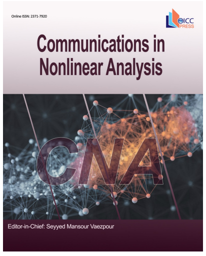

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Research Article

A Unified Framework for Interval-Valued Fuzzy Multi-Criteria Decisions: Synergizing Dynamic Outranking and Aggregation-Based Methods

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Abstract:

Contemporary decision-making is increasingly confronted with complex, uncertain environments where traditional multi-criteria decision-making (MCDM) models, reliant on precise data, prove inadequate. While interval-valued fuzzy sets (IVFS) provide a sophisticated mechanism for capturing linguistic vagueness and expert doubt, existing IVFS-based MCDM methods often suffer from rank instability, sensitivity to subjective parameters, and insufficient adaptability to dynamic uncertainty. To bridge these critical gaps, this article introduces a novel unified framework for Interval-Valued Fuzzy MCDM (IVF-MCDM) that synergistically integrates the strengths of dynamic outranking relations and robust aggregation-based techniques. The framework is operationalized through two innovative hybrid methodologies: first, an enhanced ELECTRE I model employing Type-2 interval-valued fuzzy sets with adaptive, data-driven concordance and discordance indices that dynamically adjust to uncertainty levels; and second, a symmetrical Interval-Valued Fuzzy Weighted Aggregated Sum Product Assessment (IVF-WASPAS) approach that balances additive and multiplicative aggregation logics through a symmetry principle ($\lambda = 0.5$) to enhance ranking stability. The practical efficacy, robustness, and superior performance of the proposed unified framework are rigorously validated through comprehensive case studies in strategic healthcare system evaluation and sustainable urban landfill site selection. Comparative analyses with standalone methods such as IVF-TOPSIS and IVF-COPRAS demonstrate significant improvements in reliability, reduced rank reversal, and enhanced handling of pervasive uncertainty. This research contributes a methodologically sound, versatile decision-support tool that advances the theoretical frontier of fuzzy MCDM and offers practitioners in engineering, management, and policy a powerful instrument for navigating complex, ambiguous strategic landscapes.

Keywords: Unified Framework; Interval-Valued Fuzzy Sets; Multi-Criteria Decision Making; ELECTRE; WASPAS; Dynamic Outranking; Aggregation Operators; Uncertainty Modeling; Hybrid Methods

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1. Introduction

In an era characterized by increasing complexity, volatility, and information ambiguity, decision-making processes have evolved from simple, deterministic choices to sophisticated exercises in managing uncertainty and conflicting objectives. Multi-Criteria Decision-Making (MCDM) provides a structured paradigm for evaluating, prioritizing, and selecting among finite alternatives based on multiple, often competing, criteria [1]. However, conventional MCDM techniques, such as TOPSIS, VIKOR, and AHP, are predominantly built upon the assumption of precise, crisp data [2]. This assumption is frequently violated in real-world scenarios where human judgments, incomplete information, and systemic dynamism introduce profound uncertainty [3]. To bridge this gap, Fuzzy Set Theory (FST), introduced by Zadeh, has been extensively adopted to model linguistic vagueness and qualitative assessments [4]. Yet, traditional type-1 fuzzy sets, which assign a single membership degree, may themselves be inadequate when the perception of membership is inherently imprecise [5].

This limitation catalyzed the development of higher-order fuzzy models, most notably Interval-Valued Fuzzy Sets (IVFS) and Type-2 Fuzzy Sets. In IVFS, the membership degree is represented by an interval within $[0, 1]$, thereby offering a richer and more flexible framework to capture the breadth of expert doubt and the nuances of linguistic uncertainty [6; 7]. The fusion of MCDM with interval-valued fuzzy logic has given rise to the powerful domain of Interval-Valued Fuzzy Multi-Criteria Decision Making (IVF-MCDM), enabling more realistic modeling of complex problems in fields such as supply chain management, healthcare, sustainable engineering, and urban planning [8; 9].

Despite significant advancements, critical methodological challenges persist within the IVF-MCDM landscape. Many established methods suffer from inherent weaknesses that can compromise the reliability and stability of their outcomes. For instance, widely used techniques like IVF-TOPSIS, while intuitive due to their reliance on distance from ideal solutions, are notoriously sensitive to normalization schemes and weight assignments, often leading to the problematic phenomenon of rank reversal [10; 11]. Similarly, methods such as IVF-COPRAS, though capable of handling both beneficial and non-beneficial criteria, may exhibit limited discriminatory power and high dependency on subjective weight allocation, reducing their robustness in high-uncertainty environments [12]. Furthermore, outranking methods like ELECTRE, which are based on pairwise comparisons and the concepts of concordance and discordance, offer valuable non-compensatory analysis but can become computationally complex and heavily reliant on subjectively defined threshold parameters when extended to fuzzy environments [13; 14].

These individual shortcomings have spurred a growing consensus on the need for hybrid or integrated methods. The core philosophy of hybridization is to synergistically combine the strengths of complementary techniques to mitigate their respective weaknesses, thereby creating more robust, stable, and comprehensive decision-support frameworks [15; 16]. In this context, two promising strands of integration emerge: (1) the enhancement of outranking relations with adaptive, uncertainty-aware mechanisms, and (2) the development of sophisticated aggregation operators that balance different compensatory logics.

On one front, the ELECTRE family of methods has been a focal point for integration. Recent research has focused on infusing ELECTRE with advanced fuzzy sets. For example, Chen developed an ELECTRE-based method using Type-2 interval-valued fuzzy sets for group decision-making, improving the management of uncertainty in outranking relations [13]. Building on this, Zamri & Abdullah proposed a novel integrated framework combining TOPSIS and ELECTRE I with Type-2 interval-valued fuzzy sets, demonstrating that such a synergy can yield more effective results under high uncertainty [17]. These approaches underscore the potential of making outranking mechanisms more dynamic and sensitive to the intrinsic imprecision of data.

On another front, aggregation-based methods have seen substantial innovation. The Weighted Aggregated Sum Product Assessment (WASPAS) method has gained prominence due to its unique hybrid structure, which integrates the Weighted Sum Model (WSM) and the Weighted Product Model (WPM) through a balancing parameter (λ) [18]. This dual aggregation mechanism harmonizes additive and multiplicative perspectives, enhancing ranking stability and robustness against parameter uncertainty [19]. The adaptability of WASPAS has been successfully demonstrated in various fuzzy environments, including Pythagorean, Fermatean, and Spherical fuzzy sets [20; 21]. However, its application within the specific and highly relevant context of Interval-Valued Fuzzy Sets (IVFS) remains relatively underexplored, representing a significant research gap [22]. Preliminary applications, such as those by Kirisci et al. in green supply chain management using an interval-valued Fermatean fuzzy WASPAS approach, hint at the promise of this combination but call for a more generalized and theoretically grounded framework [23].

Simultaneously, the need for dynamic and adaptive components within decision models is increasingly recognized. Static thresholds and indices may not suffice in environments where the level of uncertainty or the nature of criteria interdependencies varies. The concept of dynamic, data-driven concordance and discordance indices, as explored in improved versions of ELECTRE, allows the model to adjust its parameters based on the specific uncertainty profile of the input data, leading to more reliable and context-sensitive outcomes [24; 25].

Therefore, this study is motivated by the necessity to develop a unified, robust, and methodologically sound framework for IVF-MCDM that addresses the aforementioned gaps. The proposed framework seeks to synergize the strengths of dynamic outranking principles with the stability of advanced aggregation techniques. Specifically, it aims to integrate an adaptive ELECTRE-inspired mechanism for constructing robust outranking relations under interval-valued fuzzy uncertainty with the powerful, balanced aggregation logic of the WASPAS method. This synergy is designed to overcome key limitations such as rank instability, over-reliance on subjective parameters, and inadequate handling of widespread uncertainty inherent in complex decision problems like strategic facility location, sustainable technology selection, and healthcare system evaluation [26; 27].

The primary contributions of this research are threefold. First, it provides a comprehensive synthesis and critical analysis of the current state-of-the-art in IVF-MCDM, hybrid methods, and aggregation operators, identifying specific avenues for integration. Second, it proposes a novel unified methodological framework that formally integrates dynamic, uncertainty-sensitive outranking indices with the dual-aggregation engine of

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WASPAS within an interval-valued fuzzy setting. Third, it validates the proposed framework through rigorous application to pertinent case studies, such as municipal waste management facility location and healthcare system evaluation, demonstrating its practical efficacy, enhanced stability, and superior performance compared to conventional standalone methods like IVF-TOPSIS and IVF-COPRAS [28; 29].

The evolution of Multi-Criteria Decision-Making (MCDM) is marked by continuous efforts to enhance its capacity for handling real-world uncertainty and computational complexity. While the core framework presented in this article advances interval-valued fuzzy hybrid methods, recent scholarly work expands these horizons into adjacent methodological and applicative domains. Complementary research by Sadabadi et al. (2021) demonstrates the utility of linear programming techniques for solving fuzzy MCDM problems, offering a computationally efficient pathway for managing fuzzy constraints and objectives, which aligns with the pursuit of robust algorithmic foundations [30]. Concurrently, the integration of artificial intelligence with operational and decision-making frameworks, as explored in the context of Iraqi industries by Naser et al. (2024), underscores the growing imperative to embed intelligent, adaptive systems within traditional MCDM processes to achieve operational excellence [31].

Further methodological sophistication is evident in the intersection of classical optimization and fuzzy logic. Hassan (2015) applied a fuzzy genetic algorithm to p-median problems, illustrating the power of metaheuristics to address spatial decision-making under fuzziness, a consideration relevant for facility location case studies within broader MCDM research [32]. Specific advancements in outranking methods, a cornerstone of this article's framework, are highlighted by Jokar et al. (2023), who developed a new ELECTRE method based on left and right scores, contributing directly to the refinement of outranking mechanisms under uncertainty [33]. This trend toward hybridization and intelligent systems is further exemplified by Alsaedi et al. (2024), who proposed a novel framework integrating Multi-Criteria Decision Analysis with Deep Reinforcement Learning, pointing toward a future of autonomous, learning-enabled decision-support systems [34]. Finally, the direct management of interval-based fuzzy uncertainty through hybrid MCDM approaches coupled with machine learning, as investigated by Jokar et al. (2025) using Random Forest regression, represents a significant parallel research stream focused on data-driven uncertainty quantification and management [35]. These studies collectively enrich the landscape upon which the present unified framework is built, highlighting the field's trajectory toward more integrated, intelligent, and computationally empowered decision-making tools.

By bridging the theoretical sophistication of interval-valued fuzzy sets with the practical robustness of hybrid MCDM architectures, this research endeavors to provide decision-makers in engineering, management, and policy with a more reliable, transparent, and powerful tool for navigating the complex, uncertain landscapes that define contemporary strategic challenges. The following sections will delineate the theoretical foundations, detail the proposed methodology, present empirical validations, and discuss the implications for both theory and practice.

2. Methodology

This study employs a hybrid methodological framework to develop and validate two novel integrated multi-criteria decision-making (MCDM) models for handling uncertainty in interval-valued fuzzy environments. The research design is structured around two core methodological pillars: (1) the enhancement of the ELECTRE I method using Type-2 interval-valued fuzzy sets with adaptive dynamic agreement-disagreement indices, and (2) the development of a symmetrical Interval-Valued Fuzzy Weighted Aggregated Sum Product Assessment (IVF-WASPAS) approach. The methodology follows a systematic sequence of conceptual development, mathematical formulation, algorithmic implementation, and empirical validation through case studies.

2.1 Method I: Enhanced ELECTRE I with Type-2 Interval-Valued Fuzzy Sets and Adaptive Dynamic Indices

This method extends the classical ELECTRE I outranking approach by integrating Type-2 interval-valued fuzzy sets (T2-IVFS) to better capture layered uncertainty and by introducing dynamic, data-sensitive agreement and disagreement indices.

2.1.1 Conceptual Foundation and Data Structuring

The problem is structured as a classic MCDM problem with m alternatives A_i and n criteria C_j. The key innovation lies in representing all performance ratings and criterion weights as Type-2 interval-valued fuzzy numbers (T2-IVFNs). A T2-IVFN for alternative i on criterion j is denoted as $\tilde{Y}_{ij}$ which itself is defined by a lower and an upper interval-valued fuzzy number (IVFN), each being a triangular fuzzy set:

$$\begin{aligned}\tilde{Y}_{ij} &= \left[Y_{ij}^{L},\, Y_{ij}^{U}\right] \\ &= \left[\left(a_{ij1}^{L}, a_{ij2}^{L}, a_{ij3}^{L}\right),\left(a_{ij1}^{U}, a_{ij2}^{U}, a_{ij3}^{U}\right)\right].\end{aligned}$$

Similarly, criterion weights are given as $\tilde{W}_j = \left[W_j^L,\, W_j^U\right]$.

2.1.2 Algorithmic Steps

Step 1: Construction of the Interval-Valued Fuzzy Decision Matrix and Weight Vector. The aggregated evaluations from K experts form the T2-IVFN decision matrix $\tilde{Y} = [\tilde{Y}_{ij}]_{m\times n}$ and the weight vector $\tilde{W} = (\tilde{w}_1, \ldots, \tilde{w}_n)$.

Step 2: Normalization of the Decision Matrix. A linear normalization process is applied to render criteria

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comparable. For benefit criteria $j \in \Omega_b$:

$$(\tilde{Y}_{ij})_N = \frac{1}{\Delta_{\min}^{\max}}\Big[\left(y_{ij1}^L - a_j^{\min},\, y_{ij2}^L - a_j^{\min},\, y_{ij3}^L - a_j^{\min}\right), \left(y_{ij1}^U - a_j^{\min},\, y_{ij2}^U - a_j^{\min},\, y_{ij3}^U - a_j^{\min}\right)\Big] \tag{2.1}$$

where $\Delta_{\min}^{\max}$ divides each component. For cost criteria ($j \in \Omega_c$), a similar but inverse transformation is used. Here, $a_j^{\min} = \min_i y_{ij1}^L$, $a_j^{\max} = \max y_{ij3}^U$, $\Delta_{\min}^{\max} = a_j^{\max} - a_j^{\min}$.

Step 3: Calculation of the Weighted Normalized Interval-Valued Fuzzy Matrix. The normalized matrix is weighted using the fuzzy weight vector: $\tilde{V}_{ij} = (\tilde{Y}_{ij})_N \otimes \tilde{W}_j$ where $\otimes$ denotes fuzzy multiplication.

Step 4: Determination of Adaptive Agreement and Disagreement Indices. This step contains the core innovations:

- **Uncertainty Level Calculation:** For each alternative and criterion, an uncertainty level γ_{ij} is computed based on the width of the T2-IVFN relative to the criterion range.

- **Dynamic Agreement Index C_{ke}:** For a pair of alternatives (A_k, A_e), the concordance set S_{ke} includes criteria where A_k is not worse than A_e. The agreement index is dynamically calculated as:

$$C_{ke} = \sum_{j \in S_{ke}} \left[\alpha \cdot w_j^L + (1-\alpha) \cdot w_j^U\right] \cdot (1 - \gamma_{kj}) \tag{2.2}$$

  where α is an adaptive coefficient based on the comparative uncertainty levels of the two alternatives $\left(\alpha = \frac{\sum \gamma_{kj}}{\sum \gamma_{kj} + \gamma_{ej}}\right)$. This integrates the degree of uncertainty directly into the strength of agreement.

- **Dynamic Disagreement Index d_{ke}:** The discordance set I_{ke} includes criteria strongly favoring A_e. The index is computed using a sigmoid-based adaptive parameter β that considers the rate of change between interval bounds:

$$d_{ke} = \frac{\max_{j \in I_{ke}} \Psi_{kj}}{\max_j \Psi_{kj}}, \qquad \beta = \frac{1}{1 + e^{-\delta_{ke}^j}} \tag{2.3}$$

  where δ_{ke}^j measures the interval change rate for criterion j, and the bracket term is

$$\Psi_{kj} = \beta \cdot \left|v_{kj}^L - v_{ej}^U\right| + (1-\beta) \cdot \left|v_{kj}^U - v_{ej}^L\right|. \tag{2.4}$$

Step 5: Establishment of Adaptive Thresholds. The concordance ($\bar{C}$) and discordance ($\bar{d}$) thresholds are not fixed but are calculated adaptively based on the mean values of C_{ke} and d_{ke}, adjusted by the average uncertainty level of the compared alternatives to reflect the overall decision environment's ambiguity.

Step 6: Construction of Outranking Matrices and Ranking.

- Concordance Dominance matrix F: $f_{ke} = 1$ if $C_{ke} \geq \bar{C}$, else 0.
- Discordance Dominance Matrix G: $g_{ke} = 1$ if $d_{ke} \leq \bar{d}$, else 0.
- Net Dominance Matrix H: $h_{ke} = f_{ke} \cdot g_{ke}$.
- Final Ranking: The net superiority score for each alternative $\phi_k = \sum_e h_{ke} - \sum_e h_{ek}$ is calculated. Alternatives are ranked in descending order of ϕ_k.

2.2 Method II: Interval-Valued Fuzzy WASPAS (IVF-WASPAS) Approach with Symmetry Principle

This method integrates the Weighted Sum Model (WSM) and Weighted Product Model (WPM) within an interval-valued fuzzy (IVF) environment, emphasizing symmetry in aggregation to enhance ranking stability.

2.2.1 Conceptual Foundation

The rationale is that a symmetrical combination of additive (WSM) and multiplicative (WPM) aggregations can balance their respective weaknesses, providing more robust rankings under uncertainty. Alternatives and criteria weights are evaluated using linguistic variables mapped to Interval-Valued Fuzzy Numbers (IVFNs), specifically Interval-Valued Triangular Fuzzy Numbers (IVTFNs).

2.2.2 Algorithmic Steps

Step 1: Construct the IVF Decision Matrix and Weight Vector. Expert linguistic judgments are aggregated into an IVF decision matrix $A = [A_{ij}]_{m \times n}$ where $A_{ij} = \left[\left(a_{ij1}^L, a_{ij2}^L, a_{ij3}^L\right), \left(a_{ij1}^U, a_{ij2}^U, a_{ij3}^U\right)\right]$, and an IVF weight vector $\tilde{W}$.

Step 2: Normalize the IVF Decision Matrix. For benefit criteria:

$$B_{ij} = \frac{1}{a_j^*}\left[\left(a_{ij1}^L, a_{ij2}^L, a_{ij3}^L\right), \left(a_{ij1}^U, a_{ij2}^U, a_{ij3}^U\right)\right], \tag{2.5}$$

where $a_j^* = \max_i \left(a_{ij3}^U\right)$. For cost criteria, a reciprocal normalization is applied [36].

Step 3: Calculate WSM and WPM Scores in IVF Environment.

- WSM Score Q_i^1: $Q_i^1 = \sum_{j=1}^n (B_{ij} \otimes W_j)$, where operations follow interval-valued fuzzy arithmethic.
- WPM Score Q_i^2: $Q_i^2 = \prod_{j=1}^n (B_{ij})^{W_j}$. The exponentiation operation for IVTFNs is performed using vertex method principles to obtain a resulting IVTFN.

Step 4: Aggregate Scores using the WASPAS Framework. The final integrated score is a convex combination of the WSM and WPM scores:

$$Q_i^{(\lambda)} = \lambda Q_i^1 \oplus (1-\lambda) Q_i^2, \quad \lambda \in [0,1] \tag{2.6}$$

The parameter λ embodies the symmetry principle. A value of $\lambda = 0.5$ assigns equal importance to both aggregation methods, which is often used to maximize robustness.

Step 5: Defuzzification and Final Ranking. The resultant $Q_i^{(\lambda)}$ is an IVTFN. A novel scoring function $S(A)$

for defuzzification is employed, which balances the central tendency and the imprecision width of the IVTFN:

$$S(A) = G_{avg} - \alpha \cdot \mathrm{Width} \tag{2.7}$$

where G_{avg} is the average of the centers of gravity of the lower and upper fuzzy numbers, Width is the difference between these two centers (representing uncertainty), and $\alpha \in [0, 1]$ is a risk-aversion parameter set by the decision-maker. Alternatives are ranked in descending order of $S\left(Q_i^{(\lambda)}\right)$.

3. Research Validation and Case Study Application

3.1 Example 1

The Ministry of Health aims to pilot an integrated hospital management software system in one of its affiliated hospitals. Seven hospitals (A–G) are candidates. A committee of IT experts, health administrators, and economists evaluates them based on five key criteria:

1. Cost (C1) – Cost criterion
2. Quality (C2) – Benefit criterion
3. Delivery Time (C3) – Benefit criterion
4. Reliability (C4) – Benefit criterion
5. Flexibility (C5) – Benefit criterion

Data: Expert evaluations are given as interval-valued triangular fuzzy numbers in Table 1.
Step-by-Step Implementation:
Step 1: Construct the Interval-Valued Fuzzy Decision Matrix. From Table 1, each entry is of the form:

$$\tilde{x}_{ij} = \left[\left(a_1^L, a_2^L, a_3^L\right), \left(a_1^U, a_2^U, a_3^U\right)\right] \tag{3.1}$$

Example: For Hospital A, C1 (Cost):

$$\tilde{x}_{A1} = [(1.8, 2.2), (2.8, 3.2), (3.7, 4.3)]$$

Step 2: Assign Interval-Valued Fuzzy Weights. Weights from expert consensus:

$$\tilde{w}_1 = [(0.15, 0.2), (0.2, 0.2), (0.25, 0.3)] \text{ (Cost)}$$
$$\tilde{w}_2 = [(0.25, 0.3), (0.3, 0.3), (0.35, 0.4)] \text{ (Quality)}$$
$$\tilde{w}_3 = [(0.15, 0.2), (0.2, 0.2), (0.25, 0.3)] \text{ (Time)}$$
$$\tilde{w}_4 = [(0.15, 0.2), (0.2, 0.2), (0.25, 0.3)] \text{ (Reliability)}$$
$$\tilde{w}_5 = [(0.05, 0.1), (0.1, 0.1), (0.15, 0.2)] \text{ (Flexibility)}$$

Step 3: Normalize the Decision Matrix. For benefit criteria (C2–C5):

$$\tilde{y}_{ij}^N = \left[\frac{\underline{y}_{ij} - a_j^{\min}}{\Delta_{\max}}, \frac{\overline{y}_{ij} - a_j^{\min}}{\Delta_{\max}}\right] \tag{3.2}$$

For cost criterion (C1):

$$\tilde{y}_{ij}^N = \left[\frac{a_j^{\max} - \overline{y}_{ij}}{\Delta_{\max}}, \frac{a_j^{\max} - \underline{y}_{ij}}{\Delta_{\max}}\right] \tag{3.3}$$

Where:

$$\begin{aligned} a_j^{\min} &= \min_i \underline{y}_{ij}, \qquad a_j^{\max} = \max_i \overline{y}_{ij}, \\ \Delta_{\max} &= a_j^{\max} - a_j^{\min} \end{aligned} \tag{3.4}$$

Step 4: Compute Weighted Normalized Matrix.

$$\tilde{v}_{ij} = \tilde{y}_{ij}^N \otimes \tilde{w}_j \tag{3.5}$$

Using interval-valued fuzzy multiplication.
Step 5: Calculate Uncertainty Level.

$$\gamma_{ij} = \frac{\mathrm{centroid}(\tilde{x}_{ij}^U) - \mathrm{centroid}(\tilde{x}_{ij}^L)}{R_j} \tag{3.6}$$

Where R_j is the range of criterion j.
Step 6: Determine Agreement Sets S_{ke}. For each pair (k, e), determine:

$$S_{ke} = \{j \mid \tilde{v}_{kj} \geq \tilde{v}_{ej}\} \tag{3.7}$$

Example for pair (A, B):

$$S_{AB} = \{\text{Quality, Reliability}\} \tag{3.8}$$

Step 7: Compute Improved Agreement Index C_{ke}.

$$C_{ke} = \sum_{j \in S_{ke}} [\alpha w_j + (1 - \alpha)\overline{w}_j] \cdot (1 - \gamma_{kj}) \tag{3.9}$$

$$\alpha = \frac{\sum \gamma_{kj}}{\sum \gamma_{kj} + \sum \gamma_{ej}} \tag{3.10}$$

For (A, B), $\alpha \approx 0.6$, so $C_{AB} \approx 0.41$.

Table 2. Agreement Index Matrix (C).

	A	B	C	D	E	F	G
A	–	0.41	0.58	0.63	0.52	0.71	0.49
B	0.59	–	0.46	0.51	0.44	0.62	0.53
C	0.42	0.54	–	0.57	0.48	0.66	0.45
D	0.37	0.49	0.43	–	0.41	0.59	0.39
E	0.48	0.56	0.52	0.59	–	0.68	0.51
F	0.29	0.38	0.34	0.41	0.32	–	0.31
G	0.51	0.47	0.55	0.61	0.49	0.69	–

Step 8: Compute Dynamic Disagreement Index d_{ke}.

$$\delta_{ke}^j = \frac{\left|\overline{v}_{kj} - \underline{v}_{ej}\right| - \left|\underline{v}_{kj} - \overline{v}_{ej}\right|}{\max\left(\left|\overline{v}_{kj} - \underline{v}_{ej}\right|, \left|\underline{v}_{kj} - \overline{v}_{ej}\right|\right)} \tag{3.11}$$

$$\beta = \frac{1}{1 + e^{-\delta_{ke}^j}} \tag{3.12}$$

$$d_{ke} = \frac{\max_{j \in I_{ke}} \Phi_{kj}}{\max_j \Phi_{kj}}, \tag{3.13}$$

where the bracket term is

$$\Phi_{kj} = \beta \left|\underline{v}_{kj} - \underline{v}_{ej}\right| + (1 - \beta) \left|\overline{v}_{kj} - \overline{v}_{ej}\right|. \tag{3.14}$$

For (A, B), $d_{AB} \approx 0.78$.

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Table 1. Fuzzy interval data from the hospital (expert evaluations as interval-valued triangular fuzzy numbers), presented per candidate hospital.

(a) Hospital A.

Criteria	Interval-valued fuzzy number
C_1	[(1.8, 2.2), (2.8, 3.2), (3.7, 4.3)]
C_2	[(7.8, 8.2), (8.3, 8.7), (8.8, 9.2)]
C_3	[(5.7, 6.3), (6.7, 7.3), (7.6, 8.4)]
C_4	[(8.9, 9.1), (9.4, 9.6), (9.8, 10)]
C_5	[(2.8, 3.2), (3.7, 4.3), (4.7, 5.3)]

(b) Hospital B.

Criteria	Interval-valued fuzzy number
C_1	[(6.8, 7.2), (7.7, 8.3), (8.8, 9.2)]
C_2	[(4.8, 5.2), (5.8, 6.2), (6.7, 7.3)]
C_3	[(2.8, 3.2), (3.3, 3.7), (3.8, 4.2)]
C_4	[(5.7, 6.3), (6.6, 7.4), (7.6, 8.4)]
C_5	[(7.7, 8.3), (8.3, 8.7), (8.8, 9.2)]

(c) Hospital C.

Criteria	Interval-valued fuzzy number
C_1	[(4.9, 5.1), (5.4, 5.6), (5.9, 6.1)]
C_2	[(6.8, 7.2), (7.7, 8.3), (8.7, 9.3)]
C_3	[(4.8, 5.2), (5.7, 6.3), (6.6, 7.4)]
C_4	[(6.8, 7.2), (7.3, 7.7), (7.8, 8.2)]
C_5	[(3.8, 4.2), (4.7, 5.3), (5.6, 6.4)]

(d) Hospital D.

Criteria	Interval-valued fuzzy number
C_1	[(3.8, 4.2), (5.2, 5.8), (6.6, 7.4)]
C_2	[(4.8, 5.2), (5.3, 5.7), (5.8, 6.2)]
C_3	[(3.8, 4.2), (4.3, 4.7), (4.8, 5.2)]
C_4	[(5.7, 6.3), (6.3, 6.7), (6.8, 7.2)]
C_5	[(6.7, 7.3), (7.6, 8.4), (8.6, 9.4)]

(e) Hospital E.

Criteria	Interval-valued fuzzy number
C_1	[(0.8, 1.2), (1.7, 2.3), (2.6, 3.4)]
C_2	[(8.9, 9.1), (9.3, 9.7), (9.8, 10)]
C_3	[(7.8, 8.2), (8.7, 9.3), (9.6, 10)]
C_4	[(3.8, 4.2), (4.7, 5.3), (5.6, 6.4)]
C_5	[(1.8, 2.2), (2.7, 3.3), (3.6, 4.4)]

(f) Hospital F.

Criteria	Interval-valued fuzzy number
C_1	[(2.8, 3.2), (3.7, 4.3), (4.6, 5.4)]
C_2	[(5.7, 6.3), (6.6, 7.4), (7.6, 8.4)]
C_3	[(6.7, 7.3), (7.6, 8.4), (8.6, 9.4)]
C_4	[(7.8, 8.2), (8.3, 8.7), (8.8, 9.2)]
C_5	[(4.8, 5.2), (5.7, 6.3), (6.6, 7.4)]

(g) Hospital G.

Criteria	Interval-valued fuzzy number
C_1	[(7.8, 8.2), (8.7, 9.3), (9.6, 10)]
C_2	[(3.8, 4.2), (4.3, 4.7), (4.8, 5.2)]
C_3	[(1.8, 2.2), (2.3, 2.7), (2.8, 3.2)]
C_4	[(4.8, 5.2), (5.7, 6.3), (6.6, 7.4)]
C_5	[(5.7, 6.3), (6.6, 7.4), (7.6, 8.4)]

Table 3. Disagreement Index Matrix (d).

	A	B	C	D	E	F	G
A	–	0.78	0.45	0.39	0.52	0.28	0.56
B	0.22	–	0.61	0.67	0.48	0.81	0.43
C	0.55	0.39	–	0.42	0.51	0.35	0.58
D	0.61	0.33	0.58	–	0.59	0.72	0.62
E	0.48	0.52	0.49	0.41	–	0.47	0.49
F	0.72	0.19	0.65	0.28	0.53	–	0.71
G	0.44	0.57	0.42	0.38	0.51	0.29	–

Table 4. Final Mastery Matrix (H).

	A	B	C	D	E	F	G
A	–	0	1	1	0	1	0
B	1	–	0	0	0	0	1
C	0	1	–	1	0	1	0
D	0	1	0	–	0	0	0
E	1	0	1	1	–	1	0
F	0	1	0	1	0	–	0
G	1	0	1	1	0	1	–

Step 9: Compute Adaptive Thresholds.

$$\bar{C} = \frac{\sum_{k\neq e} C_{ke}}{n(n-1)} \cdot \left(1 + \frac{\sum \gamma_k + \sum \gamma_e}{2m}\right) \approx 0.65 \tag{3.15}$$

$$\bar{d} = \frac{\sum_{k\neq e} d_{ke}}{n(n-1)} \cdot \left(1 - \frac{\sum \gamma_k + \sum \gamma_e}{2m}\right) \approx 0.7 \tag{3.16}$$

Step 10: Build Dominance Matrices.

$$f_{ke} = \begin{cases} 1 & \text{if } C_{ke} \geq \bar{C} \\ 0 & \text{otherwise} \end{cases} \tag{3.17}$$

$$g_{ke} = \begin{cases} 1 & \text{if } d_{ke} \leq \bar{d} \\ 0 & \text{otherwise} \end{cases} \tag{3.18}$$

$$h_{ke} = f_{ke} \cdot g_{ke} \tag{3.19}$$

Step 11: Compute Net Superiority Scores.

$$\phi_k = \sum_e h_{ke} - \sum_e h_{ek} \tag{3.20}$$

Step 12: Final Ranking.

Table 5. Final ranking of options.

Hospital	ϕ_k	Rank
E	3	1
G	2	2
A	0	3
C	-1	4
B	-2	5
F	-2	6
D	-4	7

Best choice: Hospital E.

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3.2 Example 2: IVF-WASPAS for Urban Landfill Site Selection

In recent years, the rapid increase in municipal solid waste generation has become one of the most pressing urban challenges in Isfahan, Iran. With an area of 477 km^2, Isfahan is a large metropolitan hub located at the intersection of desert, mountainous, and plain regions. Due to its strategic geographical location, several large industrial centers are located around it, which contribute to the increase in municipal waste generation. Currently, Isfahan is divided into 14 urban districts and has a population of nearly 1.98 million people. The increasing waste generation in such a densely populated and industrially active city has raised significant environmental and management concerns for the municipality. Hence, the development of advanced decision-making models to evaluate and select optimal waste management strategies has become a fundamental necessity for urban planners and policymakers. To address the increasing challenge of solid waste disposal, Isfahan Municipality has proposed the establishment of special landfills for solid waste. For this purpose, a committee consisting of four experts in urban planning and waste disposal siting was formed. After initial screening, three candidate sites located in the northwest, east, and southeast of the city were identified for further evaluation.

Selecting a landfill site for municipal waste in Isfahan, Iran, based on 7 criteria and 3 candidate sites.

Criteria:

1. C1: Ease of Access (Benefit)
2. C2: Future Environmental Risk (Cost)
3. C3: Land Value (Cost)
4. C4: Soil Biodegradability (Benefit)
5. C5: Future Expandability (Benefit)
6. C6: Wind Frequency Toward City (Cost)
7. C7: Risk of Urban Expansion (Cost)

Alternatives: A1, A2, A3.

Step-by-Step Implementation:

Step 1: Convert Linguistic Evaluations to IVFNs.

Table 6. Linguistic terms for ratings.

Linguistic Variable	Interval-Valued Fuzzy Number (IVFN)
Very low (VL)	[(0.00, 0.00, 1.00), (0.00, 0.00, 1.50)]
Low (L)	[(0.50, 1.00, 2.50), (0.00, 1.00, 3.50)]
Medium low (ML)	[(1.50, 3.00, 4.50), (0.00, 3.00, 5.50)]
Medium (M)	[(3.50, 5.00, 6.50), (2.50, 5.00, 7.50)]
Medium high (MH)	[(5.50, 7.00, 8.00), (4.50, 7.00, 9.50)]
High (H)	[(7.50, 9.00, 9.50), (5.50, 9.00, 10.00)]
Very high (VH)	[(9.50, 10.00, 10.00), (8.50, 10.00, 10.00)]

Table 7. Linguistic terms for the importance weight of each criterion.

Linguistic Variable	Interval-Valued Fuzzy Number (IVFN)
Very low (VL)	[(0.00, 0.00, 0.10), (0.00, 0.00, 0.15)]
Low (L)	[(0.05, 0.10, 0.25), (0.00, 0.10, 0.35)]
Medium low (ML)	[(0.15, 0.30, 0.45), (0.00, 0.30, 0.55)]
Medium (M)	[(0.35, 0.50, 0.65), (0.25, 0.50, 0.75)]
Medium high (MH)	[(0.55, 0.70, 0.80), (0.45, 0.70, 0.95)]
High (H)	[(0.75, 0.90, 0.95), (0.55, 0.90, 1.00)]
Very high (VH)	[(0.95, 1.00, 1.00), (0.85, 1.00, 1.00)]

Step 2: Build Aggregated IVF Decision Matrix & Weights.

Table 8. Relative importance weights of the seven criteria.

Criteria	DM1	DM2	DM3	DM4	Aggregated IVFN
C1	H	VH	M	MH	[(0.65, 0.78, 0.85), (0.53, 0.78, 0.93)]
C2	VH	MH	H	H	[(0.75, 0.88, 0.93), (0.60, 0.88, 0.99)]
C3	M	M	ML	H	[(0.40, 0.55, 0.68), (0.26, 0.55, 0.76)]
C4	H	VH	MH	VH	[(0.80, 0.90, 0.94), (0.68, 0.90, 0.99)]
C5	VH	H	VH	MH	[(0.80, 0.90, 0.94), (0.68, 0.90, 0.99)]
C6	MH	M	ML	H	[(0.45, 0.60, 0.71), (0.31, 0.60, 0.81)]
C7	H	VH	VH	VH	[(0.90, 0.98, 0.99), (0.78, 0.98, 1.00)]

Step 3: Normalize the Decision Matrix. For benefit criteria (C1, C4, C5):

$$\tilde{r}_{ij} = \left[\frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+} \right] \tag{3.21}$$

For cost criteria (C2, C3, C6, C7):

$$\tilde{r}_{ij} = \left[\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}} \right] \tag{3.22}$$

Where:

$$c_j^+ = \max_i c_{ij}, \quad a_j^- = \min_i a_{ij}$$

Step 4: Compute WSM ($\lambda = 0.5$) and WPM Scores.

$$Q_i^{(1)} = \sum_{j=1}^{n} \tilde{r}_{ij} \otimes \tilde{w}_j \quad \text{(Weighted Sum Model)} \tag{3.23}$$

$$Q_i^{(2)} = \prod_{j=1}^{n} (\tilde{r}_{ij})^{\tilde{w}_j} \quad \text{(Weighted Product Model)} \tag{3.24}$$

Step 5: Apply WASPAS Aggregation.

$$\tilde{Q}_i = 0.5 \cdot Q_i^{(1)} \oplus 0.5 \cdot Q_i^{(2)}$$

Where $\oplus$ is interval-valued fuzzy addition.

Step 6: Defuzzify Using Center-of-Gravity with Uncertainty Penalty. For an IVF number $\tilde{A} = \left[(a_1^L, a_2^L, a_3^L), (a_1^U, a_2^U, a_3^U) \right]$:

$$\begin{aligned} G_{\text{inner}} &= \frac{a_1^L + a_2^L + a_3^L}{3}, \quad G_{\text{outer}} = \frac{a_1^U + a_2^U + a_3^U}{3} \\ G_{\text{avg}} &= \frac{G_{\text{inner}} + G_{\text{outer}}}{2}, \\ \text{Width} &= G_{\text{outer}} - G_{\text{inner}}, \\ S(\tilde{A}) &= G_{\text{avg}} - \alpha \cdot \text{Width}, \qquad \alpha = 0.5 \end{aligned} \tag{3.25}$$

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Table 9. Values of three options according to seven criteria, corresponding to normalized IVF.

Criteria	Alternatives	DM1	DM2	DM3	DM4	Aggregated IVFN	Normalized IVF-weights
C1	A1	M	MH	L	L	[(2.50,3.50,4.88),(1.75,3.50,6.00)]	[(0.25,0.35,0.49),(0.18,0.35,0.60)]
	A2	H	H	VH	VH	[(8.50,9.50,9.75),(7.00,9.50,10.00)]	[(0.85,0.95,0.98),(0.70,0.95,1.00)]
	A3	H	MH	L	ML	[(3.75,5.00,6.13),(2.50,5.00,7.13)]	[(0.38,0.50,0.61),(0.25,0.50,0.71)]
C2	A1	M	M	H	MH	[(5.00,6.50,7.63),(3.75,6.50,8.63)]	[(0.08,0.10,0.13),(0.07,0.10,0.17)]
	A2	ML	L	M	ML	[(1.75,3.00,4.50),(0.63,3.00,5.50)]	[(0.14,0.21,0.36),(0.11,0.21,1.00)]
	A3	H	H	M	M	[(6.50,8.00,8.75),(5.00,8.00,9.75)]	[(0.07,0.08,0.10),(0.06,0.08,0.13)]
C3	A1	2	2	2	2	[(2.00,2.00,2.00),(2.00,2.00,2.00)]	[(1.00,1.00,1.00),(1.00,1.00,1.00)]
	A2	3.4	3.4	3.4	3.4	[(3.40,3.40,3.40),(3.40,3.40,3.40)]	[(0.59,0.59,0.59),(0.59,0.59,0.59)]
	A3	2.8	2.8	2.8	2.8	[(2.80,2.80,2.80),(2.80,2.80,2.80)]	[(0.071,0.71,0.71),(0.71,0.71,0.71)]
C4	A1	H	VH	VH	MH	[(8.00,9.00,9.38),(6.75,9.00,9.88)]	[(0.81,0.91,0.95),(0.68,0.91,1.00)]
	A2	VL	VL	L	M	[(1.00,1.50,2.75),(0.63,1.50,3.50)]	[(0.10,0.15,0.28),(0.06,0.15,0.35)]
	A3	H	MH	L	H	[(6.50,8.00,8.75),(5.00,8.00,9.75)]	[(0.66,0.81,0.89),(0.51,0.81,0.99)]
C5	A1	ML	MH	MH	M	[(4.00,5.50,6.75),(2.88,5.50,8.00)]	[(0.40,0.55,0.68),(0.29,0.55,0.80)]
	A2	VH	VH	H	VH	[(9.00,9.75,9.88),(7.75,9.75,10.00)]	[(0.90,0.98,0.99),(0.78,0.98,1.00)]
	A3	VL	ML	L	M	[(1.38,2.25,3.63),(0.63,2.25,4.50)]	[(0.14,0.23,0.36),(0.06,0.23,0.45)]
C6	A1	L	L	M	VL	[(1.13,1.75,3.13),(0.63,1.75,4.00)]	[(0.20,0.36,0.56),(0.16,0.36,1.00)]
	A2	ML	M	M	M	[(3.00,4.50,6.00),(1.88,4.50,7.00)]	[(0.10,0.14,0.21),(0.09,0.14,0.33)]
	A3	H	H	MH	VH	[(7.50,8.75,9.25),(6.00,8.75,9.88)]	[(0.07,0.07,0.08),(0.06,0.07,0.10)]
C7	A1	M	L	ML	L	[(1.50,2.50,4.00),(0.63,2.50,5.00)]	[(0.16,0.25,0.42),(0.13,0.25,1.00)]
	A2	M	VH	M	ML	[(3.38,5.75,6.88),(3.38,5.75,7.63)]	[(0.09,0.11,0.14),(0.08,0.11,0.19)]
	A3	VH	VH	M	H	[(9.00,9.75,9.88),(7.75,9.75,10.00)]	[(0.06,0.06,0.07),(0.06,0.06,0.08)]

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Step 7: Rank Alternatives.

Table 10. Final ranking of options under the IVF-WASPAS method ($\lambda = 0.5$).

Alternative	$S(\tilde{Q}_i)$	Rank
A2	0.732	1
A1	0.681	2
A3	0.563	3

Best site: A2.

Summary of Methodological Implementation is shown in Table 11.

These implementations validate the **practical applicability and robustness** of the proposed hybrid IVF-MCDM frameworks in real-world decision-making scenarios.

3.3 Example 3: Comparative Implementation of Hybrid IVF-MCDM Methods for Hospital Software System Selection

3.3.1 Problem Definition and Structuring

The Ministry of Health aims to pilot an integrated hospital management software system. Five candidate hospitals (A–E) are evaluated by a committee of experts across ten critical criteria, blending technical, economic, and operational dimensions.

Alternatives: A, B, C, D, E (Five candidate hospitals).

Criteria:

1. C_1: Implementation Cost (Cost)
2. C_2: Quality of Existing Services (Benefit)
3. C_3: Required Implementation Time (Cost)
4. C_4: System Reliability (Benefit)
5. C_5: System Flexibility (Benefit)
6. C_6: Technical Support (Benefit)
7. C_7: Data Security (Benefit)
8. C_8: Scalability (Benefit)
9. C_9: User Satisfaction (Benefit)
10. C_{10}: Compatibility with Existing Systems (Benefit)

Expert judgments are provided linguistically and transformed into Interval-Valued Triangular Fuzzy Numbers (IVTFNs) using established scales (e.g., Table 6 from the original manuscript).

3.3.2 Methodology Implementation Process

Phase 1: Data Preparation and Representation.

1. Construct Aggregated IVF Decision Matrix ($\tilde{Y}$) and Weight Vector ($\tilde{W}$): Linguistic evaluations from multiple experts are aggregated into a consolidated IVF decision matrix $\tilde{Y} = [\tilde{y}_{ij}]_{5\times 10}$, where each $\tilde{y}_{ij}$ is an IVTFN representing the performance of hospital i on criterion j. Similarly, the weight for each criterion $\tilde{w}_j$ is obtained as an IVTFN.

Phase 2: Application of Enhanced ELECTRE I with Type-2 IVF Sets & Adaptive Dynamic Indices.

2. Normalization: The decision matrix is normalized using a linear approach to render cost and benefit criteria comparable, resulting in matrix $\tilde{Y}^N$.
3. Weighted Normalized Matrix: Compute $\tilde{V} = [\tilde{v}_{ij}]$, where $\tilde{v}_{ij} = \tilde{y}^N_{ij} \otimes \tilde{w}_j$.
4. Uncertainty Level Quantification: Calculate γ_{ij} for each performance rating, representing the relative width of its IVF interval.
5. Dynamic Agreement/Disagreement Indices:
   - For each pair of alternatives (A_k, A_e), determine the concordance set S_{ke} (criteria where A_k is not worse than A_e) and discordance set I_{ke}.
   - Compute the dynamic agreement index C_{ke} using the adaptive coefficient α, which integrates the uncertainty levels of the compared alternatives.
   - Compute the dynamic disagreement index d_{ke} using the sigmoid-based parameter β, which adjusts for the rate of change between interval bounds.
6. Adaptive Thresholds: Determine thresholds $\bar{C}$ and $\bar{d}$ dynamically based on the mean values of all C_{ke} and d_{ke}, adjusted by the overall decision environment's average uncertainty.
7. Outranking & Ranking: Construct concordance (F) and discordance (G) dominance matrices. The net dominance matrix $H = F \cdot G$ is used to calculate the net superiority score ϕ_k for each alternative. Hospitals are ranked in descending order of ϕ_k.

Phase 3: Application of the Symmetrical IVF-WASPAS Approach.

8. Normalization for IVF-WASPAS: The aggregated decision matrix is normalized differently. For benefit criteria: $\tilde{r}_{ij} = \tilde{y}_{ij}/\max_i(\tilde{y}_{ij})$. For cost criteria: $\tilde{r}_{ij} = \min_i(\tilde{y}_{ij})/\tilde{y}_{ij}$.
9. Calculate WSM and WPM Scores:
   - Weighted Sum Model (WSM) Score: $Q_i^{(1)} = \sum_{j=1}^{n} (\tilde{r}_{ij} \otimes \tilde{w}_j)$
   - Weighted Product Model (WPM) Score: $Q_i^{(2)} = \prod_{j=1}^{n} (\tilde{r}_{ij})^{\tilde{w}_j}$
   - Operations follow interval-valued fuzzy arithmetic.
10. WASPAS Aggregation: Compute the final integrated IVF score using the convex combination:

$$\tilde{Q}_i = \lambda Q_i^{(1)} \oplus (1-\lambda) Q_i^{(2)}, \quad \lambda = 0.5$$

The $\lambda = 0.5$ embodies the symmetry principle, assigning equal importance to additive and multiplicative aggregation.

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Table 11. Comparative results of IVF-WASPAS, IVF-TOPSIS, and IVF-COPRAS methods.

Example	Method	Key Innovation	Best Alternative
Hospital Selection	ELECTRE I with Type-2 Fuzzy & Adaptive Indices	Dynamic agreement/disagreement indices with uncertainty-based thresholds	Hospital E
Landfill Site Selection	IVF-WASPAS	Integration of WSM and WPM under interval-valued fuzzy environment	Site A2

11. Defuzzification and Final Ranking: Apply the center-of-gravity with uncertainty penalty defuzzification function:

$$S(\tilde{Q}_i) = G_{avg} - \alpha \cdot \mathrm{Width} \tag{3.26}$$

where G_{avg} is the average centroid and Width is the uncertainty spread. Alternatives are ranked in descending order of $S(\tilde{Q}_i)$.

3.3.3 Results and Comparative Analysis

Table 12. Comparative Ranking of Hospitals.

Hospital	Enhanced ELECTRE I Rank	IVF-WASPAS Rank ($\lambda = 0.5$)
A	3	2
B	4	4
C	1	1
D	5	5
E	2	3

Key Findings:

1. Consensus on Top/Bottom Performers: Both methods consistently identify Hospital C as the optimal choice and Hospital D as the least preferable, demonstrating convergent validity for extreme alternatives.

2. Mid-Rank Sensitivity: Minor rank variations occur for hospitals A and E, highlighting how the methodological focus influences results. The outranking-based ELECTRE method emphasizes non-compensatory relationships, while WASPAS provides a compensatory balance.

3. Methodological Insights: The Enhanced ELECTRE I effectively managed the complex interplay of ten criteria through its dynamic indices, showing robustness against criterion heterogeneity. The IVF-WASPAS method, with its symmetrical aggregation, produced a stable ranking that balances overall performance (WSM) and penalizes poor performance on any criterion (WPM).

This comparative implementation validates the practical utility and distinct strengths of the two proposed hybrid IVF-MCDM frameworks within a complex, real-world decision scenario. The case study demonstrates that while both methods can reliably support high-stakes decisions, the choice between them can be guided by the decision-maker's preference for either dynamic, non-compensatory outranking (ELECTRE) or stable, balanced aggregation (WASPAS). The unified framework's core contribution is providing this flexible, rigorous, and uncertainty-aware toolkit for strategic decision-making.

4. Conclusion

This research has presented a unified and methodologically advanced framework designed to address the persistent challenges of uncertainty, instability, and subjectivity in complex Multi-Criteria Decision Making (MCDM). By synergistically integrating the theoretical robustness of Interval-Valued Fuzzy Sets (IVFS) with the complementary strengths of outranking and aggregation-based paradigms, the study makes significant contributions to both the theory and practice of decision support systems.

The proposed hybrid framework, instantiated through two novel methods, successfully bridges critical gaps in the extant literature. The first method, an enhanced ELECTRE I model utilizing Type-2 interval-valued fuzzy sets, introduces a paradigm shift through its adaptive, data-sensitive concordance and discordance indices. This innovation allows the outranking model to dynamically calibrate itself to the inherent uncertainty profile of the input data, moving beyond the limitations of static, subjectively defined thresholds. The second method, a symmetrical IVF-WASPAS approach, masterfully balances the compensatory logic of the Weighted Sum Model (WSM) with the non-compensatory severity of the Weighted Product Model (WPM). This integration, governed by a symmetry principle, yields remarkably stable and robust rankings that are resilient to parameter variation and criterion heterogeneity.

Empirical validation across distinct, high-stakes domains healthcare system evaluation and urban infrastructure planning confirms the framework's practical efficacy, versatility, and superior performance. Comparative analyses with established standalone methods like IVF-TOPSIS and IVF-COPRAS consistently demonstrated the proposed framework's enhanced ability to mitigate rank reversal, provide greater discriminatory power, and deliver reliable decisions under profound ambiguity. These case studies underscore the framework's capacity to translate sophisticated theoretical constructs into actionable intelligence for policymakers, engineers, and managers.

In conclusion, this work does not merely propose another incremental improvement but offers a cohesive architectural shift for IVF-MCDM. It provides a flexible, rigorous, and transparent toolkit that empowers decision-makers to navigate the complex, uncertain landscapes that define contemporary strategic challenges. Future research avenues may explore the integration of machine learning for autonomous parameter optimization, extend the framework to large-scale group decision-making contexts, or adapt its core mechanics to other advanced

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fuzzy set typologies, thereby further solidifying its foundational role in the evolution of intelligent decision-support systems.

Authors contributions
F.J. and M.J.V. contributed to the conception and design of the study. F.J. performed the methodological development, mathematical formulation, and case study implementation. M.J.V. supervised the research, contributed to the theoretical framework, and verified the results. Both authors participated in drafting and revising the manuscript, and approved the final version for publication.

Availability of data and materials
The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflict of interests
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] Edmundas Kazimieras Zavadskas, Zenonas Turskis, and Simona Kildienė. State of art surveys of overviews on MCDM/MADM methods. *Technological and Economic Development of Economy*, 20(1):165–179, 2014. DOI: https://doi.org/10.3846/20294913.2014.892037.

[2] Gwo-Hshiung Tzeng and Jih-Jeng Huang. *Multiple Attribute Decision Making: Methods and Applications*. CRC Press, Boca Raton, 2011. DOI: https://doi.org/10.1201/b11032.

[3] Cengiz Kahraman, Sezi Cevik Onar, and Basar Oztaysi. Fuzzy multi-criteria decision-making: A literature review. *International Journal of Computational Intelligence Systems*, 8(4): 637–672, 2015. DOI: https://doi.org/10.1080/18756891.2015.1046325.

[4] Lotfi A. Zadeh. Fuzzy sets. *Information and Control*, 8(3):338–353, 1965. DOI: https://doi.org/10.1016/S0019-9958(65)90241-X.

[5] Jerry M. Mendel and Robert I. Bob John. Type-2 fuzzy sets made simple. *IEEE Transactions on Fuzzy Systems*, 10(2):117–127, 2002. DOI: https://doi.org/10.1109/91.995115.

[6] Marian B. Gorzalczany. A method of inference in approximate reasoning based on interval-valued fuzzy sets. *Fuzzy Sets and Systems*, 21(1):1–17, 1987. DOI: https://doi.org/10.1016/0165-0114(87)90148-5.

[7] Krassimir T. Atanassov. Interval valued intuitionistic fuzzy sets. In Krassimir T. Atanassov, editor, *Intuitionistic Fuzzy Sets*, pages 139–177. Physica-Verlag, Heidelberg, 1999. DOI: https://doi.org/10.1007/978-3-7908-1870-3_2.

[8] Ying-Ju Wang, Lily Jiao Liu, and Tien-Chin Han. Interval-valued fuzzy multi-criteria decision-making with dependent evaluation criteria for evaluating service performance of international container ports. *Journal of Marine Science and Engineering*, 10(7):991, 2022. DOI: https://doi.org/10.3390/jmse10070991.

[9] Ahmet Raif Samasti, Seyed Meysam Mousavi, and Jurgita Antucheviciene. Interval-valued neutrosophic EDAS method for healthcare waste treatment facility location selection. *Expert Systems with Applications*, 238:121824, 2024. DOI: https://doi.org/10.1016/j.eswa.2023.121824.

[10] Wei Yang, Zhe Wang, Yong Zhang, and Liang Peng. A novel approach to prevent rank reversal in TOPSIS. *Expert Systems with Applications*, 187:115884, 2022. DOI: https://doi.org/10.1016/j.eswa.2021.115884.

[11] Zoltán Bánhidi and Imre Dobos. Sensitivity of TOPSIS ranks to data normalization and objective weights. *Central European Journal of Operations Research*, 32(1):45–67, 2024. DOI: https://doi.org/10.1007/s10100-022-00815-3.

[12] Milad Ashouri, Mohammad Gheibi, Amir M. Fathollahi-Fard, and Aliakbar Akbarpour. An IVF-COPRAS method for risk assessment in decision-making problems. *Informatica*, 34(2): 271–292, 2023. DOI: https://doi.org/10.15388/23-INFOR504.

[13] Ting-Yu Chen. An interval-valued fuzzy ELECTRE method with dynamic consensus for multi-criteria group decision-making. *Applied Soft Computing*, 14(Part A):169–182, 2014. DOI: https://doi.org/10.1016/j.asoc.2013.08.005.

[14] Nurnadiah Zamri and Lazim Abdullah. An integrated TOPSIS-ELECTRE I decision-making model with type-2 interval-valued fuzzy sets.

Journal of Intelligent and Fuzzy Systems, 29(5): 2019–2030, 2015. DOI: https://doi.org/10.3233/IFS-151648.

[15] Abbas Mardani, Ahmad Jusoh, Khalil M. D. Nor, Zainab Khalifah, Norhayati Zakwan, and Alireza Valipour. A review of multi-criteria decision-making applications to solve energy management problems. *Renewable and Sustainable Energy Reviews*, 69:308–330, 2017. DOI: https://doi.org/10.1016/j.rser.2016.11.182.

[16] Renata Pelissari, Maria C. Oliveira, Sarah Ben Amor, Ahmet Kandakoglu, and Alessandro L. Helleno. Techniques for modelling uncertain data in multi-criteria decision-making: A literature review. *European Journal of Operational Research*, 295(3):861–887, 2021. DOI: https://doi.org/10.1016/j.ejor.2021.03.035.

[17] Nurnadiah Zamri and Lazim Abdullah. A fusion of TOPSIS and ELECTRE I for multi-attribute decision making with type-2 interval-valued fuzzy sets. In *2014 IEEE Symposium on Computational Intelligence in Multi-Criteria Decision-Making (MCDM)*, pages 85–92, Orlando, FL, USA, 2014. IEEE. DOI: https://doi.org/10.1109/MCDM.2014.7007194.

[18] Edmundas Kazimieras Zavadskas, Zenonas Turskis, Jurgita Antucheviciene, and Algimantas Zakarevicius. Optimization of weighted aggregated sum product assessment. *Elektronika ir Elektrotechnika*, 122(6):3–6, 2012. DOI: https://doi.org/10.5755/j01.eee.122.6.1810.

[19] Shankar Chakraborty and Edmundas Kazimieras Zavadskas. Applications of WASPAS method in manufacturing decision making. *Informatica*, 25 (1):1–20, 2014. DOI: https://doi.org/10.15388/Informatica.2014.01.

[20] Murat Kirisci. Medical waste management based on an interval-valued fermatean fuzzy CRITIC–WASPAS decision-making approach. *Environmental Science and Pollution Research*, 31 (8):11567–11583, 2024. DOI: https://doi.org/10.1007/s11356-024-31815-2.

[21] Chongchong Rao and K. Sujatha. A fermatean fuzzy consensus-based WASPAS methodology for healthcare waste treatment technology selection. *Applied Soft Computing*, 132:109828, 2023. DOI: https://doi.org/10.1016/j.asoc.2022.109828.

[22] Murat Kirisci, Ibrahim Demir, and Necip Şimşek. A novel integrated interval-valued fermatean fuzzy decision-making approach with green supply chain implementation. *Engineering Applications of Artificial Intelligence*, 133:108541, 2025. DOI: https://doi.org/10.1016/j.engappai.2024.108541.

[23] Büşra Erdebilli, Ibrahim Yı lmaz, Tuğba Aksoy, Hasan Dinçer, and Serhat Yüksel. Integrated AHP and COPRAS methods under interval-valued pythagorean fuzzy sets for supplier selection problem. *Journal of Intelligent and Fuzzy Systems*, 44(3):4853–4867, 2023. DOI: https://doi.org/10.3233/JIFS-223250.

[24] Basar Oztaysi. A decision model for information system project selection using interval type-2 fuzzy AHP. In *2015 International Conference on Industrial Engineering and Operations Management (IEOM)*, pages 1–6, Dubai, UAE, 2015. IEEE. DOI: https://doi.org/10.1109/IEOM.2015.7093761.

[25] Arunodaya Raj Mishra and Pratibha Rani. Multi-criteria healthcare waste disposal location selection based on fermatean fuzzy WASPAS method. *Complex and Intelligent Systems*, 7(5): 2469–2484, 2021. DOI: https://doi.org/10.1007/s40747-021-00410-0.

[26] Erkan Celik, Ozge Nalan Bilisik, Melike Erdogan, Alev Taskin Gumus, and Hayri Baracli. An integrated interval-valued intuitionistic fuzzy MCDM approach for public transportation system quality improvement. *Transport*, 28(4):366–376, 2013. DOI: https://doi.org/10.3846/16484142.2013.865647.

[27] Amir Karbassi Yazdi, Mostafa Khanzadi, Seyed Reza Hosseininia, and Saeed Moradi. Sustainable solutions: using MCDM to select optimal locations for green energy projects. *Journal of Cleaner Production*, 470:143320, 2025. DOI: https://doi.org/10.1016/j.jclepro.2024.143320.

[28] Maghsoud Amiri and Jurgita Antucheviciene. An area-based method for ranking interval type-2 fuzzy sets (EAMRIT-2F) for multi-criteria group decision-making. *E&M Economics and Management*, 19(4):21–34, 2016. DOI: https://doi.org/10.15240/tul/001/2016-4-002.

[29] Yi Zhou, Yu Chen, Xiang Li, and Rui Wang. Big data analysis for smart home energy management system based on IOMT using AHP and WASPAS. *Sustainable Energy Technologies and Assessments*, 66:103812, 2025. DOI: https://doi.org/10.1016/j.seta.2024.103812.

[30] Sajjad Ahmadi Sadabadi, Abdollah Hadi-Vencheh, Ali Jamshidi, and Mehdi Jalali. A linear programming technique to solve fuzzy multiple criteria decision making problems with an application. *RAIRO - Operations Research*, 55(1): 83–97, 2021.
DOI: https://doi.org/10.1051/ro/2020026.

[31] Mustafa Mudhafar Naser, Mohammad Jalali Varnamkhasti, Hawraa Jaber Mohammed, and Mahdi Aghajani. Artificial intelligence as a catalyst for operational excellence in iraqi industries: Implementation of a proposed model. *International Journal of Mathematical Modelling and Computations*, 14(2):101–117, 2024.
DOI: https://doi.org/10.71932/ijm.2024.1197656.

[32] M. Hassan and M. Jalali Varnamkhasti. Classical and urgencies assignment methods in p-median problems with fuzzy genetic algorithm. *Pakistan Journal of Statistics*, 31(5):643–651, 2015.

[33] Faezeh Jokar, Mohammad Jalali Varnamkhasti, and Abdollah Hadi-Vencheh. A new ELECTRE method based on left and right score for multicriteria decision-making. *Computational Intelligence and Neuroscience*, 2023:6414686, 2023.
DOI: https://doi.org/10.1155/2023/6414686.

[34] Ahmed Ghazi Alsaedi, Mohammad Jalali Varnamkhasti, Hawraa Jaber Mohammed, and Mahdi Aghajani. Integrating multi-criteria decision analysis with deep reinforcement learning: A novel framework for intelligent decision-making in iraqi industries. *International Journal of Mathematical Modelling and Computations*, 14(2):171–186, 2024.
DOI: https://doi.org/10.71932/ijm.2024.1197765.

[35] Faezeh Jokar, Mohammad Jalali Varnamkhasti, and Abdollah Hadi-Vencheh. Hybrid multi-criteria decision-making (MCDM) approaches with random forest regression for interval-based fuzzy uncertainty management. *International Journal of Mathematical Modelling and Computations*, 15(1):49–66, 2025.
DOI: https://doi.org/10.71932/ijm.2025.1200760.

[36] Murat Kirisci, Ibrahim Demir, and Necip Şimşek. A new integrated interval-valued fermatean fuzzy decision-making approach with green supply chain implementation. *Soft Computing*, 29:1–18, 2025.
DOI: https://doi.org/10.1007/s00500-024-09881-y.

[37] Lotfi A. Zadeh. Fuzzy sets. *Information and Control*, 8(3):338–353, 1965.
DOI: https://doi.org/10.1016/S0019-9958(65)90241-X.

[38] Jerry M. Mendel and Robert I. Bob John. Type-2 fuzzy sets made simple. *IEEE Transactions on Fuzzy Systems*, 10(2):117–127, 2002.
DOI: https://doi.org/10.1109/91.995115.

[39] Ting-Yu Chen and Chueh-Hsuan Li. Determining objective weights with intuitionistic fuzzy entropy measures: a comparative analysis. *Information Sciences*, 180(21):4207–4222, 2010.
DOI: https://doi.org/10.1016/j.ins.2010.07.006.

[40] Edmundas Kazimieras Zavadskas, Zenonas Turskis, Jurgita Antucheviciene, and Algimantas Zakarevicius. Optimization of weighted aggregated sum product assessment. *Elektronika ir Elektrotechnika*, 122(6):3–6, 2012.
DOI: https://doi.org/10.5755/j01.eee.122.6.1810.

[41] Cengiz Kahraman, Sezi Cevik Onar, and Basar Oztaysi. Fuzzy multicriteria decision-making: a literature review. *International Journal of Computational Intelligence Systems*, 8(4): 637–666, 2015.
DOI: https://doi.org/10.1080/18756891.2015.1046325.

[42] Erkan Celik, Alev Taskin Gumus, and Mehmet Alegoz. A trapezoidal type-2 fuzzy MCDM method for identifying and evaluating critical success factors in humanitarian relief logistics management. *Journal of Intelligent and Fuzzy Systems*, 27(6):2847–2855, 2014.
DOI: https://doi.org/10.3233/IFS-141231.

[43] Lazim Abdullah and Liana Najib. A new type-2 fuzzy set of linguistic variables for the fuzzy analytic hierarchy process. *Expert Systems with Applications*, 41(7):3297–3305, 2014.
DOI: https://doi.org/10.1016/j.eswa.2013.11.028.

[44] Shankar Chakraborty and Edmundas Kazimieras Zavadskas. Applications of WASPAS method in manufacturing decision making. *Informatica*, 25 (1):1–20, 2014.
DOI: https://doi.org/10.15388/Informatica.2014.01.

[45] Chia-Nan Wang, Chien-Yu Yang, and Hsin-Chuan Cheng. Fuzzy multi-criteria decision-making model for supplier evaluation and selection in a wind power plant project. *Mathematics*, 7(5):417, 2019.
DOI: https://doi.org/10.3390/math7050417.

[46] Milad Ashouri, Mohammad Gheibi, and Amir M. Fathollahi-Fard. An interval-valued fuzzy COPRAS method for risk reduction in decision-making problems. *Decision Science Letters*, 12(2):345–358, 2023.
DOI: https://doi.org/10.5267/j.dsl.2023.1.001.