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Analytical Solution of the Mathematical Model of Dengue Fever by the Laplace Adomian Decomposition Method

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Abstract

This paper presents and rigorously analyzes a deterministic mathematical model of the Dengue virus in a population, incorporating a nonlinear incidence function. The model considers five compartments: susceptible (S_h), symptomatic infection (I_h), asymptomatic infection (I_{hA}), recovered (R_h), and partial immunity(S_{hk}). The female mosquito population is divided into two compartments: susceptible (S_ν) and infected (I_ν).An algorithm is provided to calculate a series-type solution to the problem using the Laplace Adomian Decomposition technique. The convergence of this technique is also analyzed. Approximations of the solutionsfor various compartments are calculated using a few terms. The reliability and simplicity of the method are illustrated with numerical examples and plots The Laplace Adomian Decomposition algorithm is shown toyield very accurate approximate solutions using only a few iterations. Fourth-order Runge-Kutta solutionsare also compared with the solutions obtained by the Laplace decomposition scheme. Abstract

Keywords: Dengue fever, Existence and Uniqueness, LADM; Convergence analysis..
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1. Introduction

During the eighteenth and nineteenth centuries, the dengue virus rapidly spread to numerous large towns.across the world. Dengue fever is an ancient disease that was first identified in the Chinese region in 610 AD again in 922 AD[?]. Dengue disease has since spread from Southeast Asian nations, including Pakistan, India, SriLanka, and the Maldives[?]. This disease first appeared in its severe form in Manila between 1953 and 1954[?]. In a similar way, dengue was not recognized to exist in Africa.before 1980. Aedes aegypti and the dengue virus were both widely distributed in tropical and subtropical areas in 1997. Malaysia was reportedly one of its victims from 2000 to 2012. A survey was conducted in Thailand between 2000 and 2011 to analyze the dynamics of the dengue virus, with 40 out of 610 relevant searches meeting

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the inclusion criteria. The number of dengue cases rose in 2001, 2002, 2008, and 2010.Young children were more at risk of dying in these circumstances. Similarly, a survey conducted in Brazil between 2000 and 2010 found that 51 out of 714 reported related searches fulfilled the insertion criterion.The epidemiological modeling of dengue fever disease in Brazil is complex due to the impact of numerous socio-environmental factors. This research investigates a dengue virus incubation model in human and vector populations. The dengue virus does not immediately infect a susceptible person after being bitten by an infected vector. A person like this is referred to as an infected human at this time. The human dengue virus It takes around five days to incubate intrinsically. The susceptible vector will be an infected vector before becoming an infectious vector when it bites an infected human. However, the extrinsic incubation period for The dengue virus in the vector population is around 10 days old. Dengue fever has a major impact on morbidity.and mortality, affecting human health in addition to causing devastating economic losses. It is acknowledged.as the most dangerous infectious disease in the world. Some of the signs and symptoms of a dengue infection are mild to high fevers, headaches, slight foot swelling, skin rashes, bleeding, joint pain, circulatory failure,and death [**?**]. The Aedes aegypti mosquito, the main vector, spreads many dengue serotypes to the host.The dengue virus is spread when a mosquito bites an infected human host. If they bite another host, they are infected.Mosquitoes can transmit the dengue virus since they are contagious for the duration of their lives. Vertical Viral transmission between people and mosquitoes is extremely rare, according to previous work. People become more susceptible to hemorrhagic dengue fever or dengue shock syndrome after twelve weeks, based on the survey. According to data collected by the WHO, dengue fever affects between fifty and one hundred million people worldwide each year, with the major hosts of this viral virus being tropical and subtropical nations. One of them is Pakistan. There are four types of the dengue virus, and a person who recovers from one form of the dengue virus gains lifetime immunity to that type but not protection against the other.three types. There are 50. Each year, approximately 500,000 people with severe dengue require hospitalization, many of whom are children. The economic burden of dengue fever is also significant, with an estimated cost of 8.9 billion annually [**?**]. As such, it is critical to develop effective methods for controlling the spread of dengue fever. In addition to controlling the mosquito vector, several other approaches have been explored, including vaccines and antiviral drugs. However, the development of effective vaccines and Drug development has proven challenging due to the complexity of the dengue virus and its ability to evolve rapidly.Therefore, research in this area is ongoing, and there is a need for continued investment in efforts to develop new methods for preventing and treating dengue fever. Dengue fever is a serious infectious disease that has a significant impact on human health and the global economy. It has been spreading rapidly across the world.since the eighteenth and nineteenth centuries and is now considered the most dangerous infectious disease.in the world. Despite the significant progress made in understanding the virus and developing methods for controlling its spread, much remains to be done to effectively prevent and treat dengue fever. Continued Investment in research and public health efforts is essential to reduce the burden of this disease and improve the health and well-being of people worldwide. Our objective has been to show that approximate solutions of nonlinear differential equation systems can be easily obtained. The series solution of dengue fever is obtained by using the Laplace-Adomian Decomposition Method (LADM)[**?**]. It is clear that the Laplace-Adomian Decomposition algorithm yields very accurate, approximate solutions with only a few iterations few iterations [**?**]. The Laplace-Adomian decomposition method is a technique that combines the two concepts of the Laplace transform and the Adomian decomposition method [**?** **?**]. This method has some advantages over the previously adopted ones. We can obtain numerical solutions using this method without making any restrictive assumptions or using any discretization. This method is free from round-off errors and has a high convergence rate to the exact solution. Convergence analysis is also provided to demonstrate the efficiency of the method. Additionally, we applied the "Existence and Uniqueness" theorem to ensure that our solutions are well-defined. We also compared the LADM results with RK-4 to verify the efficiency of our proposed method."

2. Preliminaries

In this portion, we provide some definitions from [1, 3].

Definition 2.1. Laplace transform of function f(t) for t > 0 and integral over 0 to ∞. can be define as

$\mathcal{L}f(t) = F(s) = \int_0^\infty e^{st}\mathrm{f(t)dt}$

It is generally an integral transform that takes function f(t) and converts it into another function F(s) in new parameter s.

Definition 2.2. Adomian decomposition method define the solution by a series given by
$u = \sum_{k=0}^{\infty} u_k,$

and replacing the nonlinear term by the given series

$Q_u = \sum_{n=0}^{\infty} A_n,$

where A_n is Adomian polynomial and is computed as

$A_n(t) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\sum_{i=0}^{\infty}\lambda^i P_i \sum_{i=0}^{\infty}\lambda^i Q_i\right].$

Although Adomians aim is to find a method for combining linear and non-linear ordinary or partial differential equations for solving initial and boundary value problems in our paper, We shall deal only (ODE). The Adomian decomposition Method (ADM) involves separating the equation under investigation into linear and non-linear sections.

3. Model Formulation

To formulate the model, we divided the host population N_h into five epidemiological states: susceptible (S_h), symptomatic infection (I_h), asymptomatic infection (I_{hA}), and recovering (R_h). We divide the female mosquito population (N_v) into S_v and I_v. Recruitment and natural mortality of host and vector are considered μ_v and μ_v, respectively. Although the mortality rate from disease in humans is considered to be negligible, the birth and natural mortality rates of mosquitoes are assumed to have a common value of μ_v. To examine the effect of partial immunity on dengue infections, we include the susceptible class with partial immunity (S_{hk}). In order to keep the main assumptions of the proposed model, it should be noted that we do not differentiate between the infected individuals caused by various strains.
An infected vector's bites can cause a susceptible host to contract dengue. The probability of transmission, the number of bites, and the number of susceptible and infected individuals in each species all have an impact on the flows from the susceptible classes of both populations. While the per capita biting rate of mosquitoes "b" is the average number of bites per mosquito per day, the transmission probability is the chance that an infected bite would lead to a new case in a susceptible individual of a distinct species. We show the probability of transmission from (S_h) and (S_{hk}) to mosquitoes by β_h and β_{hk}, and from (S_v) to people via β_v. We suppose that $\beta_h > \beta_{hk}$ indicates that susceptible with partial immunity have a lesser chance of transmission than susceptible with no immunity. We understand $(\frac{b}{N_h})$ as the number of mosquito bites a human receives from each one, and then $(\frac{b\beta_h}{N_h}I_v)$, $(\frac{b\beta_{hk}}{N_h}I_v)$ and $(\frac{b\beta_v}{N_h}I_h)$ represent the infection rates per susceptible human and each susceptible vector. Therefore, the initial infection from susceptible S_h and the second infection from susceptible S_{hk} are both included in the infected class (I_h). A percentage of $\nu$the recovered persons (R_h) lose immunity and join the susceptible class with just partial immunity (S_{hk}). The system of differential equations that explains the dynamics of dengue virus transmission is given according to the above assumptions.

$$
\begin{aligned}
&S_h = \mu_h N_h - \frac{\beta_h b}{N_h} S_h I_v - \mu_h S_h,\\
&S_{hk} = -\frac{\beta_{hk} b}{N_h} S_{hk} I_v - \mu_h S_{hk} + \nu R_h,\\
&I_h = (1-\psi)\frac{\beta_h b}{N_h} S_h I_v - (\mu_h + \tau + \gamma) I_h + \frac{\beta_{hk} b}{N_h} S_{hk} I_v, ,\\
&I_{hA} = \psi \frac{\beta_h b}{N_h} S_h I_v - (\mu_h + \gamma) I_{hA},\\
&R_h = \gamma(I_h + I_{hA}) + \tau I_h - (\nu + \mu_h) R_h,\\
&S_v = \mu_v N_v - \frac{\beta_h b}{N_h}(I_h + I_{hA}) S_v - \mu_v S_v,\\
&I_v = \frac{\beta_v b}{N_h} I_h + I_{hA} S_v - \mu_v I_v.
\end{aligned}
\tag{3.1}
$$

with initial condition

$$
n_1 = n_1, S_{hk}(0) = n_2, I_h(0) = n_3, , I_{hA}(0) = n_4, R_h(0) = n_5, S_v(0) = n_6, I_v(0) = n_7 \tag{3.2}
$$

4. Basic properties of the model

4.1. Well-posedness

The model (3.1) possess a unique solution due to the non-negative values of its parameters along with the positive initial conditions for time $t > 0$, while the classes $S_h, S_{hk}, I_h, I_{hA}, R_h, S_v$ and I_v are all positive and bounded.

4.2. Existence and Uniqueness

Consider the model , we write it in the form of $\dot{\theta} = \pi(\theta)$.

$$
\dot{\varpi} = \begin{pmatrix} \pi_1(t,\theta)\\ \pi_2(t,\theta)\\ \pi_3(t,\theta)\\ \pi_4(t,\theta)\\ \pi_5(t,\theta)\\ \pi_6(t,\theta)\\ \pi_7(t,\theta) \end{pmatrix} = \begin{pmatrix} \mu_h N_h - \frac{\beta_h b}{N_h} S_h I_v - \mu_h S_h\\ -\frac{\beta_{hk} b}{N_h} S_{hk} I_v - \mu_h S_{hk} + \nu R_h\\ (1-\psi)\frac{\beta_h b}{N_h} S_h I_v - (\mu_h + \tau + \gamma) I_h + \frac{\beta_{hk} b}{N_h} S_{hk} I_v\\ \psi \frac{\beta_h b}{N_h} S_h I_v - (\mu_h + \gamma) I_{hA}\\ \gamma(I_h + I_{hA}) + \tau I_h - (\nu + \mu_h) R_h\\ \mu_v N_v - \frac{\beta_h b}{N_h}(I_h + I_{hA}) S_v - \mu_v S_v\\ \frac{\beta_v b}{N_h}(I_h + I_{hA}) S_v - \mu_v I_v \end{pmatrix}. \tag{4.1}
$$

Taking the first partial derivatives w.r.t the state variables of $\dot{\theta}$, which is given by:

$$
\frac{\partial}{\partial z}\dot{\theta} = \frac{\partial}{\partial z}\pi(\theta), \tag{4.2}
$$

where z is used in general, implies that:

$$
\frac{\partial}{\partial z}\pi(\theta) = \frac{\partial}{\partial z}\pi(t,\theta) \tag{4.3}
$$

Implies that,

$$\begin{pmatrix} \frac{\partial}{\partial S_h}\pi(t,\theta) \\ \frac{\partial}{\partial S_{hk}}\pi(t,\theta) \\ \frac{\partial}{\partial I_h}\pi(t,\theta) \\ \frac{\partial}{\partial I_{hA}}\pi(t,\theta) \\ \frac{\partial}{\partial R_h}\pi(t,\theta) \\ \frac{\partial}{\partial S_v}\pi(t,\theta) \\ \frac{\partial}{\partial I_v}\pi(t,\theta) \end{pmatrix} = \begin{pmatrix} -\frac{\beta_h b}{N_h} I_v - \mu_h \\ -\frac{\beta_{hk} b}{N_h} I_v - \mu_h \\ -(\mu_h + \tau + \gamma) \\ -(\mu_h + \gamma) I_{hA} \\ -(\nu + \mu_h) \\ -\frac{\beta_h b}{N_h}(I_h + I_{hA}) - \mu_v \\ -\mu_v \end{pmatrix}. \tag{4.4}$$

Clearly, see that the partial derivatives are continues such that:

$$\frac{\partial \zeta}{\partial z} \to \text{Continuous in } \mathbb{R}^n. \tag{4.5}$$

Thus, according to Theorem the model has an unique continuous solution. (3.1).

4.3. Positivity

Theorem 4.1. *Consider the model (3.1), the state variables $S_h, S_{hk}, I_h, I_{hA}, R_h, S_v$ and I_v and its none negative initial conditions are positive $\forall t > 0$.*

Proof. Consider the first equation of the model (3.1), assume the positive initial condition such that $S(0) > 0$ then it follows that,

$$\frac{dS_h}{dt} = -\left(\frac{\beta_h I_v}{N_h} + \mu_h\right) S_h. \tag{4.6}$$

Integrate the equation (4.6), implies that,

$$\ln(S_h) = -\mu_h t - \int_0^t \frac{\beta_h I_v}{N_h} dt + C, \tag{4.7}$$

Applying the initial condition,

$$S_h(t) \geq S_h(0) \exp\left(-\mu_h - \int_0^t \frac{\beta_h b I_v}{N_h} dt\right). \quad \forall t > 0 \tag{4.8}$$

Second equation implies that,

$$S_{hk}(t) \geq S_{hk}(0) \exp\left(-\mu_h t - \int_0^t \frac{\beta_{hk} b I_v}{N_h} dt\right), \quad \forall t > 0 \tag{4.9}$$

third equation,

$$\ln(I_h) \geq I_h(0) \exp\left(-(\mu_h + \tau + \gamma)t - \int_0^t \psi \frac{\beta_h b S_h}{N_h} dt\right), \quad \forall t > 0 \tag{4.10}$$

fourth equation,

$$\ln(I_{hA}) \geq I_{hA}(0) \exp\left(-(\mu_h + \gamma)t\right), \quad \forall t > 0 \tag{4.11}$$

fifth equation,

$$\ln(R_h) \geq R_h(0) \exp\left(-(\nu + \mu_h)t\right), \quad \forall t > 0 \tag{4.12}$$

sixth equation,

$$\ln(S_v) \geq S_v(0) \exp\left(-\mu_h t - \int_0^t \frac{\beta_h b}{N_h}(I_h + I_{hA}) dt\right), \quad \forall t > 0 \tag{4.13}$$

and, the last equation becomes,

$$I_v(t) \geq I_v(0) \exp\left(\mu_v\right). \quad \forall t > 0. \tag{4.14}$$

From equations (4.8)–(4.14) we concluded that the solution to the model (3.1) is positive $\forall t > 0$. This completes the proof. □

5. The Laplace Adomian Decomposition Method

This section presents the application of the Laplace-Adomian Decomposition Method to nonlinear ordinary differential systems. To start with, the Laplace transformation, denoted by L in this paper, is applied to both sides of the equation (3.1).

$$
\begin{aligned}
&\mathcal{L}\{\frac{dS_h}{dt}\} = \mathcal{L}\{\mu_h N_h\} - \mathcal{L}\{\frac{\beta_h b}{N_h} S_h I_v\} - \mathcal{L}\{\mu_h S_h\},\\
&\mathcal{L}\{\frac{dS_{hk}}{dt}\} = -\mathcal{L}\{\frac{\beta_{hk} b}{N_h} S_{hk} I_v\} - \mathcal{L}\{\mu_h S_{hk}\} + \mathcal{L}\{\nu R_h\},\\
&\mathcal{L}\{\frac{dI_h}{dt}\} = \mathcal{L}\{(1-\psi)\frac{\beta_h b}{N_h} S_h I_v\} - \mathcal{L}\{(\mu_h+\tau+\gamma) I_h\} + \mathcal{L}\{\frac{\beta_{hk} b}{N_h} S_{hk} I_v\},\\
&\mathcal{L}\{\frac{dI_{hA}}{dt}\} = \mathcal{L}\{\psi\frac{\beta_h b}{N_h} S_h I_v\} - \mathcal{L}\{(\mu_h+\gamma) I_{hA}\},\\
&\mathcal{L}\{\frac{dR_h}{dt}\} = \mathcal{L}\{\gamma(I_h + I_{hA})\} + \mathcal{L}\{\tau I_h - (\nu+\mu_h) R_h\},\\
&\mathcal{L}\{\frac{dS_v}{dt}\} = \mathcal{L}\{\mu_v N_v\} - \mathcal{L}\{\frac{\beta_h b}{N_h}(I_h\} + I_{hA}) S_v - \mathcal{L}\{\mu_v S_v\},\\
&\mathcal{L}\{\frac{dI_v}{dt}\} = \mathcal{L}\{\frac{\beta_v b}{N_h} I_h\} + \mathcal{L}\{I_{hA} S_v\} - \mathcal{L}\{\mu_v I_v\}.
\end{aligned}
\tag{5.1}
$$

Implies that

$$
\begin{aligned}
&s\mathcal{L}\{S_h\} - n_1 = \frac{\mu_h N_h}{s} - \frac{\beta_h b}{N_h}\mathcal{L}\{S_h I_v\} - \mu_h \mathcal{L}\{S_h\},\\
&s\mathcal{L}\{S_{hk}\} - S_{hk}(0) = -\frac{\beta_{hk} b}{N_h}\mathcal{L}\{S_{hk} I_v\} - \mu_h \mathcal{L}\{S_{hk}\} + \nu\mathcal{L}\{R_h\},\\
&s\mathcal{L}\{I_h\} - I_h(0) = (1-\psi)\frac{\beta_h b}{N_h}\mathcal{L}\{S_h I_v\} - (\mu_h+\tau+\gamma)\mathcal{L}\{I_h\} + \frac{\beta_{hk} b}{N_h}\mathcal{L}\{S_{hk} I_v\},\\
&s\mathcal{L}\{I_{hA}\} - I_{hA}(0) = \mathcal{L}\{S_h I_v\} - (\mu_h+\gamma)\mathcal{L}\{I_{hA}\},\\
&s\mathcal{L}\{R_h\} - R_h(0) = \gamma\mathcal{L}\{I_h\} + \gamma\mathcal{L}\{I_{hA}\} + \tau\mathcal{L}\{I_h\} - (\nu+\mu_h)\mathcal{L}\{R_h\},\\
&s\mathcal{L}\{S_v\} - S_v(0) = \frac{\mu_v N_v}{s} - \frac{\beta_h b}{N_h}\mathcal{L}\{I_h S_v) - \mathcal{L}\{I_{hA} S_v\} - \mu_v\mathcal{L}\{S_v\},\\
&s\mathcal{L}\{I_v\} - I_v(0) = \frac{\beta_v b}{N_h}\mathcal{L}\{I_h\} + \mathcal{L}\{I_{hA} S_v\} - \mu_v\mathcal{L}\{I_v\}.
\end{aligned}
\tag{5.2}
$$

Using the initial conditions 3.2, we have

$$
\begin{aligned}
&\mathcal{L}\{S_h\} = \frac{n_1}{s} + \frac{\mu_h N_h}{s^2} - \frac{\beta_h b}{sN_h}\mathcal{L}\{S_h I_v\} - \frac{\mu_h}{s}\mathcal{L}\{S_h\},\\
&\mathcal{L}\{S_{h_k}\} = \frac{n_2}{s} - \frac{\beta_{h_k} b}{sN_h}\mathcal{L}\{S_{h_k} I_v\} - \frac{\mu_h}{s}\mathcal{L}\{S_{hk}\} + \frac{\nu}{s}\mathcal{L}\{R_h\},\\
&\mathcal{L}\{I_h\} = \frac{n_3}{s} + (1-\psi)\frac{\beta_h b}{sN_h}\mathcal{L}\{S_h I_v\} - \frac{(\mu_h+\tau+\gamma)}{s}\mathcal{L}\{I_h\} + \frac{\beta_{h_k} b}{sN_h}\mathcal{L}\{S_{hk} I_v\},\\
&\mathcal{L}\{I_{hA}\} = \frac{n_4}{s} + \frac{1}{s}\mathcal{L}\{S_h I_v\} - \frac{(\mu_h+\gamma)}{s}\mathcal{L}\{I_{hA}\},\\
&\mathcal{L}\{R_h\} = \frac{n_5}{s} + \frac{\gamma}{s}\mathcal{L}\{I_h\} + \frac{\gamma}{s}\mathcal{L}\{I_{hA}\} + \frac{\tau}{s}\mathcal{L}\{I_h\} - \frac{(\nu+\mu_h)}{s}\mathcal{L}\{R_h\},\\
&\mathcal{L}\{S_v\} = \frac{n_6}{s} + \frac{\mu_v N_v}{s^2} - \frac{\beta_h b}{sN_h}\mathcal{L}\{I_h S_v) - \mathcal{L}\{I_{hA} S_v\} - \frac{\mu_v}{s}\mathcal{L}\{S_v\},\\
&\mathcal{L}\{I_v\} = \frac{n_7}{s} + \frac{\beta_v b}{sN_h}\mathcal{L}\{I_h\} + \frac{1}{s}\mathcal{L}\{I_{hA} S_v\} - \frac{\mu_v}{s}\mathcal{L}\{I_v\}.
\end{aligned}
\tag{5.3}
$$

or

$$
\begin{aligned}
\mathcal{L}\{S_h\} &= \frac{n_1}{s} + \frac{\mu_h N_h}{s^2} - \frac{\beta_h b}{sN_h}\mathcal{L}\{A\} - \frac{\mu_h}{s}\mathcal{L}\{S_h\},\\
\mathcal{L}\{S_{h_k}\} &= \frac{n_2}{s} - \frac{\beta_{h_k} b}{sN_h}\mathcal{L}\{B\} - \frac{\mu_h}{s}\mathcal{L}\{S_{hk}\} + \frac{\nu}{s}\mathcal{L}\{R_h\},\\
\mathcal{L}\{I_h\} &= \frac{n_3}{s} + (1-\psi)\frac{\beta_h b}{sN_h}\mathcal{L}\{A\} - \frac{(\mu_h+\tau+\gamma)}{s}\mathcal{L}\{I_h\} + \frac{\beta_{h_k} b}{sN_h}\mathcal{L}\{B\},\\
\mathcal{L}\{I_{hA}\} &= \frac{n_4}{s} + \frac{1}{s}\mathcal{L}\{A\} - \frac{(\mu_h+\gamma)}{s}\mathcal{L}\{I_{hA}\},\\
\mathcal{L}\{R_h\} &= \frac{n_5}{s} + \frac{\gamma}{s}\mathcal{L}\{I_h\} + \frac{\gamma}{s}\mathcal{L}\{I_{hA}\} + \frac{\tau}{s}\mathcal{L}\{I_h\} - \frac{(\nu+\mu_h)}{s}\mathcal{L}\{R_h\},\\
\mathcal{L}\{S_v\} &= \frac{n_6}{s} + \frac{\mu_v N_v}{s^2} - \frac{\beta_h b}{sN_h}\mathcal{L}\{C) - \mathcal{L}\{D\} - \frac{\mu_v}{s}\mathcal{L}\{S_v\},\\
\mathcal{L}\{I_v\} &= \frac{n_7}{s} + \frac{\beta_v b}{sN_h}\mathcal{L}\{I_h\} + \frac{1}{s}\mathcal{L}\{D\} - \frac{\mu_v}{s}\mathcal{L}\{I_v\}.
\end{aligned}
\tag{5.4}
$$

Assuming that the solutions $S_h, S_{hk}, I_h, I_{hA}, R_h, S_\nu, I_\nu$ in the form of infinite series given by

$$
S_h = \sum_{n=0}^{\infty} S_{h_n},\ S_{hk} = \sum_{n=0}^{\infty} S_{hk},\ I_h = \sum_{n=0}^{\infty} I_{h_n},\ S_{hA} = \sum_{n=0}^{\infty} S_{h_n A_n}\ , R_h = \sum_{n=0}^{\infty} R_{h_n},\ S_\nu = \sum_{n=0}^{\infty} S_{\nu_n}, I_\nu = \sum_{n=0}^{\infty} I_{\nu_n}. \tag{5.5}
$$

and the nonlinear terms are involved in the model are $S_hI_\nu, S_{hk}I_\nu, I_hS_\nu, S_\nu I_h$, are decomposed by Adomian polynomial as

$$
A = \sum_{n=0}^{\infty} A_n, \quad B = \sum_{n=0}^{\infty} B_n, \quad C = \sum_{n=0}^{\infty} C_n, \quad D = \sum_{n=0}^{\infty} D_n. \tag{5.6}
$$

where A_n, B_n, C_n, D_n are Adomian polynomials defined as

$A_n(t) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\sum_{i=0}^{\infty}\lambda^i S_{h_i}\sum_{i=0}^{\infty}\lambda^i I_{\nu_i}\right]$, $\qquad B_n(t) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\sum_{i=0}^{\infty}\lambda^i S_{hk_i}\sum_{i=0}^{\infty}\lambda^i I_{\nu_i}\right]$,

$C_n(t) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\sum_{i=0}^{\infty}\lambda^i I_{h_i}\sum_{i=0}^{\infty}\lambda^i S_{\nu_i}\right]$, $\qquad D_n(t) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\sum_{i=0}^{\infty}\lambda^i I_{hA_i}\sum_{i=0}^{\infty}\lambda^i S_{\nu_i}\right]$.

$$
\begin{aligned}
A_0 &= S_{h_0}I_{\nu_0}, & B_0 &= S_{h_0k_0}I_{\nu_0},\\
A_1 &= S_{h_0}I_{\nu_1} + S_{h_1}I_{\nu_0}, & B_1 &= S_{h_0k_0}I_{\nu_1} + S_{h_1k_1}I_{\nu_0},\\
A_2 &= S_{h_0}I_{\nu_2} + S_{h_1}I_{\nu_1} + S_{h_2}I_{\nu_0}, & B_2 &= S_{h_0k_0}I_{\nu_2} + h_1k_1I_{\nu_1} + S_{h_2k_2}I_{\nu_0},\\
A_3 &= S_{h_0}I_{\nu_3} + S_{h_1}I_{\nu_2} + S_{h_3}I_{\nu_0} + S_{h_0}I_{\nu_3}, & B_3 &= S_{h_0k_0}I_{\nu_3} + S_{h_1k_1}I_{\nu_2} + S_{h_2k_2}I_{\nu_1} + S_{h_3k_3}I_{\nu_0}\\
A_4 &= S_{h_0}I_{\nu_4} + S_{h_1}I_{\nu_3} + S_{h_2}I_{\nu_2} + S_{h_3}I_{\nu_1} + S_{h_4}I_{\nu_0}, & B_4 &= S_{h_0k_0}I_{\nu_4} + S_{h_1k_1}I_{\nu_3} + S_{h_2k_2}I_{\nu_2} + S_{h_3k_3}I_{\nu_1} + S_{h_4k_4}I_{\nu_0},
\end{aligned}
$$

$$
\begin{aligned}
C_0 &= S_{h_0}I_{\nu_0}, & D_0 &= I_{h_0A_0}S_{\nu_0},\\
C_1 &= S_{h_0}I_{\nu_1} + S_{h_1}I_{\nu_0}, & D_1 &= I_{h_0A_0}S_{\nu_1} + I_{h_1A_1}S_{\nu_0},\\
C_2 &= S_{h_0}I_{\nu_2} + S_{h_1}I_{\nu_0} + S_{h_2}I_{\nu_0}, & D_2 &= I_{h_0A_0}S_{\nu_2} + I_{h_1A_1}S_{\nu_1} + I_{h_2A_2}S_{\nu_0},\\
C_3 &= S_{h_0}I_{\nu_3} + S_{h_1}I_{\nu_2} + S_{h_2}I_{\nu_1} + S_{h_3}I_{\nu_0}, & D_3 &= I_{h_0A_0}S_{\nu_3} + I_{h_1A_1}S_{\nu_2} + I_{h_2A_2}S_{\nu_1} + I_{h_3A_3}S_{\nu_0},\\
C_4 &= S_{h_0}I_{\nu_4} + S_{h_1}I_{\nu_3} + S_{h_2}I_{\nu_2} + S_{h_3}I_{\nu_1} + S_{h_4}I_{\nu_0}, & D_4 &= I_{h_0A_0}S_{\nu_4} + I_{h_1A_1}S_{\nu_3} + I_{h_2A_2}S_{\nu_2} + I_{h_3A_3}S_{\nu_1} + I_{h_4A_4}S_{\nu_0}.
\end{aligned}
$$

Plug (5.5) & (5.6) (5.4)

$$
\begin{aligned}
&\mathcal{L}\{\sum_{n=0}^{\infty} S_{h_n}\} = \frac{n_1}{s} + \frac{\mu_h N_h}{s^2} - \frac{\beta_h b}{sN_h}\mathcal{L}\{A_n\} - \frac{\mu_h}{s}\mathcal{L}\{\sum_{n=0}^{\infty} S_{h_n}\},\\
&\mathcal{L}\{\sum_{n=0}^{\infty} S_{h_n k_n}\} = \frac{n_2}{s} - \frac{\beta_{hk} b}{sN_h}\mathcal{L}\{B_n\} - \frac{\mu_h}{s}\mathcal{L}\{\sum_{n=0}^{\infty} S_{h_n k_n}\} + \frac{\nu}{s}\mathcal{L}\{\sum_{n=0}^{\infty} R_{h_n}\},\\
&\mathcal{L}\{\sum_{n=0}^{\infty} I_{h_n}\} = \frac{n_3}{s} + (1-\psi)\frac{\beta_h b}{sN_h}\mathcal{L}\{A_n\} - \frac{(\mu_h+\tau+\gamma)}{s}\mathcal{L}\{\sum_{n=0}^{\infty} I_{h_n}\} + \frac{\beta_{hk} b}{sN_h}\mathcal{L}\{B_n\},\\
&\mathcal{L}\{\sum_{n=0}^{\infty} I_{h_n A_n}\} = \frac{I_{hA}(0)}{s} + \frac{1}{s}\mathcal{L}\{A_n\} - \frac{(\mu_h+\gamma)}{s}\mathcal{L}\{\sum_{n=0}^{\infty} I_{h_n A_n}\},\\
&\mathcal{L}\{\sum_{n=0}^{\infty} R_{h_n}\} = \frac{R_h(0)}{s} + \frac{\gamma}{s}\mathcal{L}\{\sum_{n=0}^{\infty} I_{h_n}\} + \frac{\gamma}{s}\mathcal{L}\{\sum_{n=0}^{\infty} I_{h_n A_n}\} + \frac{\tau}{s}\mathcal{L}\{\sum_{n=0}^{\infty} I_{h_n}\} - \frac{(\nu+\mu_h)}{s}\mathcal{L}\{\sum_{n=0}^{\infty} R_{h_n}\},\\
&\mathcal{L}\{\sum_{n=0}^{\infty} S_{v_n}\} = \frac{S_v(0)}{s} + \frac{\mu_v N_v}{s^2} - \frac{\beta_h b}{sN_h}\mathcal{L}\{C_n) - \mathcal{L}\{D_n\} - \frac{\mu_v}{s}\mathcal{L}\{\sum_{n=0}^{\infty} S_{v_n}\},\\
&\mathcal{L}\{\sum_{n=0}^{\infty} I_{v_n}\} = \frac{I_v(0)}{s} + \frac{\beta_v b}{sN_h}\mathcal{L}\{\sum_{n=0}^{\infty} I_{h_n}\} + \frac{1}{s}\mathcal{L}\{D_n\} - \frac{\mu_v}{s}\mathcal{L}\{\sum_{n=0}^{\infty} I_{v_n}\}.
\end{aligned}
\tag{5.7}
$$

Matching the two sides of (5.7) yields the following iterative algorithm:

$$
\begin{aligned}
&\mathcal{L}\{S_{h_0}\} = \frac{n_1}{s} + \frac{\mu_h N_h}{s^2},\\
&\mathcal{L}\{S_{h_1}\} = -\frac{\beta_h b}{sN_h}\mathcal{L}\{A_0\} - \frac{\mu_h}{s}\mathcal{L}\{S_{h_0}\},\\
&\mathcal{L}\{S_{h_2}\} = -\frac{\beta_h b}{sN_h}\mathcal{L}\{A_1\} - \frac{\mu_h}{s}\mathcal{L}\{S_{h_1}\},\\
&\mathcal{L}\{S_{h_3}\} = -\frac{\beta_h b}{sN_h}\mathcal{L}\{A_0\} - \frac{\mu_h}{s}\mathcal{L}\{S_{h_0}\},\\
&\vdots\\
&\mathcal{L}\{S_{h_{n+1}}\} = -\frac{\beta_h b}{sN_h}\mathcal{L}\{A_n\} - \frac{\mu_h}{s}\mathcal{L}\{S_{h_n}\},
\end{aligned}
\tag{5.8}
$$

$$
\begin{aligned}
&\mathcal{L}\{S_{h_0 k_0}\} = \frac{n_2}{s},\\
&\mathcal{L}\{S_{h_1 k_1}\} = -\frac{\beta_{hk} b}{sN_h}\mathcal{L}\{B_0\} - \frac{\mu_h}{s}\mathcal{L}\{S_{h_0 k_0}\} + \frac{\nu}{s}\mathcal{L}\{R_{h_0}\},\\
&\mathcal{L}\{S_{h_2 k_2}\} = -\frac{\beta_{hk} b}{sN_h}\mathcal{L}\{B_1\} - \frac{\mu_h}{s}\mathcal{L}\{S_{h_1 k_1}\} + \frac{\nu}{s}\mathcal{L}\{R_{h_1}\},\\
&\mathcal{L}\{S_{h_3 k_3}\} = -\frac{\beta_{hk} b}{sN_h}\mathcal{L}\{B_2\} - \frac{\mu_h}{s}\mathcal{L}\{S_{h_2 k_2}\} + \frac{\nu}{s}\mathcal{L}\{R_{h_2}\},\\
&\vdots\\
&\mathcal{L}\{S_{h_{n+1} k_{n+1}}\} = -\frac{\beta_{hk} b}{sN_h}\mathcal{L}\{B_n\} - \frac{\mu_h}{s}\mathcal{L}\{S_{h_n k_n}\} + \frac{\nu}{s}\mathcal{L}\{R_{h_n}\},
\end{aligned}
\tag{5.9}
$$

$$
\begin{aligned}
&\mathcal{L}\{I_{h_0}\} = \frac{n_3}{s},\\
&\mathcal{L}\{I_{h_1}\} = (1-\psi)\frac{\beta_h b}{sN_h}\mathcal{L}\{A_0\} - \frac{(\mu_h+\tau+\gamma)}{s}\mathcal{L}\{I_{h_0}\} + \frac{\beta_{hk} b}{sN_h}\mathcal{L}\{B_0\},\\
&\mathcal{L}\{I_{h_2}\} = (1-\psi)\frac{\beta_h b}{sN_h}\mathcal{L}\{A_1\} - \frac{(\mu_h+\tau+\gamma)}{s}\mathcal{L}\{I_{h_1}\} + \frac{\beta_{hk} b}{sN_h}\mathcal{L}\{B_1\},\\
&\mathcal{L}\{I_{h_3}\} = (1-\psi)\frac{\beta_h b}{sN_h}\mathcal{L}\{A_2\} - \frac{(\mu_h+\tau+\gamma)}{s}\mathcal{L}\{I_{h_2}\} + \frac{\beta_{hk} b}{sN_h}\mathcal{L}\{B_2\},\\
&\vdots\\
&\mathcal{L}\{I_{h_{n+1}}\} = (1-\psi)\frac{\beta_h b}{sN_h}\mathcal{L}\{A_n\} - \frac{(\mu_h+\tau+\gamma)}{s}\mathcal{L}\{I_{h_n}\} + \frac{\beta_{hk} b}{sN_h}\mathcal{L}\{B_n\},
\end{aligned}
\tag{5.10}
$$

$$
\begin{aligned}
&\mathcal{L}\{I_{h_0 A_0}\} = \frac{n_4}{s},\\
&\mathcal{L}\{I_{h_1 A_1}\} = \frac{1}{s}\mathcal{L}\{A_0\} - \frac{(\mu_h+\gamma)}{s}\mathcal{L}\{I_{h_0 A_0}\},\\
&\mathcal{L}\{I_{h_2 A_2}\} = \frac{1}{s}\mathcal{L}\{A_1\} - \frac{(\mu_h+\gamma)}{s}\mathcal{L}\{I_{h_1 A_1}\},\\
&\mathcal{L}\{I_{h_3 A_3}\} = \frac{1}{s}\mathcal{L}\{A_2\} - \frac{(\mu_h+\gamma)}{s}\mathcal{L}\{I_{h_2 A_2}\},\\
&\vdots\\
&\mathcal{L}\{I_{h_{n+1} A_{n+1}}\} = \frac{1}{s}\mathcal{L}\{A_n\} - \frac{(\mu_h+\gamma)}{s}\mathcal{L}\{I_{h_n A_n}\},
\end{aligned}
\tag{5.11}
$$

$$
\begin{aligned}
&\mathcal{L}\{R_{h_0}\} = \frac{n_5}{s},\\
&\mathcal{L}\{R_{h_1}\} = \frac{\gamma}{s}\mathcal{L}\{I_{h_0}\} + \frac{\gamma}{s}\mathcal{L}\{I_{h_0 A_0}\} + \frac{\tau}{s}\mathcal{L}\{I_{h_0}\} - \frac{(\nu+\mu_h)}{s}\mathcal{L}\{R_{h_0}\},\\
&\mathcal{L}\{R_{h_2}\} = \frac{\gamma}{s}\mathcal{L}\{I_{h_1}\} + \frac{\gamma}{s}\mathcal{L}\{I_{h_1 A_1}\} + \frac{\tau}{s}\mathcal{L}\{I_{h_1}\} - \frac{(\nu+\mu_h)}{s}\mathcal{L}\{R_{h_1}\},\\
&\mathcal{L}\{R_{h_3}\} = \frac{\gamma}{s}\mathcal{L}\{I_{h_2}\} + \frac{\gamma}{s}\mathcal{L}\{I_{h_2 A_2}\} + \frac{\tau}{s}\mathcal{L}\{I_{h_2}\} - \frac{(\nu+\mu_h)}{s}\mathcal{L}\{R_{h_2}\},\\
&\vdots\\
&\mathcal{L}\{R_{h_{n+1}}\} = \frac{\gamma}{s}\mathcal{L}\{I_{h_n}\} + \frac{\gamma}{s}\mathcal{L}\{I_{h_n A_n}\} + \frac{\tau}{s}\mathcal{L}\{I_{h_n}\} - \frac{(\nu+\mu_h)}{s}\mathcal{L}\{R_{h_n}\},
\end{aligned}
\tag{5.12}
$$

$$
\begin{aligned}
&\mathcal{L}\{S_{v_0}\} = \frac{n_6}{s} + \frac{\mu_\nu N_\nu}{s^2},\\
&\mathcal{L}\{S_{v_1}\} = -\frac{\beta_h b}{sN_h}\mathcal{L}\{C_0) - \mathcal{L}\{D_0\} - \frac{\mu_v}{s}\mathcal{L}\{S_{v_0}\},\\
&\mathcal{L}\{S_{v_2}\} = -\frac{\beta_h b}{sN_h}\mathcal{L}\{C_1) - \mathcal{L}\{D_1\} - \frac{\mu_v}{s}\mathcal{L}\{S_{v_1}\},\\
&\mathcal{L}\{S_{v_3}\} = -\frac{\beta_h b}{sN_h}\mathcal{L}\{C_2) - \mathcal{L}\{D_2\} - \frac{\mu_v}{s}\mathcal{L}\{S_{v_2}\},\\
&\vdots\\
&\mathcal{L}\{S_{v_{n+1}}\} = -\frac{\beta_h b}{sN_h}\mathcal{L}\{C_n) - \mathcal{L}\{D_n\} - \frac{\mu_v}{s}\mathcal{L}\{S_{v_n}\},
\end{aligned}
\tag{5.13}
$$

$$
\begin{aligned}
&\mathcal{L}\{I_{v_0}\} = \frac{n_7}{s},\\
&\mathcal{L}\{I_{v_1}\} = \frac{\beta_v b}{sN_h}\mathcal{L}\{I_{h_0}\} + \frac{1}{s}\mathcal{L}\{D_0\} - \frac{\mu_v}{s}\mathcal{L}\{I_{v_0}\},\\
&\mathcal{L}\{I_{v_2}\} = \frac{\beta_v b}{sN_h}\mathcal{L}\{I_{h_1}\} + \frac{1}{s}\mathcal{L}\{D_1\} - \frac{\mu_v}{s}\mathcal{L}\{I_{v_1}\},\\
&\mathcal{L}\{I_{v_3}\} = \frac{\beta_v b}{sN_h}\mathcal{L}\{I_{h_2}\} + \frac{1}{s}\mathcal{L}\{D_2\} - \frac{\mu_v}{s}\mathcal{L}\{I_{v_2}\},\\
&\vdots,\\
&\mathcal{L}\{I_{v_{n+1}}\} = \frac{\beta_v b}{sN_h}\mathcal{L}\{I_{h_n}\} + \frac{1}{s}\mathcal{L}\{D_n\} - \frac{\mu_v}{s}\mathcal{L}\{I_{v_n}\}.
\end{aligned}
\tag{5.14}
$$

Applying the inverse Laplace transform to the first equations in (5.8)(5.14), we have

$$
\begin{aligned}
&S_{h_0} = n_1 + \mu_h N_h t, \qquad S_{h_0 k_0} = n_2, \qquad I_{h_0} = n_3, \qquad I_{h_0 A_0} = n_4, \qquad R_{h_0} = n_5,\\
&S_{\nu_0} = n_6 + \mu_\nu N_\nu t, \qquad I_{v_0} = n_7.
\end{aligned}
\tag{5.15}
$$

Substituting these values into the second equations in (5.8)- (5.14) gives

$$
\begin{aligned}
&\mathcal{L}\{S_{h_1}\} = -\frac{\mu_h n_1}{s} - \frac{1}{s^2}(\frac{\beta_h n_1 n_7}{N_h} + \mu^2 N_h) - \frac{\beta_h b \mu_h N_h I_v(0)}{s^3 N_h},\\
&\mathcal{L}\{S_{h_1 k_1}\} = -\frac{\mu_h n_2 + n_5}{s} - \frac{\beta_{hk} b n_2 n_7}{s^2 N_h},\\
&\mathcal{L}\{I_{h_1}\} = -\frac{(\mu+\tau+\gamma) n_3}{s} + \frac{\beta_{hk} n_2 n_7}{s^2 N_h} + \frac{(1-\psi)\beta_h b n_1 n_7}{s^3 N_h} + \frac{(1-\psi)\mu_h \beta_h b n_1 n_7}{s^4 N_h},\\
&\mathcal{L}\{I_{h_1 A_1}\} = -\frac{(\mu_h+\gamma) n_4}{s} + \frac{n_1 n_7}{s^2} + \frac{\mu_h N_h I\nu(0)}{s^3},\\
&\mathcal{L}\{R_{h_1}\} = \frac{\gamma(n_3+n_4)}{s} + \frac{\tau n_3}{s} - \frac{(\gamma+\mu_h) n_5}{s},\\
&\mathcal{L}\{S_{\nu_1}\} = -\frac{n_6(n_4}{s} - \frac{\mu_\nu}{s} - \frac{\beta_h b n_2 n_7}{s^2} - \frac{\mu_\nu N_\nu n_4}{s^2} - \frac{\mu^2 N_\nu}{s^2},\\
&\mathcal{L}\{I_{v_1}\} = \frac{\beta_\nu b n_3 - \mu_\nu n_7}{s} + \frac{n_4 n_6}{s^2} + \frac{n_4 \mu_\nu N_\nu}{s^3}.
\end{aligned}
\tag{5.16}
$$

Now utilizing inverse Laplace transform in (5.16), we get few terms of series solution as

$$
\begin{aligned}
&S_{h_1} = -\mu_h n_1 - (\frac{\beta_h n_1 n_7}{N_h} + \mu^2 N_h)t - \frac{\beta_h b \mu_h N_h n_7 t^2}{N_h},\\
&S_{h_1k_1} = -(\mu_h n_2 + n_5) - \frac{\beta_{hk} b n_2 n_7 t}{N_h},\\
&I_{h_1} = -(\mu+\tau+\gamma)n_3 + \frac{\beta_{hk} n_2 n_7 t}{N_h} + \frac{(1-\psi)\beta_h b n_1 n_7 t^2}{N_h} + \frac{(1-\psi)\mu_h \beta_h b n_1 n_7 t^3}{N_h},\\
&I_{h_1A_1} = -(\mu_h+\gamma)n_4 + n_1 n_7 t + \mu_h N_h I\nu(0)t^2,\\
&R_{h_1} = \gamma(n_3+n_4) + \tau n_3 - (\gamma+\mu_h)n_5,\\
&S_{\nu_1} = -n_6(n_4 - \mu_\nu - (\beta_h b n_2 n_7 + \mu_\nu N_\nu n_4)t - \mu^2 N_\nu t,\\
&I_{v_1} = \beta_\nu b n_3 - \mu_\nu n_7 + n_4 n_6 t + n_4 \mu_\nu N_\nu t^2.
\end{aligned}
\tag{5.17}
$$

Again Substituting these values into the second equations in (5.8)- (5.14) gives

$$
\begin{aligned}
\mathcal{L}\{S_{h_2}\} = &\frac{\beta_h b \mu_h n_1}{s^2 N_h} - \frac{\beta_h b \beta_\nu n_1 n_3}{s^3 N_h} + \frac{\beta_h^2 b n_1 n_7}{s^3 N_h^2} + \frac{\beta_h b \mu_h^2 N_h}{s^3 N_h} + \frac{\beta_h b n_1 I_\nu \mu_\nu}{s^3 N_h} - \frac{\mu_h \beta_h n_1 n_7}{s^3 N_h} + \frac{\mu_h^3 N_h}{s^3}\\
&- \frac{\mu_h \beta_h b \mu_h N_h n_7}{s^4 N_h} + \frac{\beta_h b \mu_h N_h n_7}{s^4 N_h^2} - \frac{\beta_h b n_1 n_4 n_6}{s^4 N_h} - \frac{\beta_h b \mu_\nu N_\nu n_1 n_4}{s^4 N_h} - \frac{\beta_h b \mu_h N_h \beta_\nu n_3}{s^4 N_h}\\
&+ \frac{\beta_h b \mu_h \mu_\nu N_h n_7}{s^{4N_h}} - \frac{\beta_h b \mu_h N_h n_4 n_6}{s^{5N_h}} - \frac{\beta_h b \mu_h N_h \mu_\nu N_\nu n_4}{s^6 N_h},
\end{aligned}
\tag{5.18}
$$

$$
\begin{aligned}
\mathcal{L}\{S_{h_2k_2}\} = &\frac{\tau \nu n_3}{s^2} - \frac{n_7 n_5}{s^2} - \frac{(\gamma+\mu_h)\nu n_5}{s^2} + \frac{\mu_h \beta_{hk} b n_2 n_7}{s^2 N_h} + \frac{\nu \gamma n_3}{s^2} + \frac{\nu\gamma n_4)}{s^2} + \frac{\mu_h^2 n_2}{s^3} - \frac{n_5}{s^3} - \frac{\mu_h n_2 n_7}{s^3}\\
&- \frac{\beta_\nu b n_2 n_3}{s^3} - \frac{\mu_\nu n_2 n_7}{s^3} + \frac{n_2 n_4 n_6}{s^4} - \frac{\beta_{hk} b n_2 n_7^2}{s^4 N_h} + \frac{\mu_\nu N_\nu n_2 n_4}{s^5},
\end{aligned}
\tag{5.19}
$$

$$
\begin{aligned}
\mathcal{L}\{I_{h_2}\} = &\frac{(\mu+\tau+\gamma)^2 n_3}{s^2} - \frac{(1-\psi)\beta_h b \mu_h n_1}{s^2 N_h} - \frac{(1-\psi)\beta_h b \beta_\nu n_1 n_3}{s^3 N_h} - \frac{(1-\psi)\beta_h^2 b n_1 n_7}{s^3 N_h^2}\\
&- \frac{(1-\psi)\beta_h b \mu_h^2 N_h}{s^3 N_h} - \frac{(\mu_h+\tau+\gamma)\beta_{hk} n_2 n_7}{s^3 N_h} - \frac{(1-\psi)\beta_h b n_1 I_\nu \mu_\nu}{s^3 N_h} + \frac{(1-\psi)\beta_h b n_1 n_4 n_6}{s^4 N_h}\\
&- \frac{(\mu_h+\tau+\gamma)(1-\psi)\beta_h b n_1 n_7}{s^4 N_h} - \frac{(1-\psi)\beta_h b \mu_h N_h n_7}{s^4 N_h^2} + \frac{(1-\psi)\beta_h b \mu_\nu N_\nu n_1 n_4}{s^4 N_h}\\
&+ \frac{(1-\psi)\beta_h b \mu_h N_h \beta_\nu n_3}{s^4 N_h} - \frac{(1-\psi)\beta_h b \mu_h \mu_\nu N_h n_7}{s^{4N_h}} + \frac{(1-\psi)\beta_h b \mu_h N_h n_4 n_6}{s^5 N_h}\\
&- \frac{(\mu_h+\tau+\gamma)(1-\psi)\mu_h \beta_h b n_1 n_7}{s^5 N_h} - \frac{(1-\psi)\beta_h b \mu_h N_h \mu_\nu N_\nu n_4}{s^6 N_h},
\end{aligned}
\tag{5.20}
$$

$$
\begin{aligned}
\mathcal{L}\{I_{h_2A_2}\} = &\frac{\mu_h+\gamma)^2 n_4}{s^2} - \frac{\mu_h n_1}{s^2} + \frac{\beta_\nu n_1 n_3}{s^3} - \frac{\beta_h n_1 n_7}{s^3 N_h} - \frac{\mu_h^2}{s^3 N_h} - \frac{(\mu_h+\gamma)n_1 n_7}{s^3} - \frac{n_1 I_\nu \mu_\nu}{s^3}\\
&+ \frac{n_1 n_4 n_6}{s^4} + \frac{\mu_\nu N_\nu n_1 n_4}{s^4} + \frac{\mu_h N_h \beta_\nu n_3}{s^4 N_h} - \frac{\mu_h \mu_\nu N_h n_7}{s^4} - \frac{\mu_h N_h n_7}{s^4 N_h} - \frac{\mu_h(\mu_h+\gamma)N_h I\nu(0)}{s^4}\\
&+ \frac{\mu_h N_h n_4 n_6}{s^5 N_h} + \frac{\mu_h N_h \mu_\nu N_\nu n_4}{s^6},
\end{aligned}
\tag{5.21}
$$

$$\mathcal{L}\{R_{h_2}\} = -\frac{\gamma(\mu+\tau+\gamma)n_3}{s^2} - \frac{(\tau\mu+\tau+\gamma)n_3}{s^2} - \frac{(\nu+\mu_h)\gamma(n_3+n_4)}{s^2} - \frac{(\gamma\mu_h+\gamma)n_4}{s^2} - \frac{(\nu+\mu_h)\tau n_3}{s^2} - \frac{(\nu+\mu_h)(\gamma+\mu_h)n_5}{s^2} + \frac{\gamma\beta_{hk}n_2n_7}{s^3N_h} + \frac{\tau\beta_{hk}n_2n_7}{s^3N_h} + \frac{\gamma n_1n_7}{s^3} + \frac{\gamma(1-\psi)\beta_h bn_1n_7}{s^4N_h} + \frac{\gamma\mu_h N_h I\nu(0)}{s^4} + \frac{\tau(1-\psi)\beta_h bn_1n_7}{s^4N_h} + \frac{\gamma(1-\psi)\mu_h\beta_h bn_1n_7}{s^5N_h} + \frac{\tau(1-\psi)\mu_h\beta_h bn_1n_7}{s^5N_h}, \tag{5.22}$$

$$\mathcal{L}\{S_{v_2}\} = \frac{\mu_v n_6(n_4}{s^2} + \frac{\mu_\nu^2}{s^2} + \frac{\mu_v\beta_h bn_2n_7}{s^3} + \frac{\mu_\nu^2N_\nu n_4}{s^3} + \frac{\mu_v\mu_h^2N_\nu}{s^3} + \frac{n_4^2n_6}{s^3} + \frac{(\mu_h+\gamma)n_4n_6}{s^3} + \frac{\beta_h bn_2n_7n_4}{s^4} + \frac{\mu_\nu N_v I_hA(0)}{s^4} - \frac{n_1n_7n_6}{s^4} + \frac{\beta_h b\mu_\nu n_1n_7}{s^4N_h} - \frac{\beta_h b\beta_\nu n_1n_3}{s^4N_h} + \frac{\beta_h b\mu_h n_1n_7}{s^4N_h} + \frac{b\beta_h^2n_1n_7^2}{s^5N_h} + \frac{\beta_h b\mu_h^2N_h n_7}{s^5N_h} - \frac{\mu_h N_h n_7n_6}{s^5} - \beta_h b\mu_\nu n_1n_7\frac{\mu_\nu n_1n_7}{s^5N_h} - \frac{\beta_h b\mu_h N_h\beta_\nu bn_3}{s^5N_h} + \frac{\beta_h b\mu_h\mu_\nu N_h n_7}{s^5N_h} - \frac{\beta_h b\mu_\nu N_\nu n_1n_3}{s^6N_h} - \frac{\beta_h b\mu_h N_h n_4n_6}{s^6N_h} + \frac{\beta_h^2b^2\mu_h N_h n_7^2}{s^6N_h^2} - \frac{\beta_h b\mu_h N_h\mu_\nu N_\nu n_4}{s^7N_h}, \tag{5.23}$$

$$\mathcal{L}\{I_{v_2}\} = -\frac{\mu_v\beta_\nu bn_3+\mu\mu_\nu^2n_7}{s^2} - \frac{(\mu+\tau+\gamma)\beta_v bn_3}{s^2N_h} + \frac{\beta_v b\beta_{hk}n_2n_7}{s^3N_h^2} - \frac{\mu_v n_4n_6}{s^3} + \frac{(1-\psi)\beta_v b\beta_h bn_1n_7}{s^4N_h^2} - \frac{\mu_v n_4\mu_\nu N_\nu}{s^4} - \frac{(\mu_h\gamma)n_4n_6}{s^4} - \frac{n_4^2n_6}{s^4} + \frac{(1-\psi)\beta_v b\mu_h\beta_h bn_1n_7}{s^5N_h^2} - \frac{\beta_h bn_2n_7n_4}{s^5} - \frac{\mu_\nu N_v n_4}{s^5} + \frac{n_1n_7n_6}{s^5} + \frac{\mu_h N_h n_7n_6}{s^6}. \tag{5.24}$$

Applying the inverse Laplace transform to the first equations in (5.19)(5.24), we have

$$S_{h_2} = (\frac{\beta_h b\mu_h n_1}{N_h} - \mu_h^2n_1)t + (\mu_h^3N_h + \frac{\beta_h^2bn_1n_7}{N_h^2} - \frac{\beta_h b\beta_\nu n_1n_3}{N_h} + \frac{\beta_h bn_1I_\nu\mu_\nu}{N_h} + \frac{\beta_h b\mu_h^2N_h}{N_h} - \frac{\mu_h\beta_h n_1n_7}{N_h})t^2 - (\frac{\beta_h bn_1n_4n_6}{N_h} + \frac{\beta_h b\mu_\nu N_\nu n_1n_4}{N_h} + \frac{\beta_h b\mu_h N_h\beta_\nu n_3}{N_h} + \frac{\mu_h\beta_h b\mu_h N_h n_7}{N_h} - \frac{\beta_h b\mu_h N_h n_7}{N_h^2} - \frac{\beta_h b\mu_h\mu_\nu N_h n_7}{N_h})t^3 - \frac{\beta_h b\mu_h N_h n_4n_6}{N_h}t^4 - \frac{\beta_h b\mu_h N_h\mu_\nu N_\nu n_4}{N_h}t^5, \tag{5.25}$$

$$S_{h_2k_2} = (\nu\gamma n_3+\nu\gamma n_4) + \tau\nu n_3 - (\gamma+\mu_h)\nu n_5 + \frac{\mu_h\beta_{hk}bn_2n_7}{N_h})t + (\mu_h^2n_2 - n_5 + \frac{\beta_{hk}bn_7n_5}{N_h})t^2 + \frac{\beta_{hk}b\mu_h n_2n_7}{N_h} - \frac{\beta_{hk}b\beta_\nu bn_2n_3}{N_h} + \frac{\beta_{hk}b\mu_\nu n_2n_7}{N_h})t^3 + (\frac{\beta_{hk}b\beta_{hk}bn_2n_7^2}{N_h^2} - \frac{\beta_{hk}bn_2n_4n_6}{N_h})t^4 + \frac{\beta_{hk}b\mu_\nu N_\nu n_2n_4}{N_h}t^5, \tag{5.26}$$

$$
\begin{aligned}
I_{h_2} = &((\mu+\tau+\gamma)^2 n_3 - \frac{(1-\psi)\beta_h b\mu_h n_1}{N_h})t + (\frac{(1-\psi)\beta_h^2 bn_1 n_7}{N_h^2} - \frac{(1-\psi)\beta_h b\beta_\nu n_1 n_3}{N_h} + \frac{(1-\psi)\beta_h bn_1 I_\nu \mu_\nu}{N_h} \\
&- \frac{(1-\psi)\beta_h b\mu_h^2 N_h}{N_h} + \frac{(\mu_h+\tau+\gamma)\beta_{hk} n_2 n_7}{N_h})t^2 + (\frac{(1-\psi)\beta_h b\mu_h N_h n_7}{N_h^2} + \frac{(1-\psi)\beta_h b\mu_h \mu_\nu N_h n_7}{N_h} \\
&+ \frac{(\mu_h+\tau+\gamma)(1-\psi)\beta_h bn_1 n_7}{N_h} - \frac{(1-\psi)\beta_h b\mu_\nu N_\nu n_1 n_4}{N_h} - \frac{(1-\psi)\beta_h b\mu_h N_h \beta_\nu n_3}{N_h} - \frac{(1-\psi)\beta_h bn_1 n_4 n_6}{N_h})t^3 \\
&+ (\frac{(\mu_h+\tau+\gamma)(1-\psi)\mu_h \beta_h bn_1 n_7}{N_h} - \frac{(1-\psi)\beta_h b\mu_h N_h n_4 n_6}{N_h})t^4 + \frac{(1-\psi)\beta_h b\mu_h N_h \mu_\nu N_\nu n_4}{N_h} t^5,
\end{aligned}
\tag{5.27}
$$

$$
\begin{aligned}
I_{h_2 A_2} = &((\mu_h+\gamma)^2 n_4 - \mu_h n_1)t + (\beta_\nu n_1 n_3 - \frac{\mu_h^2}{N_h} - (\mu_h+\gamma)n_1 n_7 - n_1 I_\nu \mu_\nu - \frac{\beta_h n_1 n_7}{N_h})t^2 + (n_1 n_4 n_6 \\
&+ \mu_\nu N_\nu n_1 n_4 + \frac{\mu_h N_h \beta_\nu n_3}{N_h} - \mu_h \mu_\nu N_h n_7 - \frac{\mu_h N_h n_7}{N_h} - \mu_h(\mu_h+\gamma)N_h I\nu(0))t^3 + \frac{\mu_h N_h n_4 n_6 t^4}{N_h} \\
&+ \mu_h N_h \mu_\nu N_\nu n_4 t^5,
\end{aligned}
\tag{5.28}
$$

$$
\begin{aligned}
R_{h_2} = &-((\nu+\mu_h)\tau n_3 + (\nu+\mu_h)(\gamma+\mu_h)n_5 + (\nu+\mu_h)\gamma(n_3+n_4) + (\gamma\mu_h+\gamma)n_4 + \gamma(\mu+\tau+\gamma)n_3 \\
&+ (\tau\mu+\tau+\gamma)n_3)t + (\frac{\gamma\beta_{hk} n_2 n_7}{N_h} + \gamma n_1 n_7 + \frac{\tau\beta_{hk} n_2 n_7}{N_h})t^2 + (\gamma\mu_h N_h n_7 + \frac{\gamma(1-\psi)\beta_h bn_1 n_7}{N_h} \\
&+ \frac{\tau(1-\psi)\beta_h bn_1 n_7}{N_h})t^3 + (\frac{\tau(1-\psi)\mu_h \beta_h bn_1 n_7}{N_h} + \frac{\gamma(1-\psi)\mu_h \beta_h bn_1 n_7}{N_h})t^4,
\end{aligned}
\tag{5.29}
$$

$$
\begin{aligned}
S_{v_2} = &(\mu_\nu^2 + \mu_v n_6(n_4)t + (\mu_v \mu_h^2 N_\nu + n_4^2 n_6 + \mu_\nu^2 N_\nu n_4 + \mu_\nu^2 N_\nu n_4 + \mu_\nu^2 N_\nu n_4 + (\mu_h+\gamma)n_4 n_6)t^2 \\
&- (\frac{\beta_h b\beta_\nu n_1 n_3}{N_h} + \frac{\beta_h b\mu_\nu n_1 n_7}{N_h} + \beta_h bn_2 n_7 n_4 + \mu_\nu N_v n_4 + \frac{\beta_h b\mu_h n_1 n_7}{N_h} - n_1 n_7 n_6)t^3 - \frac{\beta_h b\mu_\nu n_1 n_7 \mu_\nu n_1 n_7}{N_h} \\
&+ \frac{b\beta_h^2 n_1 n_7^2}{N_h} + \frac{\beta_h b\mu_h^2 N_h n_7}{N_h} + \mu_h N_h n_7 n_6 - \frac{\beta_h b\mu_h N_h \beta_\nu bn_3}{N_h} + \frac{\beta_h b\mu_h \mu_\nu N_h n_7}{N_h})t^4 + (\frac{\beta_h^2 b^2 \mu_h N_h n_7^2}{N_h^2} \\
&- \frac{\beta_h b\mu_h N_h n_4 n_6}{N_h} - \frac{\beta_h b\mu_\nu N_\nu n_1 n_3}{N_h})t^5 - \frac{\beta_h b\mu_h N_h \mu_\nu N_\nu n_4 t^6}{N_h},
\end{aligned}
\tag{5.30}
$$

$$
\begin{aligned}
\mathcal{L}\{I_{v_2}\} = &-(\frac{(\mu+\tau+\gamma)\beta_v bn_3}{s^2 N_h} + \mu_v \beta_\nu bn_3 - \mu\mu_\nu^2 n_7)t + (\frac{\beta_v b\beta_{hk} n_2 n_7}{N_h^2} - \mu_v n_4 n_6)t^2 + (\frac{(1-\psi)\beta_v b\beta_h bn_1 n_7}{N_h^2} \\
&- (\mu_h+\gamma)n_4 n_6 - \mu_v n_4 \mu_\nu N_\nu - n_4^2 n_6)t^3 - (\beta_h bn_2 n_7 n_4 - \mu_\nu N_v n_4 + n_1 n_7 n_6 + \frac{(1-\psi)\beta_v b\mu_h \beta_h bn_1 n_7}{N_h^2})t^4 \\
&+ \mu_h N_h n_7 n_6 t^5.
\end{aligned}
\tag{5.31}
$$

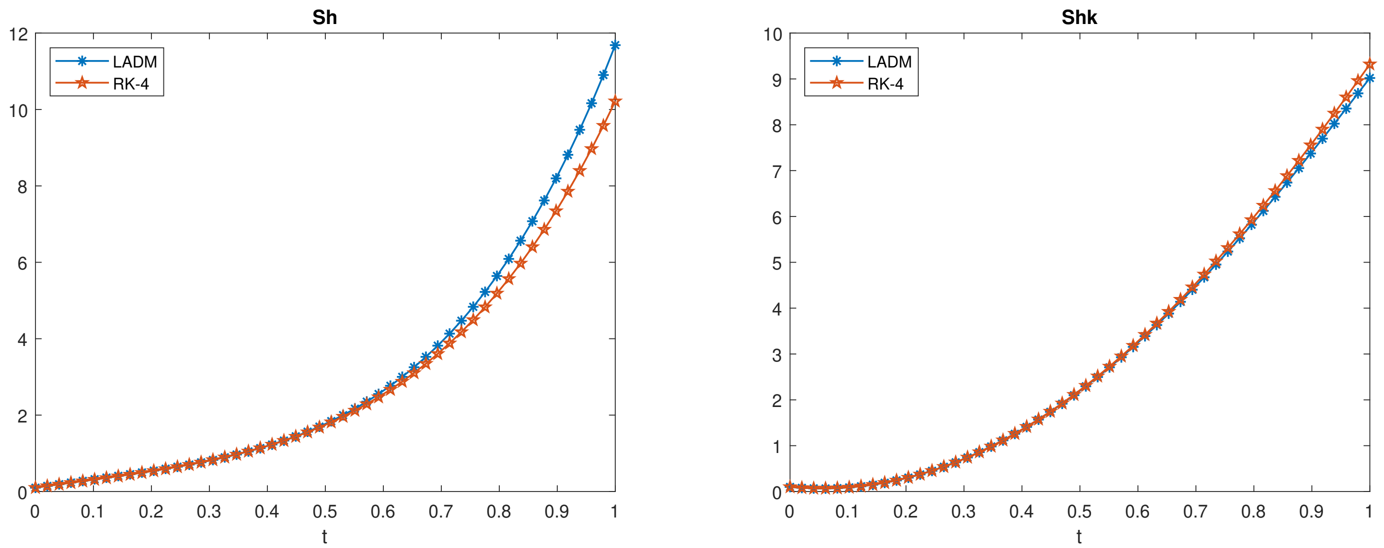
and so on. The other terms can be calculated in a similar way. With the help of these values, we are able to approximate the solutions to the above systems in the form of an infinite series as

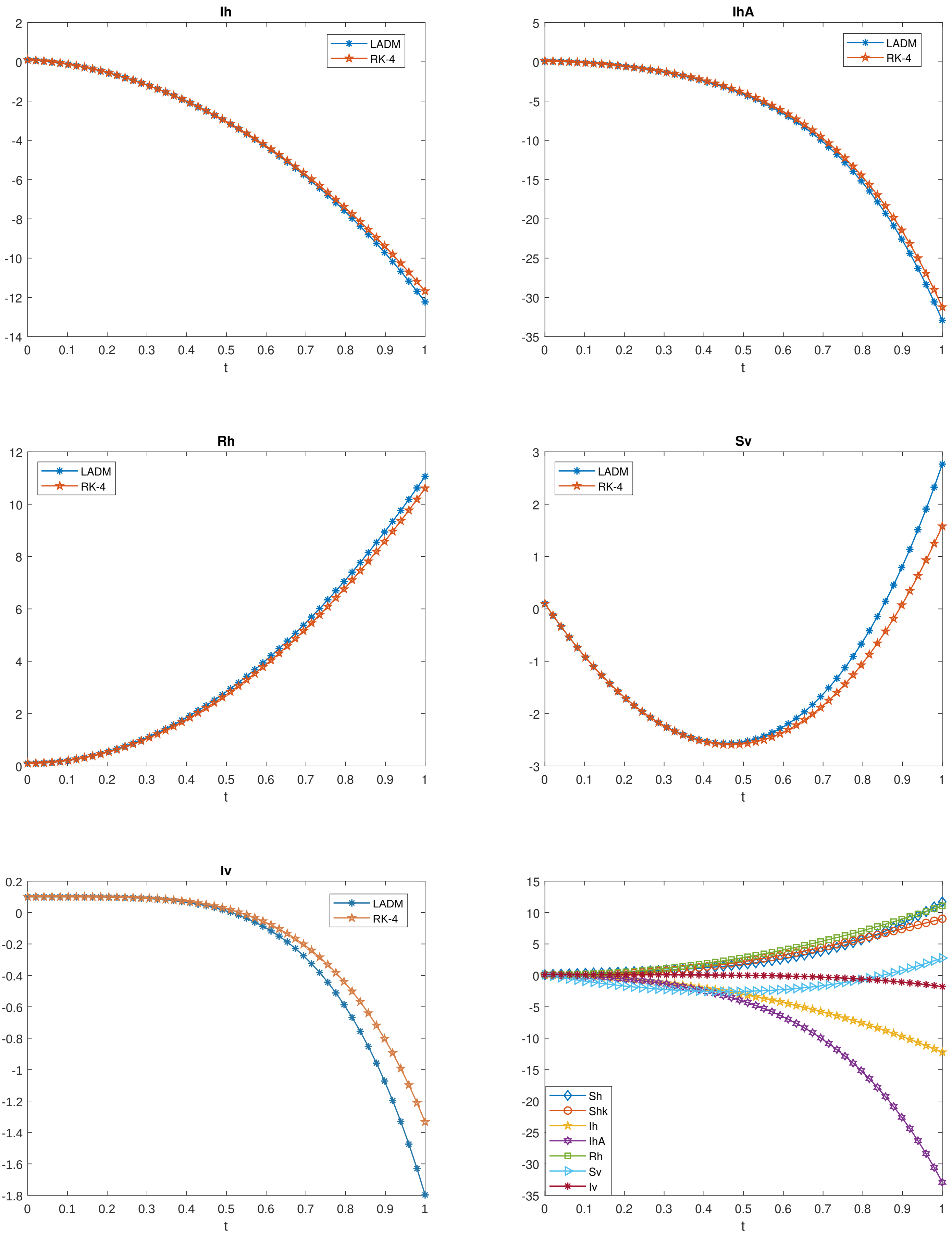
$$\begin{aligned}
S_h(t) &= S_{h_0} + S_{h_1} + S_{h_2} + S_{h_3} + S_{h_4} + S_{h_5} \cdots, \\
S_{hk}(t) &= S_{h_0k_0} + S_{h_1k_1} + S_{h_2k_2} + S_{h_3k_3} + S_{h_4k_4} + S_{h_5k_5} \cdots, \\
I_h(t) &= I_{h_0} + I_{h_1} + I_{h_2} + I_{h_3} + I_{h_4} + I_{h_5} \cdots, \\
I_{hA}(t) &= I_{h_0A_0} + I_{h_1A_1} + I_{h_2A_2} + I_{h_3A_3} + I_{h_4A_4} + I_{h_5A_5} \cdots, \\
R_h(t) &= R_{h_0} + R_{h_1} + R_{h_2} + R_{h_3} + R_{h_4} + R_{h_5} \cdots, \\
S_\nu(t) &= S_{\nu_0} + S_{\nu_1} + S_{\nu_2} + S_{\nu_3} + S_{\nu_4} + S_{\nu_5} \cdots, \\
I_\nu(t) &= I_{\nu_0} + I_{\nu_1} + I_{\nu_2} + I_{\nu_3} + I_{\nu_4} + I_{\nu_5} \cdots.
\end{aligned} \tag{5.32}$$

6. Graphical results and discussion

Utilizing the Mathematica software, the partial sums (5.5) are computed, and in particular, fifth approximations are calculated. In the mentioned equations, the initially five terms are considered for the fifth approximation because the rest of the terms changed by subsequent iterations.

$$\begin{aligned}
S_h(t) &= 0.1 + 4.314623482t5.8945261291t^2 + 5.516573849t^3 + 5.8392294t^4 + 5.809010211t^5 \\
S_{hk}(t) &= 0.1 + 1.914623482t + 2.4745261291t^2 + 2.196573849t^3 2.3932294t^4 2.219191219t^5 \\
I_h(t) &= 0.1 + 1.514623482t2.0745261291t^2 1.796573849t^3 1.9932294t^4 1.819191219t^5 \\
I_{hA}(t) &= 0.1 + 2.514623482t + 3.0945261291t^2 + 2.716573849t^3 + 0.5313294t^4 2.800000211t^5 \\
R_h(t) &= 0.1 + 3.314623482t3.8945261291t^2 + 3.516573849t^3 3.7392294t^4 + 3.819010211t^5 \\
S_\nu(t) &= 0.1 + 4.224623482t5.8945261291t^2 5.516573849t^3 + 5.8392294t^4 5.809010211t^5 \\
I_\nu(t) &= 0.1 + 4.314623482t5.8945261291t^2 + 5.516573849t^3 + 5.8392292t^4 5.829011211t^5
\end{aligned} \tag{6.1}$$




Ih
LADM
RK-4
t
IhA
LADM
RK-4
t
Rh
LADM
RK-4
t
Sv
LADM
RK-4
t
Iv
LADM
RK-4
t
Sh
Shk
Ih
IhA
Rh
Sv
Iv
t

The comparison is being made for five different iterations of the LADM method for $S_h, S_{hk}, I_h, I_{hA}, R_h, S_\nu$ and I_ν.This suggests that there may be different ways to approach a problem, and it is important to compare and evaluate these different approaches to determine which is the most effective or efficient.

7. Convergence analysis

To check the convergence that the above series (5.32) is uniformly and rapidly converges to the exact solution. We provide the following theorems to prove the convergence of the series:

Theorem 7.1. *Let $\mathbb{Z}$ be Banach space and $\mathbf{T} : \mathbb{Z} \longrightarrow \mathbb{Z}$ be a nonlinear operator $\forall\, \mathfrak{z}, \mathfrak{z}' \in \mathbb{Z}$, $\| \mathbf{T}(\mathfrak{z}) - \mathbf{T}(\mathfrak{z}') \| \leq l \| \mathfrak{z} - \mathfrak{z}' \|$, $0 < l < 1$. Then $\mathbf{T}$ has a unique point $\mathfrak{z} \ni \mathbf{T}(\mathfrak{z}) = \mathfrak{z}$, where $\mathfrak{z} = (S_h, I_h, I_{hA}, R_h, S_{hk}, S_\nu, I_\nu)$. Applying ADM, the series given in (5.32) can be defined as follows: $\mathfrak{z}_n = \mathbf{T}_{\mathfrak{z}_{n-1}}$, $\mathfrak{z}_{n-1} = \sum_{i=1}^{n-1} \mathfrak{y}_i$, $\forall n \in N$ $\mathfrak{z}_0 \in B_r(\mathfrak{z})$ where $B_r(\mathfrak{z}) = \mathfrak{z}' \in \mathbb{Z} : \| \leq l \| \mathfrak{z} - z' \| < r$, then, we have*

$$\mathfrak{z}_n \in B_r(n) \tag{7.1}$$

$$\lim_{n\to\infty} \mathfrak{z}_n = \mathfrak{z} \tag{7.2}$$

Proof. (**??**), utilize mathematical induction for $n = 1$,
$\| \mathfrak{z}_0 - \mathfrak{z} \| = \| \mathbf{T}(\mathfrak{z}_0) - \mathrm{T}(\mathfrak{z}) \| \leq l \| \mathfrak{z}_0 - \mathfrak{z} \|$
Assume that the result is true for $m-1$, then $\| \mathfrak{z}_0 - \mathfrak{z} \| \leq l^{m-1} \| \mathfrak{z}_0 - \mathfrak{z} \|$.
we have
$\| \mathfrak{z}_m - \mathfrak{z} \| = \| \mathbf{T}(\mathfrak{z}_{m-1}) - \mathbf{T}(\mathfrak{z}) \| \leq l \| \mathfrak{z}_{m-1} - \mathfrak{z} \| \leq l^m \| \mathfrak{z}_0 - \mathfrak{z} \|$
i,e.
$\| \mathfrak{z}_n - \mathfrak{z} \| \leq l^n \| \mathfrak{z}_0 - \mathfrak{z} \| \leq l^n r < r$
which implies that
$\mathfrak{z}_n \in B_r(n)$
(**??**) Since $\| \mathfrak{z}_n - \mathfrak{z} \| \leq l^n \| \mathfrak{z}_0 - \mathfrak{z} \|$ implies that $\lim_{n\to\infty} l^n = 0$
$\therefore \lim_{n\to\infty} \| \mathfrak{z}_n - \mathfrak{z} \| = 0 \Rightarrow \lim_{n\to\infty} \mathfrak{z}_n = \mathfrak{z}$ □

8. Conclusion

The objective of this paper is to demonstrate that nonlinear differential equation systems can be solved using approximate solutions obtained through the Laplace-Adomian Decomposition Method (LADM). The LADM is a hybrid technique that combines the Laplace transform and the Adomian decomposition method. This method is advantageous over other methods as it does not require any restrictive assumptions or discretization and is free from round-off errors. Additionally, it has a high convergence rate to the exact solution. The convergence analysis provided in this paper demonstrates the efficiency of the LADM. Furthermore, the method yields accurate solutions with only a few iterations. The numerical solutions obtained through this method are superior to those obtained through other methods that involve an extra parameter on which the solutions depend. We also demonstrate the existence and uniqueness of the solutions obtained through the LADM. The uniqueness theorem for the initial value problem is applicable to the nonlinear differential equation systems solved through this method, which provides a unique solution for a given set of initial conditions. The existence of the solution is guaranteed by its continuous dependence on the initial conditions. We compared the results of our proposed method with those obtained using the K-4 method and found that our method is highly efficient with RK-4 and other numerical methods. Therefore, the LADM can be used as a reliable method for obtaining accurate solutions for nonlinear differential equation systems.

Conflict of Interest

The authors have no conflict of interest regarding the publication of this article.

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