (Figure 5). This derived square exhibited geometric congruence with the overall geometry of the wall thicknesses. Subsequently, square RSTU was generated through identical procedural iteration. All of the aforementioned steps were systematically applied to the plan of Hasht Behesht Palace in Isfahan, and its geometric analysis was conducted using the square-growth system. The intersections of this square with the plan's boundary lines determined portico widths on three sides (VW, ZZ', X'Y'), while establishing the principal span width (XY) of the northern portico (Iwvan) (Figure 6). Ultimately, the inward progression of the base square revealed the diagonal alignments governing the central pool's geometry. Within this regulatory geometric framework, the core pattern transitions to an eight pointed star grid foundation. By drawing concentric squares and connecting key points within the geometric framework, finer subdivisions are organized in accordance with the base pattern. This process generates regulating lines based on the eight pointed star, in which successive sides run parallel to each other and follow a 1:√2 proportional relationship. Applying reverse engineering to reconstruct the architect's design process, this analytical sequence extends the prior geometric decoding: diagonals W'O' and Z'V' of square W'Z'O'V' were constructed, with their intersections on the inscribed circle defining square M'N'P'Q' proportional to RSTU and parallel to W'Z'O'V' thereby forming an eight pointed stellar composite through the M'N'P'Q'–RSTU dyad. This procedural sequence was executed through concentric eight pointed star grids superimposed on the palace plan. Circle OR and square R'S'T'U' demonstrated precise alignment with the architectural chamfers (Figure 7). This inward procedural extension definitively confirmed the governing diagonals' alignment with the central pool's geometry (Figure 8). Geometric analysis is a fundamental principle in architectural studies; without acknowledging that historical architects applied geometric principles, attributing geometry to their works is unjustifiable. Previous studies examining geometric structures in prominent architectural works have emphasized that understanding proportional systems forms the foundation for architectural creation [22, 51]. This study, focusing on the geometric analysis of the plan of Hasht Behesht Palace in Isfahan and aligned with the research objective, seeks to analyze the geometric structure of the palace's plan to demonstrate that its planimetric organization is based on the square-growth proportional system. In other words, this study has revealed the formative process of the geometric order underlying the Safavid-era Hasht Behesht Palace through geometric analysis based on the proposed regulatory geometric system. Furthermore, the analysis indicates that the primary geometric forms employed in the design are the square and the circle, and that the square growth system operates through the ratios of 1:√2 and √2.

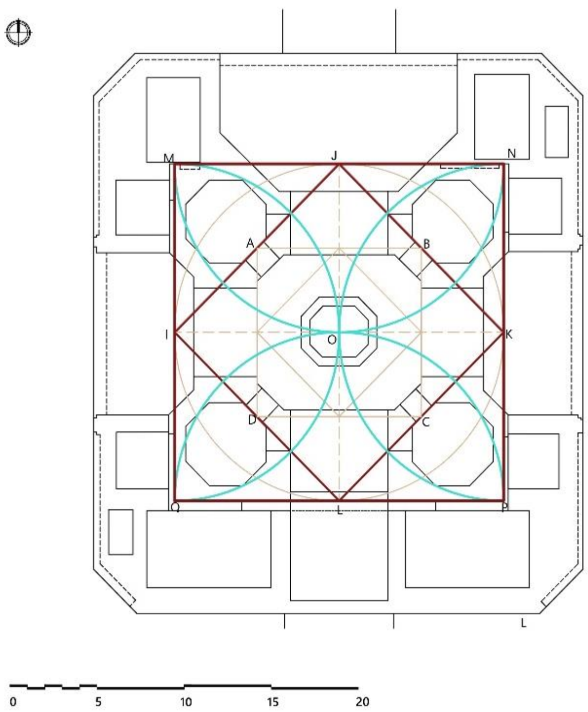

Figure 5. Expansion of regulatory geometric framework based on squares and square root ratios Wall thickness profiles

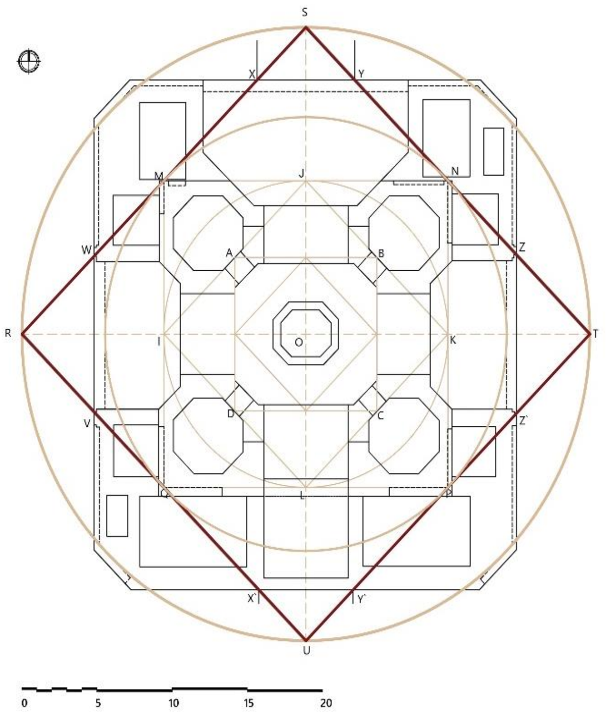

Figure 6. Expansion of regulatory geometric framework based on squares and square root ratios Porticos (ayvan)

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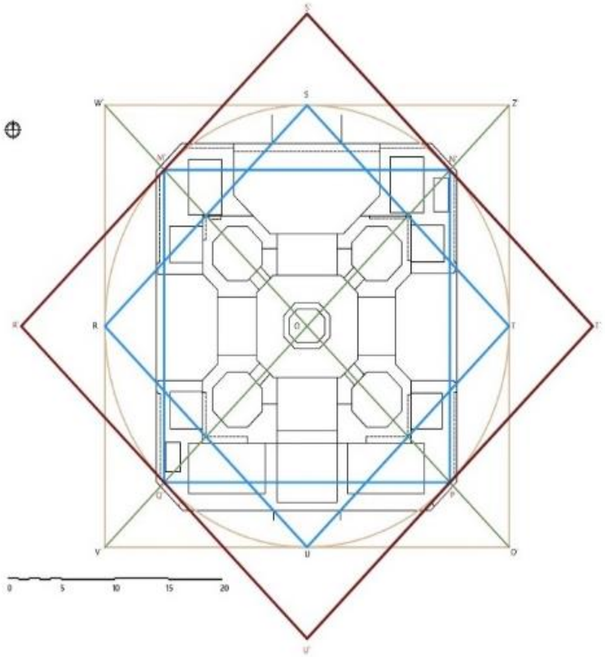

Figure 7. Foundational eight pointed stellar grid within the square based regulatory geometric framework

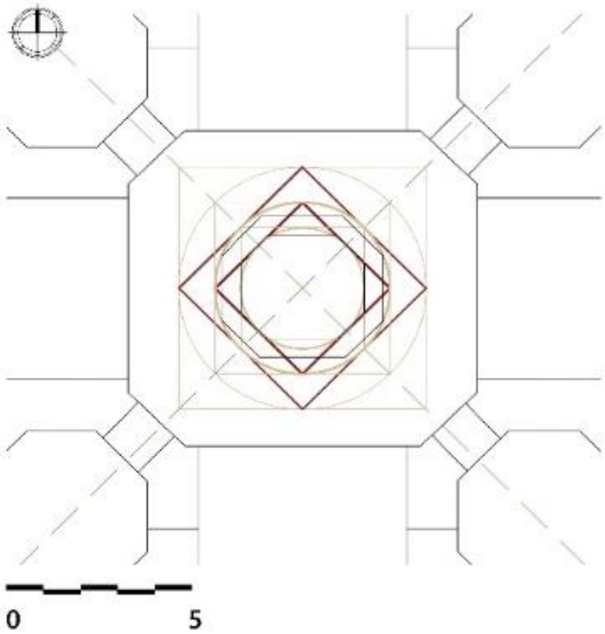

Figure 8[17]. Expansion of regulatory geometric framework based on squares and square root ratios Central pool

These findings are consistent with prior research emphasizing the essential role of geometry in establishing structural order. Hence, understanding proportional systems is essential in architectural creation. One aspect of this geometry appears explicitly in the overall form, while another remains implicit, embedded in hidden structures that influence other forms, an aspect thoroughly addressed in this study.

7. Conclusion

Geometry, as the fundamental essence of architecture, plays a fundamental role in organizing architectural planimetry. Previous studies on geometric structures in prominent architectural works have demonstrated geometry's central role in shaping architectural conception and establishing order and coherence in the design process [59, 65, 66]. This research, focusing on the geometric analysis of the plan of Hasht Behesht Palace in Isfahan, aimed to decipher the geometric system governing its design. The findings clearly indicate that the planimetric organization of the palace follows the square-growth system governed by the key proportional ratios of 1:√2 and √2.

Comparison of the results with similar studies on prominent monuments of the Safavid and Islamic architectural periods [20, 28, 52] confirms the existence of a pre-designed hidden geometric order as a foundational principle in this domain. This consistency, both in analytical methodology and case studies selection, indicates that architects of this era utilized a common and systematic geometric language for planimetric composition.

Based on the conducted analysis, the design process began with the integration of the square and circle, and through systematic geometric replication according to the square-growth system, determined the placement of principal elements including the central hall and porticos. Ultimately, the iterative application of this pattern established the spatial coherence and compositional balance of the plan.

These findings provide a direct response to the main research question, demonstrating that the geometric order was not fortuitous but formed an integral intellectual and structural foundation throughout the design process an assertion corroborated by field investigations and reverse engineering analysis.

The study faced certain limitations, including deterioration and partial destruction of the building, discrepancies among historical drawings, and restricted access to several sections. However, these challenges were largely addressed through a combination of precise field measurements, comparison with verified archival documents, and modeling in AutoCAD. The results of this analytical method indicate that the planar structure of significant historical buildings is derived from a

[17] Figure 8 is cropped sections derived from the researcher-generated verified AutoCAD plan (based on 2024 field measurements and archival validation) to illustrate each analytical step of the square-growth geometric system.

mathematical and geometric substructure, adhering to a hidden and deliberate order.

This approach proves particularly valuable for historical structures that have undergone partial destruction or lack sufficient documentation for accurate restoration. Consequently, this method can serve as an effective analytical model for examining other historical buildings with similar central-plan layouts. Thus, this research fills a critical scholarly gap regarding hidden geometric systems in Safavid architecture, establishing a methodological paradigm for future studies in this domain.

Authors Contribution

All the authors have participated sufficiently in the intellectual content, conception and design of this work or the analysis and interpretation of the data (when applicable), as well as the writing of the manuscript.

Availability of data and materials

The data that support the findings of this study are available from the corresponding author, upon reasonable request.

Conflict of interests

The author states that there is no conflict of interest.

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