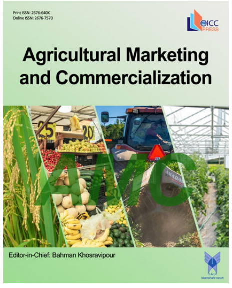

Volume 9, Issue 2, 092507 (178-188)

Agricultural Marketing and Commercialization (AMC)

https://doi.org/10.57647/j.amc.2025.090207

Analyzing the Performance of Symmetrical and Asymmetrical GARCH Models and Presentation of the Optimal Model in Predicting Price Index Fluctuations and Cash Return of Tehran Stock Exchange

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Original Article

Received:
2025-06-05

Revised:
2025-07-17

Accepted:
2025-9-08

Published in Issue:
2025-12-30

Abstract:

This study aims to present an optimal model for modeling the volatility of the Tehran Stock Exchange Price and Dividend Index during the period 2013 to 2024. It investigates the factors contributing to the superiority of GARCH models and seeks to determine whether certain models systematically outperform others, and under what conditions such superiority occurs. To this end, an evolutionary process based on the conventional steps of identification, estimation, and diagnostic testing, as outlined in the Box-Jenkins methodology, was employed. To evaluate model performance, a set of criteria was utilized, including log-likelihood, three information criteria, the Mincer-Zarnowitz time series test, and various loss functions for assessing both in-sample and out-of-sample performance. The findings of the study indicate that market conditions (periods of stability or crisis) significantly affect the performance of GARCH models. During crisis periods, asymmetric GARCH models demonstrate superior performance in forecasting market volatility and are considered more suitable. In contrast, under stable market conditions, the simple GARCH model, with its adequate efficiency and lower cost, is deemed a more economical choice. Furthermore, the results suggest that incorporating external variables and employing nonlinear models improves the quality of modeling in crisis situations. This improvement is evident not only in the accuracy of in-sample estimates but also in the out-of-sample forecasting power. Although in times of crisis the implied volatility index provides more information than the standard GARCH model, using heavy-tailed distributions such as the Student's t-distribution does not lead to a significant improvement in model performance compared to the normal distribution. Therefore, based on the results of this study, it can be concluded that the standard GARCH model is an efficient choice during stable periods. However, during crisis conditions, the GJR asymmetric GARCH model especially when augmented with external variables can be considered an optimal model for estimating and forecasting the volatility of the Tehran Stock Exchange Price and Dividend Index.

Keywords: Tehran Stock Exchange Price and Dividend Index, Generalized Autoregressive Conditional Heteroskedasticity (GARCH), Symmetric and Asymmetric GARCH Models, Common Financial Market Patterns, In-Sample and Out-of-Sample Estimation, Box-Jenkins Approach

Cite this article: Torabi, D., Bagheri, A., Fatehi Dabanloo, M. H., Houshmand Neghabi, Z., Khoshsim, R. (2025). Analyzing the performance of symmetrical and asymmetrical GARCH models and presentation of the optimal model in predicting price index fluctuations and cash return of Tehran Stock Exchange. Agricultural Marketing and Commercialization, 9(2), 178-188. https://doi.org/10.57647/j.amc.2025.090207

INTRODUCTION

One of the limitations of the ARCH model is that it requires the estimation of many parameters. Therefore, Bollerslev (1986) extended the ARCH model and transformed it into the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. This extension was done by converting the autoregressive (AR) process into an autoregressive moving average (ARMA) process, in order to benefit from its better performance in forecasting inflation rate uncertainty. The GARCH model defines conditional variance as a function of lagged values and the squared lagged residuals Sharma and Brooks Believe that Both the ARCH and GARCH models are consistent with empirical facts, but the GARCH model requires fewer estimated parameters and is thus more economical. (Sharma and _ 2015) Another empirical fact that is well addressed by standard GARCH models is the presence of volatility clustering and positive kurtosis. (Mandelbrot 1963) This indicates that large movements in stock volatility are usually followed by further large changes, and small movements tend to be followed by small ones.

Another established empirical feature is that the returns of financial markets not only exhibit negative skewness and heavy-tailed distributions, but also that their volatility does not respond symmetrically to positive and negative shocks (Black 1976). This phenomenon is sometimes referred to as the leverage effect and sometimes as the risk premium effect (Engle and Patton 2007). However, this asymmetry has not been accounted for in standard ARCH and GARCH models, as these models only deal with the magnitude of shocks and not with their sign. One response to the failure of standard GARCH models to capture asymmetry has been the development of newer and more complex versions of GARCH. These models have been improved to address various aspects of complexity in estimating and forecasting volatility(Brooks 2008). The newer versions discussed in this study include the Exponential GARCH model proposed by Nelson (1991) and the GJR-GARCH model introduced by Glosten et al. (1993). Other versions, such as the APARCH model (Ding, Granger et al. 1993).

A valid question here is: what makes these different models more appropriate and accurate in modeling and forecasting volatility? This study aims to evaluate the performance of various selected GARCH-type models in estimating and forecasting the monthly volatility of the Tehran Stock Exchange during the period 2013–2024. The output of this study ultimately provides recommendations for enhancing model performance in estimation and forecasting. Furthermore, the Tehran Exchange Dividend and Price Index (TEDPIX) is selected as a representative of the Tehran Stock Exchange, and this study attempts to examine whether there are models that systematically outperform others, and if such models exist, what conditions contribute to the improved performance of certain models over others. The results of this research include recommendations for the model that most accurately and effectively incorporates all optimizing factors proposed in this study for forecasting volatility of the above-mentioned indices.

THEORETICAL FOUNDATIONS

Symmetric and Asymmetric GARCH Models:

The financial and investment literature classifies risk and volatility modeling into several different forms: autoregressive moving average models, stochastic volatility models, regime-switching models, threshold models, and autoregressive conditional heteroskedasticity (ARCH) models (Brooks, 2019). Given the objectives of this research, this chapter focuses on the last category, namely ARCH models and their generalized form known as GARCH models. The focus in reviewing the literature is on volatility in financial markets, with a particular emphasis on its forecasting.

According to the literature, the first-order GARCH model has been widely used for modeling return volatility (Franses & van Dijk, 1996; Bohumic & Wang, 2019), as it is superior in terms of simplicity and resource efficiency compared to higher-order models. Therefore, the selected order for this study is GARCH(1,1). According to the study by Brooks (2019), Equation (3-2) represents the conditional variance equation that must be estimated for use in the GARCH model.

Equation (1) $$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

Where σ^2_t is the conditional variance, u^2_{t-1} represents the lagged squared residuals (the ARCH term), and σ^2_{t-1} is the lagged conditional variance (the GARCH term), with the conditions $\alpha i>0$, $\beta i>0$ being satisfied. To generate the error terms used in Equation (1), the conditional mean equation is defined as a simple equation with a constant term.

GJR-GARCH model:

Standard GARCH models treat positive and negative market movements equally and symmetrically. By adding additional terms to the standard model, asymmetry can be addressed. This approach allows the reaction to negative shocks to differ from positive shocks, which is observed in real market behavior. In this study, two asymmetric models have been used: the GJR-GARCH and the Exponential GARCH (EGARCH). Based on the studies of Glosten et al. (1993) and Brooks (2019), the GJR-GARCH model is specified by Equation (2).(Glosten, Jagannathan et al. 1993)

Equation (2) $$\sigma_t^2 = \alpha_t + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1}$$

Where σ^2_t is the conditional variance, u^2_{t-1} represents the lagged squared residuals (the ARCH term), and σ^2_{t-1} is the lagged conditional variance (the GARCH term) and $\gamma u^2_{t-1} I_{t-1}$ is the asymmetric term.

By including the indicator function in the model, asymmetry is taken into account. Therefore, responses to negative shocks differ from positive shocks, which aligns more closely with the empirical facts of stock market volatility. When shocks are positive, equals zero, and the GJR model reduces to the standard GARCH model. Conversely, when shocks are negative, equals one, and due to the leverage effect, volatility is pushed toward higher values.

EXPONENTIAL GARCH MODEL:

According to the study by Nelson (1991), the Exponential GARCH model can be specified by Equation (3).

Equation(3) $$n(\sigma_t^2) = \alpha_0 + \beta_1 \ell n\sigma_{t-1}^2 + \gamma \frac{u_{t-1}}{\sigma_{t-1}} + \theta \frac{|u_{t-1}|}{\sigma_{t-1}}$$

Where σ^2_t is the conditional variance, $\theta \frac{|u_{t-1}|}{\sigma_{t-1}}$ represents the ARCH term, $\beta_1\ ln\sigma^2_{t-1}$ is the GARCH term, and $\gamma \frac{u_{t-1}}{\sigma_{t-1}}$ is the asymmetry term. When $\gamma i<0$ gamma $ut-i<0$ and simultaneously (i.e., a negative shock occurs), the third term becomes larger, pushing volatility to higher values due to the asymmetric effect. Once again, this behavior is more realistic and better captures the characteristics of stock market volatility.

Complexity or Simplicity: The Trade-off Between Asymmetric GARCH and Classical GARCH:

An important debate in the literature of this study is whether more complex models with a larger number of parameters offer better performance compared to simpler models. The literature seeks to compare and evaluate the accuracy and forecasting power of these different models. Several studies have been conducted to assess the predictive performance of GARCH-type models in terms of both in-sample and out-of-sample accuracy. Hansen and Lunde (2005) compared 330 ARCH and GARCH models to evaluate their ability to forecast one-day-ahead conditional variance. Using exchange rate and stock market return data, the models were assessed based on their out-of-sample performance.(Hansen and Lunde 2005)

The criteria used to evaluate the performance of the models included six different loss functions. In the analysis of exchange rates, they found no evidence that complex and asymmetric models outperformed the standard GARCH model. However, in the case of stock market data, models that accounted for the leverage effect performed better than the GARCH model.(Granger and Poon 2001). This finding indicates that the volatility of different markets exhibits distinct characteristics, and the selection of the most suitable models should primarily be based on the type and conditions of the market. (Bhowmik and Wang 2020)

To summarize the literature review section, it should be noted that forecasting volatility from a forward-looking perspective is not a straightforward task, and evaluating forecasting performance can be a challenging matter. In general, theoretically, it appears that using more complex models by incorporating asymmetric variables can lead to better modeling of volatility. This notion is not only theoretically plausible but also supported by numerous empirical studies. However, considering empirical findings that contradict the hypothesis that complex models are superior, the question arises: What is the source of this difference among various models? The next part of the literature review aims to answer this question as the main inquiry of the current study. A volatility forecasting model, as the name suggests, should be able to accurately model and forecast volatility. Theoretically, the more accurately a model understands the common features of financial time series, the more accurately it is expected to perform. This is why most studies in the literature are dedicated to examining the consistency of models with the stylized facts observed in financial time series. According to the existing literature and from a theoretical standpoint, on the one hand, all studies reviewed in the domestic literature support the superiority of asymmetric GARCH models in forecasting volatility. On the other hand, for a volatility forecasting model to be accurate, first, it must properly fit the in-sample data (i.e., it should estimate the market characteristics during the sample period with minimal estimation errors), and second, the mentioned model must also perform well for any other category of data (i.e., out-of-sample data). In other words, the model must have predictive power.

RESEARCH BACKGROUND

Foreign Research Background:

- Abedini et al. (2024), in a study, optimized the stock portfolio of various industries in the stock market using the Orthogonal GARCH model. Weekly data from 2010 to 2020 were analyzed for six industries. The results of the dynamic mean-variance model showed that the highest weights were allocated to the pharmaceutical and chemical industries, and that the Orthogonal GARCH model demonstrated high capability in estimating volatility and optimizing the portfolio.(Abedini, Abounoori et al. 2024)

- Bagherzadeh Sohrabi et al. (2025), in an article published in the *Journal of Financial Engineering and Portfolio Management*, forecasted the volatility of cryptocurrency returns (Bitcoin and Ethereum) using the Hidden Markov and Markov-GARCH methods. The results showed that the Markov-GARCH method improved trend forecasting accuracy by up to 25% for Ethereum and 5% for Bitcoin compared to the Hidden Markov model, although it performed slightly worse in terms of absolute value prediction accuracy.(Bagherzadeh Sohrabi, Mombeini et al. 2025)

- Zhang, Zhe, Chu, et al. (2024), in an article titled *"A Comparative Study on Interval Forecasting Using GARCH Models under Symmetric and Asymmetric Distributional Assumptions,"* argue that interval forecasting enables decision-makers to make more informed decisions by providing a range of possible future values along with their associated probabilities. The results of this study can be summarized as follows: In pairwise comparisons, GARCH models under asymmetric distributional assumptions exhibit higher accuracy in interval forecasting compared to GARCH models with symmetric distributions. Moreover, among the various types of GARCH models, the GJR-GARCH model shows better accuracy in interval forecasting, whereas the SGARCH and EGARCH models demonstrate similar performance.(Zhang, Choo et al. 2024)

- Khodadadi et al. (2024), using a conditional Copula-GARCH approach, examined the relationship between stock price crashes and rational price bubbles in data from 30 listed companies during the years 2011 to 2022. The results showed that there is no significant relationship between stock price crashes and bubbles based on rational structure, and that volatility dependence can be modeled using the copula distribution. (Khodadadi, Lashgarara et al. 2024)

- Alao et al. (2023), in an article titled "Symmetric and Asymmetric GARCH Estimates of the Impact of Oil Price Uncertainty on Output Growth: Evidence from G7 Countries,"argue that crude oil is a fundamental source of energy. Without access to energy, output growth would be impossible. As a result of this connection, fluctuations in oil prices can lead to instability in output in both developed and developing economies. The findings indicate differences in the intensity of positive and negative (asymmetric) effects of oil price shocks on output growth. Furthermore, the results show that past news and previous volatility have a significant effect on current conditional volatility of output growth in G7 countries. In conclusion, the study finds that the impact of oil price volatility on output growth in the examined economies is asymmetric in nature, highly persistent and clustered, and that asymmetric GARCH models outperform symmetric ones.(Alao, Alhassan et al. 2023)

- Barzegar et al. (2022), in a study aimed at designing a spillover model of financial distress probability in Iran's banking system, examined data from listed banks during the years 2016 to 2020 using multivariate DCC-GARCH models and the KMV model. The findings indicated a significant relationship between the risk of financial distress in banks and its contagion across the banking network.(Barzegar, Fallah Shams et al. 2023)

- Ansari-Nasab and Rahimi (2021), in a study aimed at modeling the volatility of OPEC crude oil prices using long memory and exponential nonlinear behavior, utilized daily data from the period 1986 to 2017. The results showed that the combined EGARCH-ARFIMA (1,1)-(3,0.09,4) model had the best performance in forecasting oil price volatility, and that ignoring exponential features and long memory leads to inaccurate analysis.(Ansarinasab and Rahimi 2021)

- Another study examines the performance of four first-order GARCH models including GARCH, Exponential GARCH (EGARCH), GJR-GARCH, and APARCH using three distributions: Normal, Student's t, and Skewed Student's t, across two major European stock market indices: the FTSE 100 (London Stock Exchange) and the DAX 30 (Frankfurt Stock Exchange). The results indicate that the GJR-GARCH and APARCH models, using daily data over a 15-year period in both markets, provide the best volatility forecasts Peters, (2001) and Bohumic & Wang, (2020).(Peters 2001)

Research Gap and the Role of This Study in Addressing It:

Review of the research background shows that previous studies have not sufficiently compared the efficiency of symmetric and asymmetric GARCH models using various criteria over a relatively long period under both normal and crisis conditions. This gap can be considered a research deficiency relative to the present study, which addresses the following points and thus fills this gap:

1. Based on theoretical foundations and previous empirical works, this study selects two widely accepted asymmetric GARCH models and aims to compare their performance with each other and with the simple classical GARCH model. It evaluates them using various criteria over a relatively long period to identify the reasons and drivers of the most accurate performance. In the search for sources of performance differences among models, the current research compares the selected models' performance in both normal market conditions and crisis periods.
2. This study also examines the idea of adding an external variable (the implied volatility index) to the GARCH family models to see whether this can be a potential source of difference.
3. Furthermore, this research tests the implementation of the models using different distributions by comparing their performance under the normal distribution and the Student's t-density function. The findings of this study include recommendations for improving GARCH models in forecasting the volatility of the Tehran stock market. The next chapter of this research deals with the methodology according to which the study is conducted.

METHODOLOGY

This study, in terms of its objective, is classified as applied research; regarding the nature of the data, it is ex-post facto and quantitative (causal), and it is descriptive in nature.

Population and Sample:

The statistical population of the study consists of monthly time series data of the overall stock market index over a twelve-year period from 2013 to the end of 2024. The primary data source will be the official website of the Tehran Stock Exchange. The study period is divided into two sub-periods: the first 7 years (2013 to the end of 2019) are used for estimation (identifying the best in-sample fitted models under stable conditions), and the remaining 5 years (2020 to the end of 2024) are used for out-of-sample evaluation (crisis period). Studies show that after a relatively long and stable period with a gradual upward trend, the Tehran Stock Exchange index experienced a very rapid increase during the years 2018 and 2019, followed by several years of intense volatility. The stable period (early 2013 to the end of 2019) is considered the normal period, while the rapid rise and subsequent severe volatility period (early 2020 to the end of 2024), influenced by currency prices and foreign sanctions, is regarded as the crisis period in this study.

Data Analysis Method:
In addition to the monthly values of the stock index, another time series tested to improve model performance is the global implied volatility index (VIX). This index is a forward-looking indicator reflecting investors' expectations of future volatility. It is sometimes known as the "fear index" .(Chronopoulos, Papadimitriou et al. 2018) In this study, the monthly values of this index will be introduced as an exogenous variable into the GARCH models to evaluate whether incorporating this external variable can significantly enhance and optimize the GARCH models. The time period for including this exogenous variable in the models under investigation spans from 2020 to the end of 2024, which corresponds to the crisis years defined in this research. This study aims to evaluate the performance of selected GARCH family models in estimating and forecasting the monthly volatility of the Tehran stock market during the period 2013–2024. The study seeks to determine whether there are models that systematically outperform others and, if so, to identify the conditions under which some models have superiority over others. The strategy employed in this research is quantitative analysis using existing datasets from the TSETMC website concerning the Tehran Stock Exchange Price and Dividend Index (TEDPIX). Based on the existing literature and theoretical background in this field, the first two stages in specifying the volatility model involve examining the data and testing the evident facts revealed in financial market volatility. The goal of these stages is to assess the fit of models with the selected data throughout the study period. The next stage involves comparing different models among those compatible, using in-sample data and out-of-sample forecasting performance. The defined process includes collecting monthly observations of the Tehran Stock Exchange Price and Dividend Index and calculating its monthly logarithmic returns using equation (4).

Equation (4) $$r_t = logI_t - logI_{t-1}$$

In summary, this study calculates the monthly returns of the Tehran Stock Exchange Price and Dividend Index (TEDPIX) as continuously compounded returns using equation (3-1) over the period from $t-1$ to t. An evolutionary process based on the Box-Jenkins methodology is employed for identification, estimation, and diagnostic checking. Before estimation and during the identification phase, the dataset is tested for revealed facts regarding volatility and ARCH effects. In the estimation phase, selected GARCH family models are implemented to test goodness of fit over the study period. The period is then divided into two sub-periods: one for estimation and the other for forecasting. EViews 13 software is used for estimation and forecasting. Following methodological references from Damodar (2004) and Brooks (2019), maximum likelihood estimation is preferred over ordinary least squares (OLS).(Gujarati 2004)
To evaluate model performance both in-sample and out-of-sample, several criteria are employed, including log-likelihood, AIC, SBC, and HQC. For out-of-sample evaluation, alongside other metrics (RMSE, MAE, MAPE, and Symmetric MAPE), a one-month-ahead static forecast is conducted. This means that after each sub-period of estimation, a one-month-ahead volatility forecast is produced. Actual data are then added to the dataset, and the one-month-ahead forecast is repeated. This process continues until the end of the out-of-sample forecasting sub-period. Subsequently, a series of Mincer-Zarnowitz regressions are conducted to statistically assess the differences between the realized actual volatility and the model's predicted volatility. To achieve the study's objectives and ultimately provide recommendations for improving volatility forecasting models, tests are repeated after removing crisis periods to evaluate model performance during normal times. The models are also tested with the addition of an external variable named VIX to examine whether this inclusion improves model performance. Finally, various tests using the Student's t-distribution are performed to assess model performance under different distributional assumptions. In the diagnostic checking phase of the Box-Jenkins methodology, several common tests are conducted to ensure that the models appropriately capture the behavioral characteristics of volatility. This process is repeated for all hypothesis tests. Chapter Four performs the above-mentioned tests and presents the findings, analysis, and discussion.

FINDINGS

In this section, we implement the three stages of the Box-Jenkins approach. Accordingly, the first part examines the descriptive statistics of the Tehran Stock Exchange Price and Dividend Index and tests the revealed facts about financial time series to identify the overall suitability of GARCH family models in estimating and forecasting the volatility of Iran's capital market. In other words, this part attempts to determine whether GARCH family models are appropriate for modeling and forecasting the volatility of the price and dividend index (Identification Phase). The second part applies the models under various conditions (Estimation Phase), presents the test results, and aims to analyze them thoroughly based on the theoretical background and literature review.

IDENTIFICATION PHASE

Using Equation1, the monthly logarithmic returns of the Tehran Stock Exchange Price and Dividend Index over the period 2013–2024 were calculated in order to transform the time series into a stationary and standardized process. The first step before applying GARCH models is to test for the presence of stylized facts in the data to determine whether the dataset is appropriate for estimating models from the GARCH family.
Data Normality Test: An examination of this study shows that the index has an average monthly return of 1.29 percent. Additionally, the monthly standard deviation is 4.03 percent, which is significantly higher than the average. The median of the returns is also a positive number, but

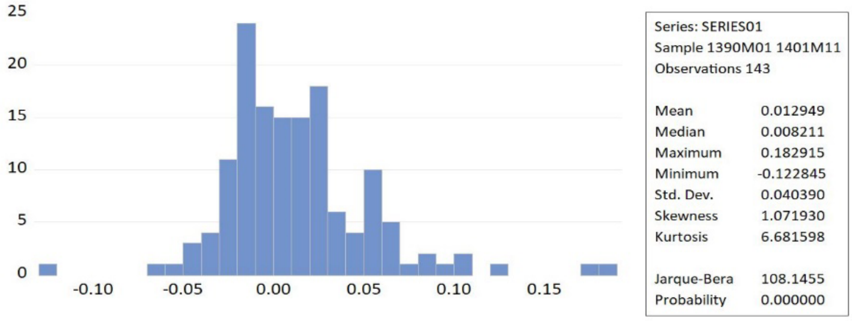

Figure 1. Descriptive Statistics and Histogram

lower than the mean. The kurtosis coefficient, which indicates the heaviness of the tails in distribution functions, is 6.68 a considerably high value (a normal distribution has a kurtosis of 3). This is a common feature in stock market return data. In the continuation of the study, the Jarque-Bera test was conducted to assess the normality of the data distribution. The critical values of the chi-square distribution with 2 degrees of freedom at the 1%, 5%, and 10% significance levels are 9.2, 5.99, and 4.605, respectively. Therefore, the null hypothesis of data being normally distributed is rejected. This is the same result that was expected based on the stylized facts about financial data.

Stationarity Test: The Augmented Dickey-Fuller (ADF) test was employed to examine the stationarity of the data (Table 1), yielding a test statistic of -8.30. This value is lower than the critical values at all significance levels (−3.44 at 1%, −2.86 at 5%, and −2.57 at 10%). Therefore, the null hypothesis of the presence of a unit root is rejected, and it can be concluded that the time series is stationary over the study period. This provides further justification for the appropriateness of applying GARCH models.

Heteroskedasticity Test: Using the Lagrange Multiplier (LM) test, both the F-statistic and the $T \times R^2$ coefficient are statistically significant, as their values are relatively large (20 and 17.8, respectively) and their p-values are close to zero. Given these test statistics, there is strong evidence to reject the null hypothesis. Therefore, a significant ARCH effect is observed in the time series of the Tehran Stock Exchange index. This confirms the appropriateness of applying conditional heteroskedastic autoregressive models in the present dissertation for the period under study.

Autocorrelation Test: According to Brooks (2019), an appropriate approach to detecting autocorrelation is to construct confidence intervals at different significance levels. At the 5% significance level, the confidence interval is calculated as follows:

$$\pm 1.96 \times \frac{1}{\sqrt{T}} = \pm 1.96 \times \frac{1}{\sqrt{143}} = \pm 0.1639$$

In the above formula, T represents the number of observations.The calculated values in this research should be compared with the value obtained from the above formula. This comparison indicates the presence of evidence for autocorrelation at lag two. It should be noted that at lag one, it is natural for the values to fall outside the confidence interval. The presence of autocorrelation once again confirms the necessity of modeling stock market volatility in Iran using ARCH/GARCH models. In the above sections, the identification phase, which is the first phase of the Box-Jenkins approach in modeling volatility, has been completed, and the descriptive statistics along with the general characteristics of the time series of Iran's capital market returns have been examined. Overall, the obtained evidence confirms the presence of heteroskedasticity in the Tehran Stock Exchange return series. Based on the theoretical literature, modeling the volatility of such time series requires the use of ARCH/ GARCH family models. The next part of the study will focus on implementing selected GARCH models across different time periods.

Estimation Phase (Hypothesis Testing of the Research)

In order to achieve the main objective of this study and to answer the research questions, the main hypotheses of the research are defined as follows:

Hypothesis 1: Asymmetric GARCH models perform better than their symmetric counterparts in measuring and forecasting the volatility of Iran's capital market.

Hypothesis 2: Market conditions (stability or crisis) have a significant impact on the performance of GARCH models in modeling the volatility of the Tehran Stock Exchange.

Hypothesis 3: Asymmetric GARCH models are more optimal for forecasting volatility in markets under crisis conditions.

Hypothesis 4: By altering the models (through the use of different probability distributions and the addition of explanatory variables to the standard models), the performance of asymmetric GARCH models will significantly improve.

Estimation of the Model and Analysis of the First Hypothesis Test:

To test the first hypothesis, the estimation of symmetric and asymmetric GARCH models will be conducted in two scenarios: in-sample (stable period) and out-of-sample (crisis period). The results of these estimations will reveal whether the first hypothesis of the research is confirmed or not. To test the first hypothesis which seeks to determine the best model among the selected GARCH family models, including the standard GARCH, Exponential GARCH, and GJR-GARCH, for modeling and forecasting monthly volatility of the Tehran Stock Exchange's total return

Table 1. Estimation Results of Selected Models in the Estimation Sub-Period (2013–2019)

P-value	Estimated Coefficients	Coefficients	Specifications	Models	
0.2860	7.390	α_0	Constant term		
0.0812	0.2834	α_1	u^2_{t-1}	$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2$	Symmetric GARCH
0.0036	0.61058	β_1	σ^2_{t-1}		
0.1032	0.000147	α_0	Constant term		
0.0690	0.453304	α_1	u^2_{t-1}	$\sigma_t^2 = \alpha_t + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1}$	GJR GARCH
0.0214	0.4677	β_1	σ^2_{t-1}		
0.1400	-0.43099	γ	$u^2_{t-1} I_{t-1}$		
0.0328	-2.6983	α_0	Constant term		
0.001	0.6914	β_1	$ln\sigma^2_{t-1}$	$\ell n(\sigma_t^2) = \alpha_0 + \beta_1 \ell n\sigma_{t-1}^2 + \gamma \frac{u_{t-1}}{\sigma_{t-1}} + \theta \frac{\lvert u_{t-1}\rvert}{\sigma_{t-1}}$	Exponential GARCH
0.0889	0.2613	γ	$\frac{u_{t-1}}{\sigma_{t-1}}$		
0.1627	0.4020	θ	$\frac{\lvert u_{t-1}\rvert}{\sigma_{t-1}}$		

Table 2. Comparison of In-Sample Performance of Models in the Estimation Subperiod

Exponential GARCH	GJR GARCH	Symmetric GARCH	Measurement
198.933	198.3660	197.3015	Log-Likelihood
-4.6730	-4.6594	-4.6578	Akaike Information Criterion (AIC)
-4.5273	-4.5137	-4.5412	Schwarz Criterion (SC)
-4.6175	-4.60	-4.6110	Hannan–Quinn Criterion (HQC)

and price index the main 12-year period of the study was divided into different sub-periods. All tests conducted in the previous section of this dissertation were also repeated for these sub-periods to ensure the suitability of the GARCH family models during these times as well. The test results indicate the appropriateness of these models for the mentioned sub-periods. The three selected GARCH models were implemented for both sub-periods, and estimation and forecasting errors were evaluated using the previously mentioned criteria.The two main sub-periods of this study are:

- Estimation Sub-period: The first 7 years (from the beginning of 2013 to the end of 2019), used to identify the best in-sample fitting models.
- Forecasting Sub-period: The remaining 5 years (from the beginning of 2020 to the end of 2024), used to evaluate the out-of-sample forecasting ability of the models.

The results of the estimation of symmetric and asymmetric GARCH models during the sub-period of stability (2013–2019) are presented in Table 1. All coefficients of the standard GARCH model are statistically significant and important, leading to the conclusion that, despite the fact that in many global financial markets asymmetric GARCH models better capture the complexities inherent in financial market data, in the Tehran Stock Exchange the standard GARCH models perform better in estimating market volatility. Among the asymmetric models, the exponential GARCH (EGARCH) models exhibit lower estimation errors compared to their counterpart, the GJR-GARCH model. The best model in terms of statistical fit is the one that shows higher log-likelihood and lower values in information criteria (AIC, Schwarz Criterion, and Hannan-Quinn Criterion).

Unlike the asymmetric GARCH models where some estimated coefficients have less significance, all estimated coefficients in the simple GARCH models are statistically significant. This result confirms that standard GARCH models are more efficient and consistent for estimating volatility in the Tehran stock market. This finding is consistent with Åstrand (2020) but contradicts the study by Bohumík and Wang (2020). Simple GARCH models outperform nonlinear asymmetric models in estimating volatility in the Tehran Stock Exchange.

Therefore, the first hypothesis of the study (Hypothesis

1: Asymmetric GARCH models perform better than their symmetric GARCH counterparts in measuring and forecasting the volatility of the Iranian capital market) is not supported. Although the asymmetric GARCH models exhibited lower in-sample forecasting errors in estimating the volatility of the Tehran Stock Exchange based on performance evaluation criteria, this study did not obtain sufficient and reliable evidence regarding the statistical significance of the estimated coefficients and asymmetry parameters in the asymmetric models.

Estimation of the Model and Analysis of the Second Hypothesis Test:

To test the second hypothesis which aims to understand whether nonlinear GARCH models can systematically perform better than the standard GARCH model, and if so, which of the two nonlinear GARCH models is optimal the necessary estimations were performed.

In this research, we aim to address one of the weaknesses of many previous studies. Many findings from prior research are only valid for the data within the study period. In this thesis, by evaluating the out-of-sample performance of the models, we seek to ensure that the model with the best performance during the estimation period also maintains acceptable performance with data outside the estimation period. In other words, we want to determine whether a model suitable for estimation can also be appropriate for out-of-sample forecasting. The first criterion in this study for evaluating out-of-sample performance is to compare one-step-ahead forecasts from the simple GARCH model with the realized (actual) market variance. The realized market variance is calculated using Equation 3-5 of this dissertation. The comparison results show that the differences between the realized variance over time and the values predicted by the model fluctuate around zero, except for certain periods in 2020 when the differences exceed one percent. The next step is to conduct a statistical test to assess whether the observed differences are statistically significant. To examine this, we regress the realized values of the Tehran Stock Exchange's monthly return variance on the model's predicted monthly variance. This is achieved through a series of Mincer-Zarnowitz regressions. Performing this series of regressions and comparing the models' performance in forecasting Tehran's market volatility reveals:

1- All three models were able to forecast the volatility of the price and cash return index from the beginning of 2020 to the end of 2025 in a way that the difference between realized and predicted volatilities is statistically indistinguishable from zero indicating strong predictive power for all three models.

2- Although all three models perform well in forecasting volatility within the Mincer-Zarnowitz framework, when forecast error functions are introduced into the model comparison, the GARCH-JJR model achieves the highest ranking among the models. (However, the difference is not statistically significant)

3- Therefore, the first hypothesis is rejected in the context of out-of-sample evaluation as well, since, contrary to the first hypothesis of this study, symmetric (standard) GARCH models outperform asymmetric GARCH-family models in measuring and forecasting the volatility of Iran's capital market.

Estimation of the Model and Analysis of the Third Hypothesis Test:

The third hypothesis explores whether the accuracy and precision of the models differ between normal economic periods and financial crisis periods. To address this question, additional sub-periods were defined. The period from early 2013 to the end of 2019 was considered a normal economic period, while the period from early 2020 to the end of 2024 was considered a crisis period. For testing the third hypothesis, the sub-period from the beginning of 2020 to the end of 2024 which includes a sharply rising market index followed by intense market volatility was selected as a representative of crisis conditions. The three selected models Standard GARCH, Exponential GARCH, and GJR-GARCH were applied over the aforementioned sub-period and evaluated using four performance criteria. The results of the model performance comparison are presented in Table 3.

Therefore, the third hypothesis of the study tests the assumption that asymmetric GARCH models are more optimal for forecasting market volatility under crisis conditions. Contrary to the stable market periods in Iran, during which the standard GARCH model performs better, asymmetric GARCH models are more suitable for modeling volatility in the Tehran Stock Exchange during periods of sharp changes and high market turbulence.

Among the tested models, the GJR-GARCH model demonstrates the best fit for capturing volatility in Iran's capital market under crisis conditions. Although the Log-Likelihood criterion slightly favors the Exponential GARCH model, the remaining three evaluation criteria show the lowest estimated error for the GJR-GARCH

Table 3. Comparison of In-Sample Performance of Models in the Estimation Subperiod

Exponential GARCH	GJR GARCH	Symmetric GARCH	Measurement
99.2768	99.1999	96.8529	Log-Likelihood
-3.142	-3.139	-3.095	Akaike Information Criterion (AIC)
-2.968	-2.965	-2.955	Schwarz Criterion (SC)
-3.074	-3.071	-3.0404	Hannan–Quinn Criterion (HQC)

model. More importantly, the estimated coefficients of the GJR-GARCH model variables are generally more statistically significant and meaningful compared to those of the Exponential and Standard GARCH models.

Unlike the stability sub-period analyzed earlier in this study, during the crisis sub-period—marked by a sharp depreciation of the national currency and subsequent political and economic turmoil—the Tehran Stock Exchange exhibits clear signs of asymmetry and leverage effects. Therefore, asymmetric GARCH family models are more capable of effectively capturing the volatility structure of Iran's capital market during such turbulent times.

Estimation of the Model and Analysis of the Fourth Hypothesis Test:

The fourth hypothesis aims to understand the possible source of superiority of certain models and explore how these models can be improved. To this end, three evaluations were conducted:

1. The selected GARCH family models were evaluated and compared after incorporating the exogenous variable VIX.
2. The same models were evaluated and compared under two different distribution assumptions: Normal distribution and Student's t-distribution.
3. The effects of simultaneously incorporating the exogenous variable VIX and using the Student's t-distribution were evaluated.

The sub-periods used for including the exogenous variable and running tests with different distributions coincide with the crisis period. In this way, the study attempts to assess the potential for enhancing model performance during crises, and the results will be presented in the form of general recommendations to end-users of the study. According to the findings of this section can be summarized as follows:

• This study provides evidence that incorporating the implied volatility index (VIX) as an independent variable into the variance equation improves the log-likelihood value of all three models. However, there is no significant reduction in the forecast error for the standard GARCH and Exponential GARCH models.

• Only the GARCH-JJR model shows a statistically significant improvement across all evaluation criteria (increased maximum likelihood and reduced forecast error) when this independent variable is added to its variance equation. This finding is consistent with previous studies.

• The economic implication of this result is that the VIX index contains information not captured by the basic model specifications, and thus contributes to a better modeling of volatility in the Tehran Stock Exchange. Theoretically, this result appears reasonable, since VIX is defined as a proxy for expected volatility and offers a forward-looking measure of global market uncertainty.

• When the models were run under heavier-tailed distributions (such as the Student's t-distribution), the results were less conclusive. Using the Student's t-density function compared to the baseline models did not lead to a significant improvement in model performance. This indicates that, contrary to many previous studies, the Student's t-distribution does not provide a meaningful advantage over the normal distribution used in the GARCH model. This finding is supported by studies like Lo and Hong (2010) and Lin (2018), which argue that incorporating asymmetric components in the model is more important than the choice of error distribution. At the same time, it contradicts the research by Alberg et al. (2008), Wenström (2014), and Wang et al. (2020), who suggest that using a heavy-tailed error distribution significantly improves model performance and that asymmetric GARCH models under the Student's t-distribution better capture volatility patterns.

Overall, the evidence obtained from this section indicates that the GJR-GARCH model is an efficient model for capturing the volatility of the Tehran Stock Exchange during crisis periods, while the standard GARCH model proves to be effective during stable periods. This key finding of the study supports and complements the results of previous research in this area.

DISCUSSION AND CONCLUSION

In this study, the performance of three selected GARCH models including the standard (symmetric) GARCH and its two asymmetric variants, namely the GJR-GARCH and the Exponential GARCH was evaluated in estimating and forecasting the monthly volatility of the Tehran Stock Exchange Price and Return Index over a 12-year period from 2013 (1392) to the end of 2024 (1403).

- **Hypothesis 1 Result:** According to the obtained results, the conditional volatility of the Tehran Price and Return Index is persistent, with a half-life of approximately 7.125 months, and also exhibits mean-reverting behavior. This study did not find sufficient evidence to support Hypothesis 1, which stated that asymmetric GARCH models consistently and systematically outperform their symmetric counterparts in modeling and forecasting volatility in Iran's capital market under all conditions. **(Hypothesis 1 rejected)**

- **Hypothesis 2 Result:** This hypothesis posits that market conditions (stable vs. crisis) have a significant effect on the performance of GARCH models in modeling volatility in Iran's stock market. This hypothesis was supported. More specifically, there was insufficient evidence of a consistent leverage effect or asymmetry in the volatility of the Tehran Stock Exchange. In other words, the commonly observed **phenomenon** in global markets where negative events impact volatility more than positive ones was not confirmed under stable conditions in Tehran's market. However, when the models were applied to the crisis sub-period, the results were distinctly different and significant. This indicates that market crises significantly influence the performance of GARCH models in modeling Tehran's stock market volatility, compared to stable periods. **(Hypothesis 2 confirmed for crisis conditions).**

- **Hypothesis 3 Result:** The empirical results of this study showed that the GJR-GARCH model, an asymmetric variant of the GARCH model, is the best-performing model in terms of in-sample performance during crisis conditions. This further confirms the findings of other studies in this field: during crisis periods, more complex models with additional parameters that account for asymmetry are better at capturing **volatility** behavior. However, during stable periods, there is no need to complicate models by including asymmetry variables. Additionally, under the Mincer-Zarnowitz regression framework and using various criteria, the evaluation of GARCH-family models' forecasting performance for Tehran's market showed that all models performed well in out-of-sample forecasting during the study period. **(Hypothesis 3 confirmed)**

- **Hypothesis 4 Result:** The statistical results indicated that introducing the exogenous VIX variable into the models improved the log-likelihood values for all three models. However, there was no significant reduction in forecast error for the standard GARCH and Exponential GARCH models.The most optimized model in terms of increased likelihood after introducing the exogenous implied volatility index variable was the GJR-GARCH model, where all model indicators improved significantly. This dissertation also found no strong evidence that using heavy-tailed distributions improves model performance. Furthermore, the simultaneous use of an exogenous variable and a heavy-tailed distribution did not significantly enhance model performance. Although some metrics improved after this simultaneous modification, the improvement was not systematic or statistically significant. Therefore, regarding Hypothesis 4, it can be concluded that modifying models (by using different probability distributions or adding variables to standard models) leads to meaningful improvements only in the performance of the asymmetric GJR-GARCH model. **(Hypothesis 4 rejected for symmetric and exponential asymmetric GARCH models; confirmed only for the asymmetric GJR-GARCH model)**

RECOMMENDATIONS BASED ON RESULTS

Based on the findings of this study, the following suggestions are offered to improve the process of modeling and forecasting volatility:

• According to the results obtained from testing the first and second hypotheses of this research, along with theoretical evidence and existing empirical findings, the use of nonlinear and asymmetric GARCH models is recommended during crisis periods in the Tehran Stock Exchange. Incorporating asymmetry variables into the models significantly improves both in-sample and out-of-sample performance of volatility models during crises. As many studies confirm, complex nonlinear models are well capable of capturing various dimensions of volatility. Since positive and negative shocks in Tehran's stock market have different impacts on volatility, it is recommended to use nonlinear asymmetric models to estimate and forecast the volatility of the total return and price indices of the Tehran Stock Exchange in times of crisis.

• Based on the results from testing the third hypothesis, among the evaluated asymmetric models in this research, the use of the GJR-GARCH model is advised during crises due to its more efficient performance in estimating and forecasting volatility in Tehran's stock market. A good volatility model should be able to understand and replicate the revealed facts about conditional volatility. This means that unique features such as positive skewness, clustering, asymmetry, and mean reversion tendency should be embedded in the model (Hossein et al., 2019). The GJR-GARCH model, which enables the use of asymmetry variables, is more successful in capturing this reality for Tehran's stock market during crisis periods. The findings also showed that the simple standard GARCH model can adequately handle the estimation and forecasting of capital market volatility in stable periods, with acceptable in-sample and out-of-sample performance. Therefore, it is more economical to use simpler models with fewer parameters during periods of market stability.

• Including an exogenous independent variable in the GARCH variance equation can significantly improve the performance of the GJR-GARCH model during crisis periods. This is supported by previous studies, and the rationale behind this recommendation is that if an exogenous variable can introduce additional information into the GARCH model, it enhances the model's performance. Thus, adding an implied volatility index as an independent variable to the GJR-GARCH model is recommended to improve its performance in modeling market volatility during crisis periods in Iran's capital market.

• Although it has been proven that the unconditional distribution of capital return volatility usually exhibits heavy tails, the findings of this study suggest that using a Student's t-distribution with heavier tails compared to a model using a normal distribution does not produce a meaningful advantage.

RECOMMENDATIONS FOR FUTURE STUDIES

- Using multi-step forecasting instead of one-step-ahead forecasting.

- Employing volatility models that allow for regime shifts and structural breaks, such as Markov-Switching GARCH models.

- Analyzing the volatility of other financial markets in Tehran alongside the price and return index of the stock exchange. This can significantly aid in interpreting volatility spillover effects from one market to another or between different sectors.

Authors' Contributions

Afshin Aminipour contributed to the development of the theoretical framework and literature review. Akbar Bagheri was responsible for statistical analyses. The third author (Mohammad Hossein Fatehi Dabanloo) assisted in data collection and organizing the research findings. Also, Zahra Houshmand Neghabi and Reza Khoshsim contributed to editing, refining the manuscript, and aligning it with publication requirements. All authors reviewed and approved the final version of the manuscript.

Availability of Data and Materials
The datasets generated or analyzed during the current study can be obtained from the corresponding author on a reasonable request.

Conflict of interest
The authors state that there is no conflict of interest.

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